

10.1.1 e.m.f., internal resistance and the V-I graph

Name: _____ Class: _____ Date: _____

Total: 30 marks

KEY WORDS resistance 电阻 • internal resistance 内阻 • potential difference 电势差
• terminal p.d. 端电压 • circuit symbols 电路符号

Objective

Sheet 10.1 is the thinnest sheet in this subject – four one-mark definitions – against a unit that the exam asks eleven-mark questions on. This sheet is the rest of 10.1: reading and **drawing circuit diagrams** 电路图 with the standard symbols, the **energy** definitions of **e.m.f.** 电动势 and **potential difference** 电势差 and the difference between them, what the **internal resistance** 内阻 of a source does to the **terminal p.d.** 端电压, and the V - I experiment that measures both at once. This sheet covers syllabus 10.1.

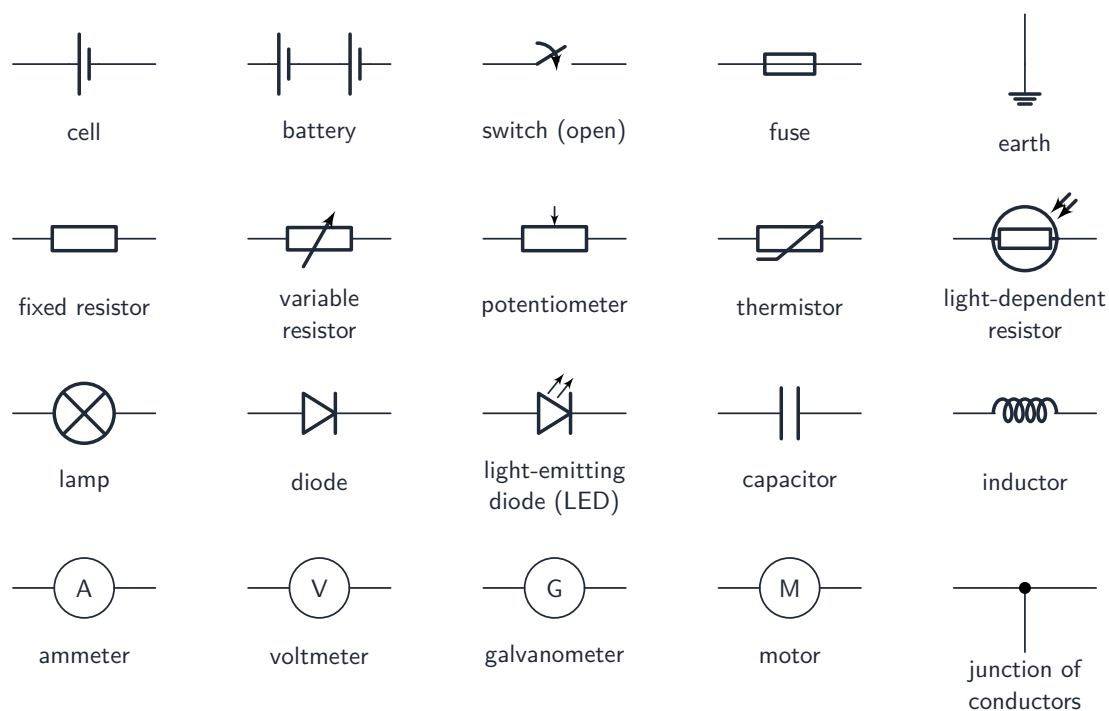
You must be able to:

- recall, draw and interpret the standard **circuit symbols** 电路符号 and circuit diagrams
- define the e.m.f. of a source as the **energy transferred per unit charge** in driving charge round a complete circuit
- distinguish between e.m.f. and potential difference in terms of energy
- understand the effect of the internal resistance of a source on the terminal potential difference

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

- The symbols, and how a diagram is marked



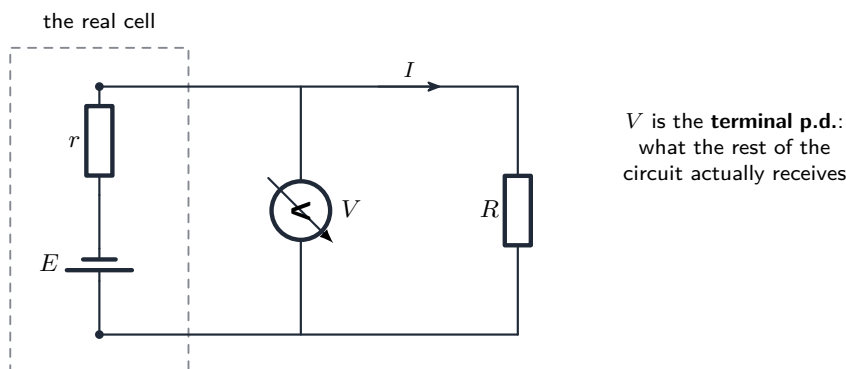
- Draw with a **ruler**, with straight wires and square corners; a wire that simply ends is a lost mark.
- An **ammeter** goes **in series** with the component whose current you want; a **voltmeter** goes **in parallel** across the component whose p.d. you want. Swapping them is the commonest diagram error.
- Junctions that are connected get a **dot**; wires that cross without connecting do not.

■ Two definitions, and they differ only in direction

	definition	where the energy goes
electromotive force (e.m.f.) E	the energy transferred to each unit of charge, from some other form into electrical, in driving it around a complete circuit	chemical $\rightarrow$ electrical, inside the source
p.d. V	the energy transferred from each unit of charge into another form, in a component	electrical $\rightarrow$ thermal, light, kinetic, in the component

- Both are **joules per coulomb**, so both are measured in **volts**. That is why the distinction has to be made in words about **energy**, and why “e.m.f. is the voltage of the battery” earns nothing.
- E.m.f. is **not a force**, despite its name.

■ A real cell: an ideal source with a resistance in series



Each coulomb is given E joules by the source, gives up Ir joules inside the cell and V joules outside it, so $E = V + Ir$, or $V = E - Ir$. Nothing is lost – the missing energy is dissipated *inside* the cell, which is why a battery gets warm.

Follow one coulomb round the circuit:

$$E = V + Ir \quad \Longleftrightarrow \quad V = E - Ir \quad \Longleftrightarrow \quad E = I(R + r)$$

- It is given E joules by the source.
- It gives up Ir joules **inside** the cell, warming it – these are sometimes called the “lost volts”.
- It gives up V joules **outside**, in the rest of the circuit. V is the **terminal p.d.**: what the circuit actually receives.

Two limiting cases the exam likes:

situation	what happens
no current drawn ($I = 0$)	$V = E$: the terminal p.d. equals the e.m.f., because nothing is lost inside
terminals shorted ($R = 0$)	$I = \frac{E}{r}$, the largest current the cell can deliver, and all of the energy is wasted inside it

Worked example. A cell of e.m.f. 1.5 V and internal resistance $0.50\ \Omega$ is connected to a $2.5\ \Omega$ resistor. Find the current and the terminal p.d.

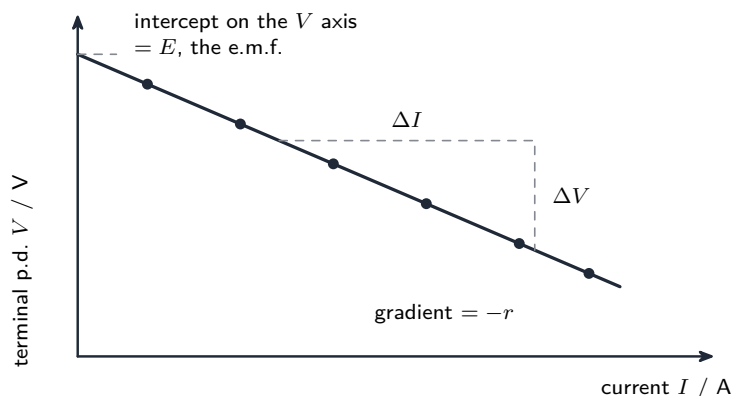
- the e.m.f. drives the current through **both** resistances

$$I = \frac{E}{R + r} = \frac{1.5\text{ V}}{2.5\ \Omega + 0.50\ \Omega} = 0.50\text{ A}$$

- then the terminal p.d. is what is left after the internal drop

$$V = E - Ir = 1.5\text{ V} - 0.50\text{ A} \times 0.50\ \Omega = \mathbf{1.25\text{ V}}$$

■ The experiment: one graph, two answers



Vary the external resistance, plot V against I , and $V = E - Ir$ gives a straight line: the **intercept** is the e.m.f. and the **gradient** is minus the internal resistance. Extrapolating back to $I = 0$ is how you measure an e.m.f. you can never read directly, because any real voltmeter draws some current.

Vary the external resistance, record V and I each time, and plot V against I . Since

$$V = E - rI$$

is $y = c + mx$:

- the **intercept** on the V axis is the **e.m.f.** E ;
- the **gradient** is $-r$, so the internal resistance is **minus** the gradient, and it is positive.
- This is how an e.m.f. is measured, because a real voltmeter always draws a **small current**, so the reading straight across the cell is always slightly **less** than E . The graph extrapolates to the $I = 0$ case that cannot quite be reached.
- **Power:** the source supplies EI in total, of which I^2r is wasted inside and VI is delivered outside.

2 Practice

Now use the methods above.

2.1 Define the **e.m.f.** of a source.

[2]

2.2 Explain, in terms of energy, the difference between the e.m.f. of a cell and the potential difference across a resistor. [2]

2.3 A cell of e.m.f. 1.5 V and internal resistance $0.40\ \Omega$ delivers a current of 0.60 A. Calculate the terminal p.d. [2]

2.4 Explain why the terminal p.d. of a cell falls as the current drawn from it increases. [2]

2.5 State the terminal p.d. of a cell when **no** current is drawn from it, and explain your answer. [2]

2.6 Calculate the largest current the cell in **2.3** could deliver, and state the condition under which this happens. [2]

3 Exam-style questions

3.1 As the current drawn from a cell increases, the terminal potential difference [1]

A	B
C	D

A increases, because more energy is transferred each second.

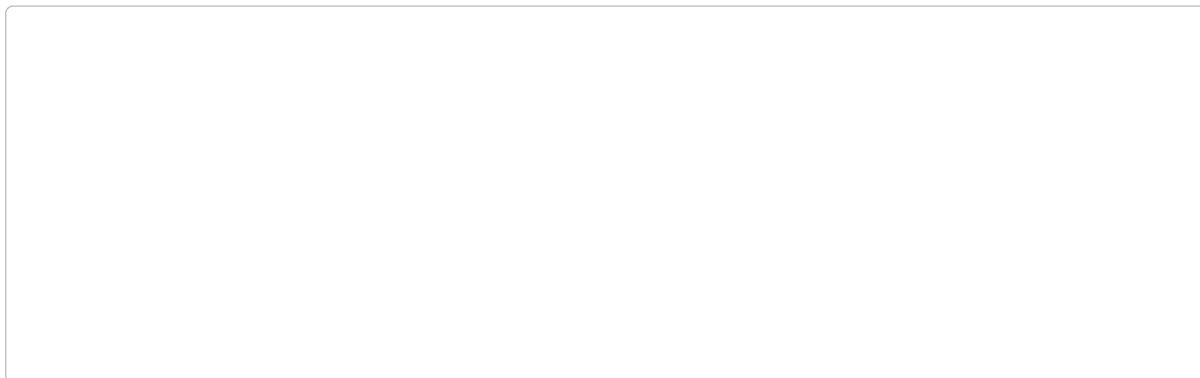
B stays the same, because the e.m.f. of the cell is fixed.

C decreases, because more energy is dissipated in the internal resistance.

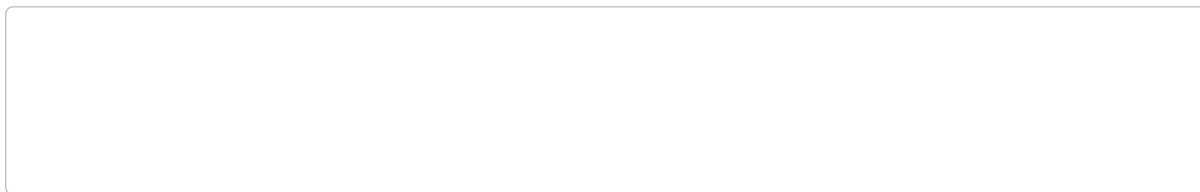
D decreases, because the e.m.f. of the cell falls.

3.2 A battery of e.m.f. 12 V and internal resistance $0.50\ \Omega$ is connected to a $2.5\ \Omega$ resistor in series with a parallel combination of a $6.0\ \Omega$ and a $3.0\ \Omega$ resistor.

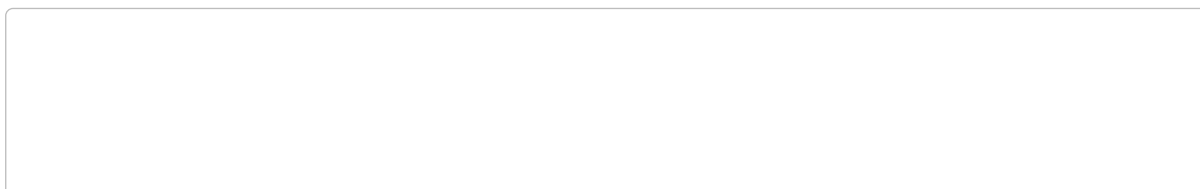
(a) Draw a circuit diagram of this arrangement, including an ammeter to measure the current from the battery and a voltmeter to measure the terminal p.d. [2]



(b) Calculate the total resistance of the external circuit. [2]



(c) Calculate the current delivered by the battery. [2]



3.3 A student varied the resistance of the external circuit and recorded the terminal p.d. V and the current I .

I / A	0.20	0.45	0.70	0.95	1.20
V / V	5.72	5.45	5.18	4.91	4.64

(a) State the equation relating V and I for this circuit, and state what the gradient and the intercept of a graph of V against I represent. [3]

(b) Use the first and last pairs of readings to calculate the internal resistance of the source. [3]

(c) Calculate the e.m.f. of the source. [2]

(d) Calculate the power dissipated inside the source when the current is 1.20 A. [2]

(e) Explain why connecting a voltmeter directly across the terminals of the source does not give exactly its e.m.f. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the **energy transferred to each unit of charge** from another form into electrical form
- in driving that charge **around a complete circuit**

2.2

- the **e.m.f.** is the energy given **to** each coulomb by the source, as chemical energy becomes electrical
- the **p.d.** is the energy given **up** by each coulomb in the component, as electrical energy becomes thermal or another form

2.3

- use $V = E - Ir$

$$V = 1.5 \text{ V} - 0.60 \text{ A} \times 0.40 \text{ } \Omega = \mathbf{1.26 \text{ V}}$$

2.4

- a larger current means a larger p.d. Ir across the **internal resistance**
- the e.m.f. is fixed, so with more of it used inside the cell, less is left for the external circuit and $V = E - Ir$ falls

2.5

- the terminal p.d. **equals the e.m.f.**
- with $I = 0$ there is no p.d. across the internal resistance, so nothing is lost inside the cell

2.6

- the largest current flows when the external resistance is **zero** – the terminals are short-circuited

$$I_{\max} = \frac{E}{r} = \frac{1.5 \text{ V}}{0.40 \text{ } \Omega} = \mathbf{3.75 \text{ A}}$$

3.1 C

- the internal resistance is unchanged, but a larger current gives a larger Ir inside the cell, so less of the fixed e.m.f. is left as terminal p.d.
- **D** is the standard error: the e.m.f. does **not** change with current

3.2

(a)

- the battery, the $2.5 \text{ } \Omega$ resistor and the parallel pair correctly in **series**, with the $6.0 \text{ } \Omega$ and $3.0 \text{ } \Omega$ in **parallel** with each other
- the **ammeter in series** with the battery and the **voltmeter in parallel** across its terminals, with correct symbols

(b)

- combine the parallel pair first

$$\frac{1}{R_p} = \frac{1}{6.0 \, \Omega} + \frac{1}{3.0 \, \Omega} = \frac{1}{2.0 \, \Omega}, \quad R_p = 2.0 \, \Omega$$

$$R_{\text{ext}} = 2.5 \, \Omega + 2.0 \, \Omega = \mathbf{4.5}$$

(c)

- the e.m.f. drives the current through the external resistance **and** the internal resistance

$$I = \frac{E}{R_{\text{ext}} + r} = \frac{12 \, \text{V}}{4.5 \, \Omega + 0.50 \, \Omega} = \mathbf{2.4 \, A}$$

3.3

(a)

- $V = E - Ir$
- the **intercept** on the V axis is the **e.m.f.** E
- the **gradient** is $-r$, so the internal resistance is minus the gradient

(b)

- the internal resistance is minus the gradient of V against I

$$\text{gradient} = \frac{4.64 \, \text{V} - 5.72 \, \text{V}}{1.20 \, \text{A} - 0.20 \, \text{A}} = -1.08 \, \Omega$$

$$r = \mathbf{1.08}$$

(c)

- substitute one pair of readings into $E = V + Ir$

$$E = 5.72 \, \text{V} + 0.20 \, \text{A} \times 1.08 \, \Omega = \mathbf{5.94 \, V}$$

(d)

- the power dissipated inside the source is $I^2 r$

$$P = (1.20 \, \text{A})^2 \times 1.08 \, \Omega = \mathbf{1.6 \, W}$$

(e)

- a real voltmeter draws a small current, so there is a small p.d. Ir inside the cell and the reading is slightly **less** than the e.m.f.