A-LEVEL PHYSICS • EXERCISE SHEET

1.4 Scalars and vectors

Name: ____________ Class: ________ Date: __________ **Total: 34 marks**

KEY WORDS vector 矢量 • scalar 标量 • component 分量 • perpendicular 垂直 • magnitude 大小

Objective

Build the skills to answer the real exam questions on **scalars and vectors**. In the exam, 1.4 is tested by **multiple-choice** questions and by short parts inside longer questions: classify a quantity, find a **resultant**, or **resolve** a vector into components.

You must be able to:

- tell a **scalar 标量** from a **vector 矢量** and complete a classification table
- find the **resultant 合矢量** of two **perpendicular 垂直** vectors (size and direction)
- give the direction of $\vec{X}+\vec{Y}$ and $\vec{X}-\vec{Y}$
- find the resultant of **two equal forces** at an angle
- **resolve 分解** any vector (a force *or* a velocity) into two perpendicular **components 分量**
- add three or more **coplanar 共面** vectors by components
- **sketch** each component of a projectile's velocity against time

1 Worked examples

Study these first. Each one shows the method for an exam question type used below — read them and you can do the Practice and Exam-style questions yourself.

A — Scalar or vector?

A scalar has size only; a vector has size **and** direction. **Quick test:** if "in which direction?" makes sense, it is a vector. In the exam you often tick a table:

Quantity	Scalar	Vector
speed	✓	
acceleration		✓
energy	✓	
momentum		✓

(Common scalars: mass, time, temperature, energy, distance, speed. Common vectors: displacement, velocity, acceleration, force, momentum.)

B — Add two perpendicular vectors

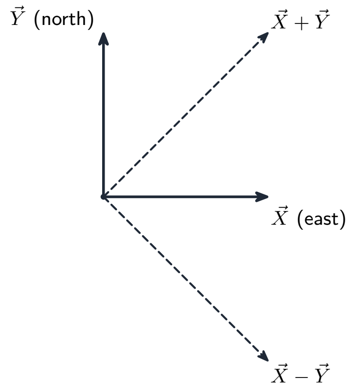
Draw them **tip to tail**; the resultant closes the triangle. For two vectors at 90° the size is $|\vec{R}|=\sqrt{X^2+Y^2}$ and the direction is $\tan\theta = Y/X$.

Worked example. A swimmer heads north at 1.2 m s^{-1}; the river flows east at 0.5 m s^{-1}:

$$|\vec{R}|=\sqrt{1.2^2+0.5^2}=1.3\text{ m s}^{-1},\qquad \tan\theta=\tfrac{0.5}{1.2}\Rightarrow\theta\approx 23^\circ\text{ east of north.}$$

C — Direction of $\vec{X}+\vec{Y}$ and $\vec{X}-\vec{Y}$

To **add**, place the arrows tip to tail. To **subtract**, reverse the second arrow first: $\vec{X}-\vec{Y}=\vec{X}+(-\vec{Y})$, where $-\vec{Y}$ is the same size as $\vec{Y}$ but points the opposite way.



With $\vec{X}$ east and $\vec{Y}$ north: $\vec{X}+\vec{Y}$ points **north-east**, and $\vec{X}-\vec{Y}$ points **south-east**.

C2 — Adding three or more coplanar vectors

Coplanar 共面 vectors all lie in the same flat plane — every vector in this course is coplanar. For three or more, drawing triangles gets messy, so **use components**: add all the x-parts, add all the y-parts, then recombine.

Worked example. Three coplanar forces act at a point: 2.0 N east, 3.0 N north, 4.0 N west.

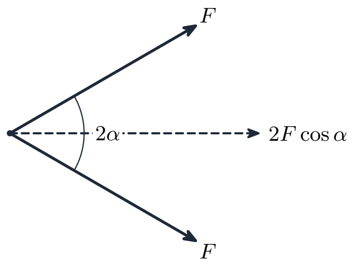
$$\sum x = 2.0-4.0=-2.0\text{ N},\qquad \sum y=+3.0\text{ N}$$

$$R=\sqrt{2.0^2+3.0^2}=3.6\text{ N},\qquad \tan\theta=\tfrac{3.0}{2.0}\Rightarrow\theta=56^\circ\text{ north of west}$$

Take one direction as positive and stay with it — a lost minus sign is the usual error.

■ D — Two equal forces at an angle

Two equal forces of size F with an angle 2α between them have a resultant of size $2F\cos\alpha$, along the line that **cuts the angle in half**.

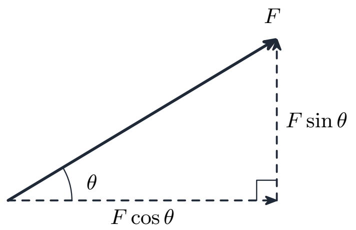


Worked example. Two 6.0 N forces act with 60° between them. Here $2\alpha = 60°$, so $\alpha = 30°$:

$$R = 2F\cos\alpha = 2(6.0)\cos 30° = 10.4 \text{ N}.$$

■ E — Resolve a vector into components

Any vector $\vec{v}$ at angle θ to the horizontal splits into a horizontal part $v\cos\theta$ and a vertical part $v\sin\theta$ — this works for a force **or** a velocity 速度.



Force example. A 50 N force at 30°: horizontal $= 50\cos 30° = 43$ N, vertical $= 50\sin 30° = 25$ N.

Velocity example. A ball kicked at 28 m s^{-1}, 34° above the horizontal: $v_H = 28\cos 34° = 23$ m s^{-1}, $v_V = 28\sin 34° = 16$ m s^{-1}.

On a slope of angle θ, the weight splits into $W\sin\theta$ **along** the slope and $W\cos\theta$ **into** the slope.

■ F — The two components of a projectile, against time

Once the components are found, each one has its own simple graph — and the exam asks you to sketch both on one grid:

- **horizontal** v_H: a **horizontal straight line** (no horizontal force, so it never changes).
- **vertical** v_V: a **straight line sloping down**, starting at $+v\sin\theta$, crossing zero at the top of the flight, and reaching $-v\sin\theta$ on landing (taking up as positive).

The gradient of that second line is $-g$, and the time to the top is v_V/g. Label each line, and use the whole time axis you are given.

2 Practice

Now apply the methods above.

2.1 Complete the table by ticking **scalar** or **vector** for each quantity. [3]

Quantity	Scalar	Vector
distance		
force		
temperature		
velocity		
power		
acceleration		

2.2 Forces of 3.0 N east and 4.0 N north act at a point. Find the **magnitude** 大小 of the resultant. [2]

2.3 Resolve a 50 N force acting at 30° to the horizontal into its horizontal and vertical components. [2]

2.4 A ball leaves the ground at 20 m s^{-1} at 60° above the horizontal. Find the **horizontal** and **vertical** components of its velocity. [2]

2.5 Two forces, each 5.0 N, act at a point with 90° between them. Find the **magnitude** of the resultant. [2]

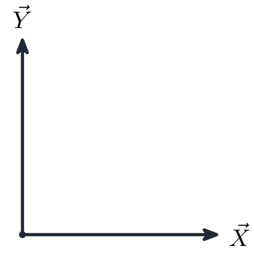
2.6 Three **coplanar** forces act at a point: 6.0 N east, 2.0 N west and 5.0 N north. Find the **magnitude** and the **direction** of the resultant. [3]

3 Exam-style questions

3.1 Which of these is a **vector** quantity? [1]

- **A** energy
- **B** power
- **C** displacement
- **D** temperature

3.2 Two vectors $\vec{X}$ and $\vec{Y}$ are shown: $\vec{X}$ points east and $\vec{Y}$ points north. What are the directions of $\vec{X}+\vec{Y}$ and $\vec{X}-\vec{Y}$? [1]



- **A** $\vec{X}+\vec{Y}$ north-east; $\vec{X}-\vec{Y}$ south-east
- **B** $\vec{X}+\vec{Y}$ north-east; $\vec{X}-\vec{Y}$ north-west
- **C** $\vec{X}+\vec{Y}$ south-west; $\vec{X}-\vec{Y}$ south-east
- **D** $\vec{X}+\vec{Y}$ south-east; $\vec{X}-\vec{Y}$ north-east

3.3 Two forces of 6.0 N and 8.0 N act at a point at right angles to each other. What is the magnitude of the resultant force? [1]

- **A** 2.0 N
- **B** 10 N
- **C** 14 N
- **D** 48 N

3.4 A stone is thrown with a velocity of 15 m s^{-1} at 40° above the horizontal.

(a) Find the **horizontal** and **vertical** components of this velocity. [2]

(b) Air resistance is negligible. State which component stays **constant** during the flight, and why. [2]

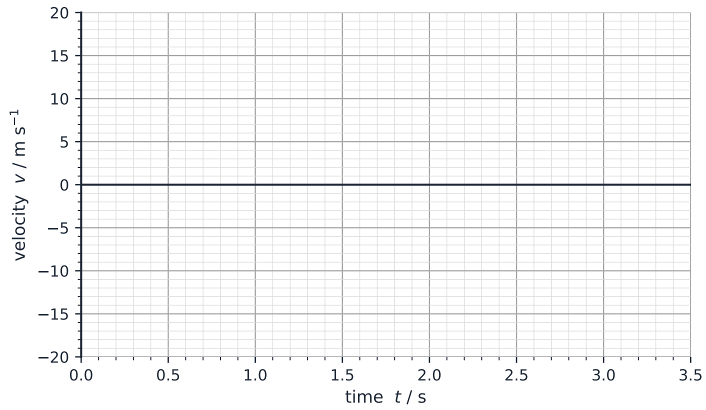
3.5 Two forces, each 7.0 N, act at a point with 80° between them. Find the **magnitude** of the resultant. [2]

3.6 A player kicks a ball from level ground with a velocity of 21 m s^{-1} at 55° to the horizontal. Air resistance is negligible.

(a) **Calculate** the horizontal component v_H and the vertical component v_V of the velocity of the ball as it leaves the ground. [2]

(b) **Show that** the ball reaches its maximum height at time $t = 1.75$ s. [1]

(c) The ball lands at time $t = 3.5$ s. On the grid below, **sketch** the variation with time t of v_H and of v_V, from $t = 0$ to $t = 3.5$ s. Take the upward direction as positive. **Label** your lines H and V. [4]



(d) **Determine** the magnitude and the direction of the velocity of the ball just before it lands. [3]

(e) State why the horizontal component of the velocity does not change during the flight. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- scalar: size only; vector: size **and** direction
- distance **scalar**; force **vector**; temperature **scalar**; velocity **vector**; power **scalar**; acceleration **vector**

2.2

- the forces are perpendicular, so use Pythagoras

$$R = \sqrt{(3.0\text{ N})^2 + (4.0\text{ N})^2} = \sqrt{25\text{ N}^2} = \mathbf{5.0}\text{ N}$$

2.3

- horizontal is the adjacent side: $F\cos\theta$; vertical is the opposite side: $F\sin\theta$
- horizontal: $50\text{ N} \times \cos 30° = \mathbf{43}\text{ N}$
- vertical: $50\text{ N} \times \sin 30° = \mathbf{25}\text{ N}$

2.4

- $v_\text{H} = v\cos\theta$, $v_\text{V} = v\sin\theta$
- $v_\text{H} = 20\text{ m s}^{-1} \times \cos 60° = \mathbf{10}\text{ m s}^{\mathbf{-1}}$
- $v_\text{V} = 20\text{ m s}^{-1} \times \sin 60° = \mathbf{17}\text{ m s}^{\mathbf{-1}}$

2.5

- equal forces at 90°: Pythagoras, or $R = 2F\cos 45°$

$$R = \sqrt{(5.0\text{ N})^2 + (5.0\text{ N})^2} = \mathbf{7.1}\text{ N}$$

2.6

- take east and north as positive
- $\sum x = 6.0\text{ N} - 2.0\text{ N} = 4.0\text{ N}$ east; $\sum y = 5.0\text{ N}$ north

$$R = \sqrt{(4.0\text{ N})^2 + (5.0\text{ N})^2} = \mathbf{6.4}\text{ N}$$

$$\tan\theta = \frac{5.0\text{ N}}{4.0\text{ N}} \Rightarrow \theta = \mathbf{51}° \text{ north of east}$$

3.1 C

- a vector needs a **direction** as well as a size
- **displacement** is "how far, and which way"
- **A**, **B**, **D** (energy, power, temperature) are scalars — *distance* is the scalar trap against displacement

3.2 A

- $\vec{X}$ east and $\vec{Y}$ north: tip-to-tail sum points **north-east**
- $\vec{X} - \vec{Y} = \vec{X} + (-\vec{Y})$: reverse $\vec{Y}$ to south, so the result points **south-east**
- **B** keeps $\vec{Y}$ pointing north when subtracting

3.3 B

- perpendicular forces: Pythagoras

$$R = \sqrt{(6.0\text{ N})^2 + (8.0\text{ N})^2} = \sqrt{100\text{ N}^2} = \mathbf{10}\text{ N}$$

- **A** is the difference; **C** is the sum; **D** is the product

3.4

(a) resolve at 40° to the horizontal

- $v_\text{H} = 15\text{ m s}^{-1} \times \cos 40° = \mathbf{11}\text{ m s}^{\mathbf{-1}}$
- $v_\text{V} = 15\text{ m s}^{-1} \times \sin 40° = \mathbf{9.6}\text{ m s}^{\mathbf{-1}}$

(b) the **horizontal** component stays constant

- no air resistance $\Rightarrow$ no horizontal force $\Rightarrow$ no horizontal acceleration

3.5

- two equal forces: $2\alpha = 80°$ so $\alpha = 40°$

$$R = 2F\cos\alpha = 2 \times 7.0\text{ N} \times \cos 40° = \mathbf{10.7}\text{ N}$$

3.6

(a)

- $v_\text{H} = 21\text{ m s}^{-1} \times \cos 55° = \mathbf{12}\text{ m s}^{\mathbf{-1}}$
- $v_\text{V} = 21\text{ m s}^{-1} \times \sin 55° = \mathbf{17}\text{ m s}^{\mathbf{-1}}$

(b) at the top, $v_\text{V} = 0$, so $t = v_\text{V}/g$

$$t = \frac{17.2\text{ m s}^{-1}}{9.81\text{ m s}^{-2}} = \mathbf{1.75}\text{ s}$$

(c)

- **H**: horizontal straight line at $v = 12\text{ m s}^{-1}$ across the whole grid, labelled H
- **V**: straight line of negative gradient, labelled V
- starts at $+17\text{ m s}^{-1}$ and ends at -17 m s^{-1}
- crosses $v = 0$ at $t = 1.75$ s

(d) just before landing, $v_\text{H} = 12\text{ m s}^{-1}$ and $v_\text{V} = -17\text{ m s}^{-1}$

$$v = \sqrt{(12\text{ m s}^{-1})^2 + (17\text{ m s}^{-1})^2} = \mathbf{21}\text{ m s}^{\mathbf{-1}}$$

$$\tan\theta = \frac{17\text{ m s}^{-1}}{12\text{ m s}^{-1}} \Rightarrow \theta = \mathbf{55}° \text{ below the horizontal}$$

(e) air resistance is negligible, so there is **no horizontal force** and therefore no horizontal acceleration