A-LEVEL PHYSICS • EXERCISE SHEET

1.3 Errors and uncertainties

Name: ____________ Class: ________ Date: __________ **Total: 36 marks**

KEY WORDS uncertainty 不确定度 • percentage uncertainty 百分比不确定度 • absolute uncertainty 绝对不确定度 • accuracy 准确度 • systematic error 系统误差

Objective

Build the skills to answer exam questions on **errors and uncertainties** — error types, precision vs accuracy, and combining uncertainties.

You must be able to:

- explain **systematic** (incl. **zero**) and **random** errors
- distinguish **precision** 精密度 from **accuracy** 准确度
- combine **absolute** and **percentage** uncertainties in a derived quantity
- **justify** the number of significant figures in a calculated result
- decide, from the uncertainties, whether two results **agree**

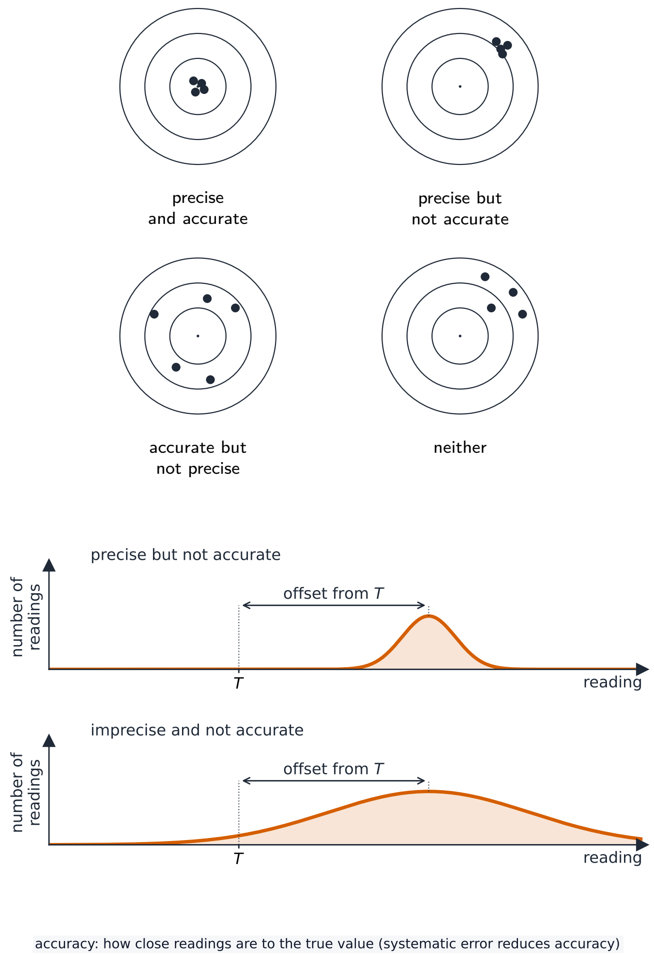
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Errors, precision and accuracy

- A **systematic error** shifts every reading the **same way** (a **zero error**, a wrong calibration). Repeating does not reveal it; it harms **accuracy**.
- A **random error** scatters readings **both ways**. Taking the **mean** of many repeats reduces it; it harms **precision**.

Precision = how close repeats are to each other; **accuracy** = how close they are to the true value:



Precision contrasted with accuracy

■ Uncertainty in one quantity

Write a measurement as $x \pm \Delta x$. The **percentage uncertainty** is $\dfrac{\Delta x}{|x|} \times 100\%$. E.g. (2.50 ± 0.05) m $\rightarrow \dfrac{0.05}{2.50} \times 100 = 2\%$.

■ **Combining uncertainties**

- **Add / subtract** → add the **absolute** uncertainties: $y = a \pm b \Rightarrow \Delta y = \Delta a + \Delta b$.
- **Multiply / divide** → add the **percentage** uncertainties.
- **Power** $y = a^n$ → **multiply** the percentage by $|n|$.

Worked example. $R = V/I$ with $V = (6.0 \pm 0.1)$ V, $I = (2.0 \pm 0.1)$ A: $R = 3.0\ \Omega$; $\%V = 1.7\%$, $\%I = 5.0\%$, so $\%R = 6.7\%$, i.e. $R = (3.0 \pm 0.2)\ \Omega$.

■ **Significant figures in a calculated answer**

Give a calculated result to the **same number of significant figures as the least precise measurement** it came from — no more. $\rho = \dfrac{m}{V}$ from a mass of 31.3 g (3 s.f.) and a side of 1.53 cm (3 s.f.) is quoted to **3 s.f.**; writing 8.7194×10^3 claims a precision the measurements do not have. When a question says **justify**, say which measurement set the limit.

■ **Do two results agree?**

An answer with an uncertainty is a **range**, not a point. Two results **agree** when their ranges **overlap**:

$$\rho_{\mathrm{A}} = 8.7 \times 10^3 \text{ kg m}^{-3} \pm 4\% \Rightarrow 8.35 \times 10^3 \text{ to } 9.05 \times 10^3$$

$$\rho_{\mathrm{B}} = 9.2 \times 10^3 \text{ kg m}^{-3} \pm 6\% \Rightarrow 8.65 \times 10^3 \text{ to } 9.75 \times 10^3$$

The two ranges share the band 8.65–9.05 × 10^3, so the samples **could** be the same material. Write both ranges down — that comparison is where the marks are, not the verdict on its own.

2 Practice

Now apply the methods above.

2.1 State the difference between a **systematic** error and a **random** error. [2]

2.2 A balance reads 0.3 g with **nothing** on it. Name this error and state how to correct a reading. [2]

2.3 A length is (2.50 ± 0.05) m. Find its **percentage uncertainty**. [1]

2.4 State the difference between **precision** and **accuracy**. [2]

2.5 A student measures a length with a metre rule as 84.2 cm. **Estimate** the absolute uncertainty in this reading, and explain your choice. [2]

2.6 A speed is calculated as 3.472 m s^{-1} from a distance of 12.5 m and a time of 3.6 s. State how many significant figures the answer should be given to, and **justify** your answer. [2]

3 Exam-style questions

3.1 Repeated readings are **close together** but **far from** the true value. They are: [1]

- **A** accurate but not precise
- **B** precise but not accurate
- **C** both accurate and precise
- **D** neither

3.2 Two lengths are $a = (40 \pm 1)$ mm and $b = (25 \pm 1)$ mm. **Determine** $a - b$ with its **absolute uncertainty**. [2]

3.3 A resistance is found from $R = V/I$, with $V = (6.0 \pm 0.1)$ V and $I = (2.0 \pm 0.1)$ A. Find R and its **absolute uncertainty**. [4]

3.4 A cube has side $L = (2.00 \pm 0.02)$ cm. Its volume is $V = L^3$. Find the **percentage uncertainty** in V. [2]

3.5 Two samples of metal, A and B, are tested to find the density of the material each is made from.

(a) **Explain** what is meant by the **accuracy** of a measured value. [1]

(b) Sample A is a sphere. Its diameter is $d = (2.40 \pm 0.02)$ cm and its mass is $m = (68.5 \pm 0.5)$ g. The volume of a sphere of diameter d is $V = \dfrac{\pi d^3}{6}$.

Show that the density of the material of sample A is about 9.5×10^3 kg m^{-3}. [3]

(c) **Justify** the number of significant figures you have given for the density in **(b)**. [1]

(d) **Calculate** the percentage uncertainty in the density of the material of sample A. [3]

(e) **Determine** the absolute uncertainty in the density of sample A, and write the density as $\rho \pm \Delta\rho$. [2]

(f) The density of the material of sample B is measured as 9.0×10^3 kg m$^{-3} \pm 3\%$. **State and explain** whether A and B could be made from the same material. [2]

3.6 A student times **20** complete swings of a pendulum with a stopwatch and records 31.4 s. Her reaction time makes each timing uncertain by about 0.2 s.

(a) **Estimate** the percentage uncertainty in her value for the **period** of one swing. [2]

(b) Explain why timing 20 swings gives a smaller percentage uncertainty than timing one. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- a **systematic** error shifts every reading the **same way**; repeating does not remove it
- a **random** error scatters readings **both ways**; the **mean** of repeats reduces it

2.2

- the balance reads a mass with nothing on it, so this is a **zero error** (systematic)
- **subtract** 0.3 g from every reading

2.3

- percentage uncertainty = $(\Delta x/|x|) \times 100\%$

$$\frac{0.05\text{ m}}{2.50\text{ m}} \times 100\% = \mathbf{2}\%$$

2.4

- **precision**: how close repeats are to **each other**
- **accuracy**: how close a reading (or the mean) is to the **true value**

2.5

- a metre rule is marked in millimetres, so one end is ± 0.05 cm
- a length uses **two** ends, so $\Delta L = \pm 0.1$ cm is also accepted if that is explained

2.6

- 3.6 s is given to **2 s.f.**; 12.5 m is 3 s.f.
- a calculated answer cannot be more precise than the least precise input, so quote **2 s.f.**: 3.5 m s^{-1}

3.1 B

- repeats that are **close together** are **precise**
- they sit **far from** the true value, so they are **not accurate**
- that pattern is a **systematic** error (e.g. a zero error)
- **A** swaps the two words

3.2

- subtract the lengths: $a - b = 40\text{ mm} - 25\text{ mm} = 15\text{ mm}$
- for a difference, add the **absolute** uncertainties: $\Delta = 1\text{ mm} + 1\text{ mm} = 2\text{ mm}$
- $(\mathbf{15} \pm \mathbf{2})$ mm

3.3

- $R = V/I$

$$R = \frac{6.0\text{ V}}{2.0\text{ A}} = 3.0\ \Omega$$

- for a quotient, add the **percentage** uncertainties

$$\%V = \frac{0.1\text{ V}}{6.0\text{ V}} \times 100\% = 1.7\%, \qquad \%I = \frac{0.1\text{ A}}{2.0\text{ A}} \times 100\% = 5.0\%$$

- $\%R = 1.7\% + 5.0\% = 6.7\%$

$$\Delta R = 0.067 \times 3.0\ \Omega = 0.2\ \Omega \Rightarrow R = (\mathbf{3.0} \pm \mathbf{0.2})$$

3.4

$$\%L = \frac{0.02\text{ cm}}{2.00\text{ cm}} \times 100\% = 1\%$$

- $V = L^3$, so multiply the percentage by 3: $\%V = 3 \times 1\% = \mathbf{3}\%$

3.5

(a) **accuracy** is how close the measured value is to the **true value**

(b) convert **both** quantities to SI first: $d = 2.40\text{ cm} = 2.40 \times 10^{-2}\text{ m}$, $m = 68.5\text{ g} = 68.5 \times 10^{-3}\text{ kg}$

$$V = \frac{\pi d^3}{6} = \frac{\pi(2.40 \times 10^{-2}\text{ m})^3}{6} = 7.24 \times 10^{-6}\text{ m}^3$$

$$\rho = \frac{m}{V} = \frac{68.5 \times 10^{-3}\text{ kg}}{7.24 \times 10^{-6}\text{ m}^3} = 9.46 \times 10^3\text{ kg m}^{-3} \approx \mathbf{9.5 \times 10^3}\text{ kg m}^{\mathbf{-3}}$$

(c) both d and m are given to **3 s.f.**, so ρ cannot be quoted more precisely than **3 s.f.**

(d)

$$\%m = \frac{0.5\text{ g}}{68.5\text{ g}} \times 100\% = 0.73\%$$

$$\%d = \frac{0.02\text{ cm}}{2.40\text{ cm}} \times 100\% = 0.83\%$$

- $V \propto d^3$, so $\%V = 3 \times 0.83\% = 2.5\%$
- $\%\rho = \%m + \%V = 0.73\% + 2.5\% = \mathbf{3.2}\%$

(e)

$$\Delta\rho = 0.032 \times 9.46 \times 10^3\text{ kg m}^{-3} = 0.30 \times 10^3\text{ kg m}^{-3}$$

- $\rho = (\mathbf{9.5} \pm \mathbf{0.3}) \times \mathbf{10^3}\text{ kg m}^{\mathbf{-3}}$

(f) write both as ranges

- A: $9.46 \times 10^3 \pm 3.2\% \Rightarrow 9.16 \times 10^3$ to 9.76×10^3 kg m^{-3}
- B: $9.0 \times 10^3 \pm 3\% \Rightarrow 8.73 \times 10^3$ to 9.27×10^3 kg m^{-3}
- the ranges **overlap**, so A and B **could** be the same material

3.6

(a) the 0.2 s is the uncertainty in the **total** time, not in one swing

$$\%T = \frac{0.2\text{ s}}{31.4\text{ s}} \times 100\% = \mathbf{0.6}\%$$

(b) the absolute uncertainty stays ≈ 0.2 s (reaction time) however many swings you time

- 20 swings make the **measured time 20 times larger**, so the same Δt is a much smaller **fraction** of it