

1.2 SI units

Name: _____ Class: _____ Date: _____

Total: 27 marks

KEY WORDS current 电流 • temperature 温度 • derived unit 导出单位 • prefix 词头
• si units 国际单位制

Objective

Build the skills to answer exam questions on **SI units** 国际单位制 — base units, derived units, homogeneity and prefixes.

You must be able to:

- recall the **five SI base units** (kg, m, s, A, K) and complete an exam table of them
- express a **derived unit** in SI base units, and find the units of an unknown constant
- check the **homogeneity** of an equation, and use it to find an unknown **power**
- convert a **compound unit** that carries **prefixes**

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Base and derived units

The **base quantities**: mass (kg), length (m), time (s), current (A), temperature (K).
Build a **derived unit** from the equation that defines it:

$$\begin{array}{ccccccc}
 \text{force} & = & \text{mass} & \times & \text{acceleration} & & \\
 \downarrow & & \downarrow & & \downarrow & & \\
 \text{kg m s}^{-2} & = & \text{kg} & \times & \text{m s}^{-2} & &
 \end{array}$$

so $1 \text{ N} = 1 \text{ kg m s}^{-2}$

So $1 \text{ N} = 1 \text{ kg m s}^{-2}$; the **joule** = $\text{kg m}^2 \text{ s}^{-2}$; the **watt** = $\text{kg m}^2 \text{ s}^{-3}$; the **pascal** = $\text{kg m}^{-1} \text{ s}^{-2}$.

■ Units of an unknown constant

Rearrange for the constant, then substitute base units. For $F = kAv$, where A is an area and v a speed:

$$k = \frac{F}{Av} = \frac{\text{kg m s}^{-2}}{\text{m}^2 \times \text{m s}^{-1}} = \text{kg m}^{-2} \text{ s}^{-1}$$

Write the fraction first, then cancel one line at a time — that first line is the method mark.

■ Checking homogeneity

An equation is **homogeneous** when **every term** has the **same base units**. Check $v^2 = u^2 + 2as$:

- LHS: $(\text{m s}^{-1})^2 = \text{m}^2 \text{ s}^{-2}$
- u^2 : $\text{m}^2 \text{ s}^{-2}$; $2as$: $(\text{m s}^{-2})(\text{s}) = \text{m}^2 \text{ s}^{-2}$

All three are $\text{m}^2 \text{ s}^{-2}$ — homogeneous. Matching units do **not** prove an equation fully correct (a wrong number, like the 2, would still match), but **mismatched** units prove it wrong. Two terms that are **added** must have the same units, so $F = kv^2 + m$ can never be right.

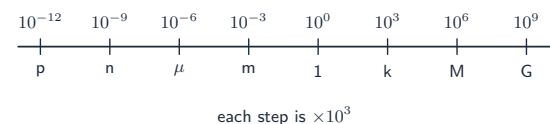
■ Finding an unknown power

If the equation is homogeneous, the powers must balance. In $T = 2\pi \left(\frac{L}{g}\right)^n$, with T a time, L a length and g an acceleration:

$$\left[\frac{L}{g}\right] = \frac{\text{m}}{\text{m s}^{-2}} = \text{s}^2 \Rightarrow (\text{s}^2)^n = \text{s}^1 \Rightarrow 2n = 1 \Rightarrow n = \frac{1}{2}$$

The 2π has no units, so it never enters the check.

■ Prefixes, and compound units



p	n	μ	m	c	d	k	M	G	T
10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^{-1}	10^3	10^6	10^9	10^{12}

To convert a **compound** unit, replace **each** prefix by its power of ten — and remember a prefix inside a power is raised to that power too:

$$0.25 \text{ kN mm}^{-2} = 0.25 \times \frac{10^3 \text{ N}}{(10^{-3} \text{ m})^2} = 0.25 \times \frac{10^3}{10^{-6}} \text{ N m}^{-2} = 2.5 \times 10^8 \text{ N m}^{-2}$$

$(10^{-3})^2 = 10^{-6}$, not 10^{-3} — that squaring is where most marks are lost.

2 Practice

Now apply the methods above.

2.1 Name the **five** SI base quantities and give each unit. [2]

2.2 Write the SI **base units** of **force**, and of **pressure**. [2]

2.3 Write 5 mA in **amperes** and 2.0 GHz in **hertz**, both in standard form. [2]

2.4 Use $W = Fs$ to express the **joule** in SI base units. [2]

2.5 A density is 3.0 g cm^{-3} . Express it in kg m^{-3} . [2]

2.6 In the equation $P = k\rho v^3$, P is a power, ρ a density and v a speed. **Determine** the SI base units of k . [2]

3 Exam-style questions

3.1 Which of these is **not** an SI base unit? [1]

- A ampere
- B coulomb
- C kelvin
- D kilogram

3.2 Which quantity has the same SI base units as **force**? [1]

- A $\frac{\text{energy}}{\text{distance}}$
- B $\frac{\text{energy}}{\text{time}}$
- C momentum $\times$ distance
- D momentum $\times$ time

3.3 What is 6.0 kN cm^{-2} expressed in N m^{-2} ? [1]

- A 6.0×10^1
- B 6.0×10^5
- C 6.0×10^7
- D 6.0×10^9

3.4 The speed v of a wave on the surface of a liquid is given by $v = k \left(\frac{p}{\rho} \right)^n$, where p is a pressure, ρ is a density and k has no units. The equation is homogeneous. What is the value of n ? [1]

- A -2
- B $-\frac{1}{2}$
- C $\frac{1}{2}$
- D 2

3.5 An engineering team releases an unmanned **submersible** (an underwater craft) from rest at the surface of a lake. Its volume is 0.25 m^3 and its weight is 2.1 kN . The density of the lake water is 1030 kg m^{-3} .

(a) The team records three quantities during the dive. **Complete** the table by writing the SI base units of each quantity, and ticking whether it is a scalar or a vector. [3]

quantity	SI base units	scalar	vector
water temperature	_____	☐	☐
water pressure	_____	☐	☐
drag force on the submersible	_____	☐	☐

(b) **Show that** the upthrust on the fully submerged submersible is about 2.5 kN . (Upthrust = weight of the water displaced.) [2]

(c) **Determine** the magnitude and the direction of the initial acceleration of the submersible when it is released. [3]

(d) The drag force F on the submersible is modelled as $F = kv^2$, where v is its speed. **Determine** the SI base units of k . [2]

(e) A student writes the drag equation as $F = kv^2 + m$, where m is the mass of the submersible. Use **units** to explain why this equation cannot be correct. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the five **SI base quantities** and their units:
- mass — kg; length — m; time — s; current — A; temperature — K

2.2

- force from $F = ma$: $\text{kg} \times \text{m s}^{-2} = \mathbf{kg \, m \, s^{-2}}$
- pressure from $p = F/A$: $\text{kg m s}^{-2}/\text{m}^2 = \mathbf{kg \, m^{-1} \, s^{-2}}$

2.3

- $m = 10^{-3}$, so $5 \text{ mA} = 5 \times 10^{-3} \text{ A} = \mathbf{5 \times 10^{-3} \, A}$
- $G = 10^9$, so $2.0 \text{ GHz} = \mathbf{2.0 \times 10^9 \, Hz}$

2.4

- $W = Fs$ and $1 \text{ N} = 1 \text{ kg m s}^{-2}$

$$1 \text{ J} = 1 \text{ N m} = (\text{kg m s}^{-2})(\text{m}) = \mathbf{kg \, m^2 \, s^{-2}}$$

2.5

- replace each prefix, and raise a prefix inside a power

$$3.0 \text{ g cm}^{-3} = 3.0 \times \frac{10^{-3} \text{ kg}}{(10^{-2} \text{ m})^3} = 3.0 \times \frac{10^{-3}}{10^{-6}} \text{ kg m}^{-3} = \mathbf{3.0 \times 10^3 \text{ kg m}^{-3}}$$

2.6

- rearrange: $k = P/(\rho v^3)$; P is a power so $[P] = \text{kg m}^2 \text{ s}^{-3}$

$$k = \frac{\text{kg m}^2 \text{ s}^{-3}}{(\text{kg m}^{-3})(\text{m s}^{-1})^3} = \frac{\text{kg m}^2 \text{ s}^{-3}}{\text{kg s}^{-3}} = \mathbf{m^2}$$

3.1 B

- kg, m, s, A, K are the **SI base units**
- the **coulomb** is derived: $1 \text{ C} = 1 \text{ A s}$
- A, C, D** are base units

3.2 A

- force has base units kg m s^{-2}
- energy / distance:

$$\frac{\text{kg m}^2 \text{ s}^{-2}}{\text{m}} = \mathbf{kg \, m \, s^{-2}}$$

- B** is power ($\text{kg m}^2 \text{ s}^{-3}$); **C** is $\text{kg m}^2 \text{ s}^{-1}$; **D** is kg m

3.3 C

- $k = 10^3$, $c = 10^{-2}$; square the prefix on cm

$$6.0 \text{ kN cm}^{-2} = 6.0 \times \frac{10^3 \text{ N}}{(10^{-2} \text{ m})^2} = 6.0 \times \frac{10^3}{10^{-4}} \text{ N m}^{-2} = \mathbf{6.0 \times 10^7 \text{ N m}^{-2}}$$

3.4 C

- $[p] = \text{kg m}^{-1} \text{s}^{-2}$, $[\rho] = \text{kg m}^{-3}$

$$\left[\frac{p}{\rho} \right] = \frac{\text{kg m}^{-1} \text{s}^{-2}}{\text{kg m}^{-3}} = \text{m}^2 \text{s}^{-2}$$

- v is a speed, so $(\text{m}^2 \text{s}^{-2})^n = \text{m s}^{-1}$
- $2n = 1 \Rightarrow n = \frac{1}{2}$

3.5

(a) complete the table

- water temperature: K, **scalar**
- water pressure: $\text{kg m}^{-1} \text{s}^{-2}$, **scalar**
- drag force: kg m s^{-2} , **vector**

(b) upthrust = weight of water displaced = $\rho V g$

$$U = (1030 \text{ kg m}^{-3})(0.25 \text{ m}^3)(9.81 \text{ m s}^{-2}) = 2526 \text{ N} \approx \mathbf{2.5 \text{ kN}}$$

(c) weight $W = 2.1 \text{ kN} = 2100 \text{ N}$

- resultant $F = U - W = 2526 \text{ N} - 2100 \text{ N} = 426 \text{ N}$ **upwards**

$$m = \frac{W}{g} = \frac{2100 \text{ N}}{9.81 \text{ m s}^{-2}} = 214 \text{ kg}$$

$$a = \frac{F}{m} = \frac{426 \text{ N}}{214 \text{ kg}} = \mathbf{2.0 \text{ m s}^{-2}}$$

upwards

(d) rearrange $F = kv^2$

$$k = \frac{F}{v^2} = \frac{\text{kg m s}^{-2}}{(\text{m s}^{-1})^2} = \mathbf{\text{kg m}^{-1}}$$

(e) kv^2 has the units of force, kg m s^{-2} ; m has the unit kg

- terms that are **added** must have the same units, so the equation is not homogeneous