Exercise walk-through

9.3.1
Resistance, I-V characteristics and resistivity

A-Level Physics

You must be able to

- define resistance and state **Ohm's law 欧姆定律**
- recall and use $V = IR$
- sketch the I–V characteristics of a metallic conductor at constant temperature, a **semiconductor diode 二极管** and a **filament lamp 灯丝灯泡**
- explain that the resistance of a filament lamp increases as the current increases because its temperature increases
- recall and use $R = \rho L/A$, including a wire that is stretched and a block measured between different faces
- understand that the resistance of a light-dependent resistor decreases as the light intensity increases
- understand that the resistance of a thermistor decreases as the temperature increases

1 The method

Method — Resistance, the ohm and Ohm's law

Resistance is the ratio of the **potential difference 电势差** across a component to the **current 电流** in it:

$$R = \frac{V}{I}$$

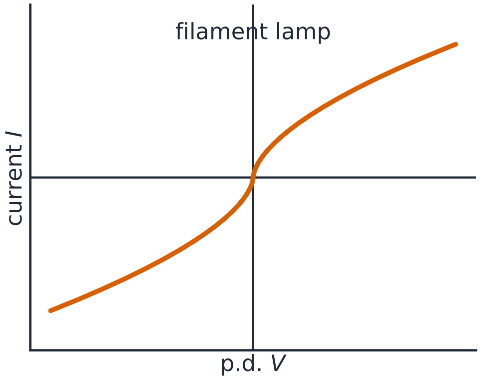
One **ohm 欧姆** is one volt per ampere. Ohm's law says that the current in a metallic conductor is proportional to the p.d. across it, **provided its temperature stays constant**.

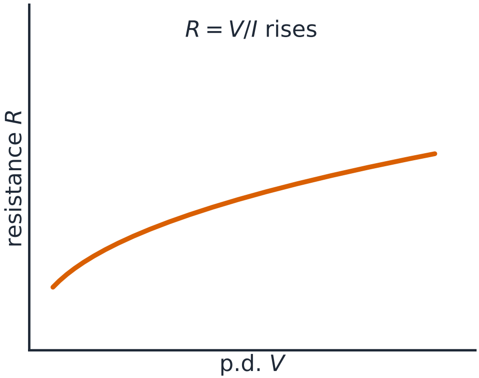
- $R = V/I$ is always true: it is the definition of resistance.
- $I \propto V$ is only true when the I–V graph is a straight line through the origin. A component whose graph curves does not obey Ohm's law.
- A component takes 0.40 A at 6.0 V and 0.25 A at 3.0 V. Its resistance is $6.0/0.40 = 15\ \Omega$ and $3.0/0.25 = 12\ \Omega$: the resistance changes, so the component is not ohmic.

Method — The three characteristics

- **Metallic conductor at constant temperature:** a straight line through the origin, with the same gradient for both directions of current. Its resistance is constant.
- **Filament lamp:** an S-shaped curve through the origin that becomes less steep as the p.d. rises. A larger current heats the filament, its temperature rises, and so its resistance rises. The graph of R against V therefore rises.
- **Semiconductor diode:** almost no current in the reverse direction; in the forward direction almost no current until about 0.6 V, and then the current rises steeply, so the resistance falls.

Method — The three characteristics





Method — Resistivity

$$R = \frac{\rho L}{A}$$

ρ is the resistivity of the **material**, measured in $\Omega\,\mathrm{m}$; L is the length the current travels and A is the **cross-sectional area 横截面积** it travels through.

A constantan wire has $\rho = 4.9 \times 10^{-7}\ \Omega\,\mathrm{m}$, a length of 3.0 m and a diameter of 0.40 mm.

1. Radius $= 0.20$ mm $= 2.0 \times 10^{-4}$ m, so $A = \pi r^2 = 1.26 \times 10^{-7}\ \mathrm{m}^2$.
2. $R = \dfrac{(4.9 \times 10^{-7})(3.0)}{1.26 \times 10^{-7}} = 12\ \Omega$.

- Resistivity belongs to the material; resistance also depends on the shape of the sample.

Method — The same wire, a new shape

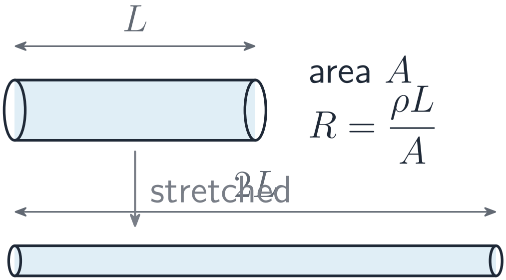
Stretching a wire does not change the volume of metal, so $A \times L$ stays the same.

1. If the length doubles, the area halves, and $R = \rho L/A$ becomes 4 times larger: $R \propto L^2$.
2. A wire is stretched until its diameter is 90% of its old value. The area falls to $0.90^2 = 0.81$ of its old value, so the length rises to $1/0.81 = 1.23$ times its old value.
3. $R' = \dfrac{\rho(1.23L)}{0.81A} = 1.5\,R$.

- Cutting a wire in half is different: the volume changes, and each half has half the resistance.

Method — The same wire, a new shape

Stretching a wire does not change the volume of metal, so … stays the same



area A

$$R = \frac{\rho L}{A}$$

area $A/2$

$$R' = \frac{\rho \cdot 2L}{A/2} = 4R$$

same volume, so $A \times L$ is fixed: $R \propto L^2$

Method — Which faces does the current use?

A block of copper ($\rho = 1.7 \times 10^{-8}\ \Omega\,\mathrm{m}$) has edges of 2.0 cm, 4.0 cm and 8.0 cm.

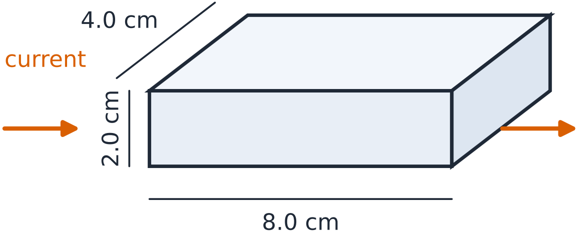
1. In $R = \rho L/A$, L is the distance between the two faces the current enters and leaves, and A is the area of one of those faces.
2. Smallest resistance: the current crosses the shortest edge, $L = 0.020$ m, through the largest faces, $A = 0.040 \times 0.080 = 3.2 \times 10^{-3}$ m^2:

$$R = \frac{(1.7 \times 10^{-8})(0.020)}{3.2 \times 10^{-3}} = 1.1 \times 10^{-7}\ \Omega$$

3. Largest resistance: $L = 0.080$ m and $A = 0.020 \times 0.040 = 8.0 \times 10^{-4}$ m^2, giving $1.7 \times 10^{-6}\ \Omega$, which is 16 times larger.

Method — Which faces does the current use?

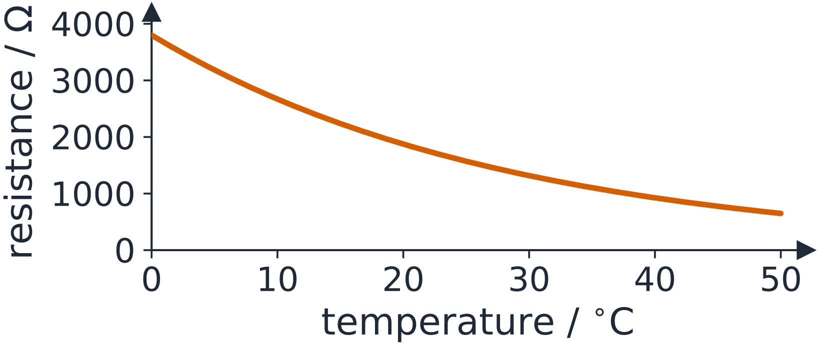
A block of copper (...) has edges of ... , ... and



Method — Thermistor and light-dependent resistor

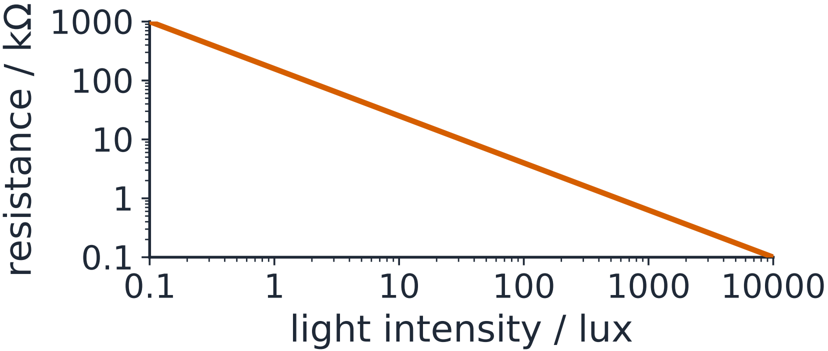
- The resistance of a thermistor **falls** as its temperature rises: on this graph from about $3800\ \Omega$ at $0\ ^{\circ}$C to about $650\ \Omega$ at $50\ ^{\circ}$C.
- The resistance of an LDR **falls** as the light on it gets brighter, from about 1000 kΩ in near darkness to about 0.1 kΩ in bright light.
- Both changes are large, which is what makes these components useful in sensing circuits.

Method — Thermistor and light-dependent resistor



NTC: R falls as T rises

Method — Thermistor and light-dependent resistor



LDR: more light $\Rightarrow$ lower R

2 Practice

Practice 2.1 ▪ 2 marks

A component has a p.d. of 12 V across it and a current of 0.25 A in it. Calculate its resistance.

Solution 2.1

Method: Resistance, the ohm and Ohm's law — p.4

Worked steps

- $R = V/I$ **M1**

$$R = \frac{12}{0.25} = \mathbf{48}\ \Omega$$ **A1**

Practice 2.2 ▪ 2 marks

The resistivity of copper is $1.7 \times 10^{-8}\ \Omega\,\mathrm{m}$. Calculate the resistance of a copper wire of length 5.0 m and cross-sectional area 1.5×10^{-6} m^2.

Solution 2.2

Method: Resistivity — p.7

Worked steps

- $R = \rho L/A$ **M1**

$$R = \frac{(1.7 \times 10^{-8})(5.0)}{1.5 \times 10^{-6}} = \mathbf{0.057\ \Omega}$$ **A1**

Practice 2.3 ▪ 2 marks

A wire has a diameter of 0.80 mm. Calculate its cross-sectional area.

Solution 2.3

Method: Resistivity — p.7

Worked steps

- radius $= 0.40$ mm $= 4.0 \times 10^{-4}$ m, and $A = \pi r^2$ M1

$$A = \pi(4.0 \times 10^{-4})^2 = \mathbf{5.0 \times 10^{-7}}\ \mathrm{m}^{\mathbf{2}}$$ A1

Practice 2.4 ▪ 1 mark

State what is meant by the resistivity of a material.

Solution 2.4

Method: Resistivity — p.7

Worked steps

- the resistivity is $\rho = RA/L$: the property of the material that, with the length and the cross-sectional area, fixes the resistance of a sample **B1**

Practice 2.5 ▪ 2 marks

A second wire is made of the same material and has the same length as the first, but twice the diameter. State how its resistance compares with the first, and why.

Solution 2.5

Method: The same wire, a new shape — p.8

Worked steps

- twice the diameter gives four times the area, since $A = \pi d^2/4$ B1
- $R = \rho L/A$, so the resistance is **one quarter** of the first wire's B1

Practice 2.6 ▪ 2 marks

State how the resistance of a thermistor changes as it is warmed, and how the resistance of an LDR changes as the light on it gets brighter.

Solution 2.6

Method: Thermistor and light-dependent resistor — p.12

Worked steps

- the resistance of the thermistor **decreases** as it is warmed **B1**
- the resistance of the LDR **decreases** as the light gets brighter **B1**

3 Exam-style

Exam-style 3.1 ▪ 1 mark

Two wires are made of the same material and have the same mass. Wire X is twice as long as wire Y. What is $\frac{\text{resistance of X}}{\text{resistance of Y}}$?

A $\frac{1}{4}$

B $\frac{1}{2}$

C 2

D 4

Solution 3.1

A $\frac{1}{4}$

B $\frac{1}{2}$

C 2

D 4 ✓

Worked steps

D

- the same mass of the same material means the same volume, so twice the length gives half the area **B1**
- $R = \rho L/A$ then rises by $2 \times 2 = 4$

Exam-style 3.2 ▪ 1 mark

The p.d. across a filament lamp is doubled, and the current in it rises to 1.6 times its previous value. By what factor does the resistance of the lamp change?

A 0.80

B 1.25

C 1.6

D 3.2

Solution 3.2

- **A** 0.80
- **B** 1.25 ✓
- **C** 1.6
- **D** 3.2

Worked steps

B

- $R = V/I$, so the factor is $\dfrac{2}{1.6} = 1.25$ **B1**
- the resistance rises because the filament is hotter

Exam-style 3.3 ▪ 8 marks

A student measures the resistance R of several different lengths L of the same wire, and plots R against L. The graph is a straight line through the origin with a gradient of $4.8\ \Omega\,\mathrm{m}^{-1}$. The wire has a diameter of 0.36 mm.

Exam-style 3.3 (a) ▪ 2 marks

Explain why the graph is a straight line through the origin.

Solution 3.3 (a)

Worked steps

$R = \rho L/A$, and ρ and A are the same for every length **B1**

- so R is proportional to L: a straight line whose intercept is zero, because a wire of zero length has no resistance **B1**

Exam-style 3.3 (b) ▪ 2 marks

Calculate the cross-sectional area of the wire.

Solution 3.3 (b)

Worked steps

radius $= 0.18$ mm $= 1.8 \times 10^{-4}$ m **M1**

$$A = \pi(1.8 \times 10^{-4})^2 = \mathbf{1.0 \times 10^{-7}}\ \mathrm{m}^{\mathbf{2}}$$ **A1**

Exam-style 3.3 (c) ▪ 3 marks

Determine the resistivity of the metal.

Solution 3.3 (c)

Worked steps

the gradient is ρ/A **M1**

- $\rho = \text{gradient} \times A = 4.8 \times 1.02 \times 10^{-7}$ **M1**
- $\rho = \mathbf{4.9 \times 10^{-7}}\ \Omega\,\text{m}$ **A1**

Exam-style 3.3 (d) ▪ 1 mark

State one advantage of finding the resistivity from the gradient rather than from one pair of readings.

Solution 3.3 (d)

Worked steps

the gradient uses every reading, so a single wrong reading matters less, and any constant error in measuring the length shows up as an intercept instead of changing the gradient **B1**

Exam-style 3.4 ▪ 6 marks

A rectangular block of carbon has edges of 1.0 cm, 2.0 cm and 5.0 cm. The resistivity of the carbon is $3.5 \times 10^{-5}\ \Omega\,\mathrm{m}$.

Exam-style 3.4 (a) ▪ 3 marks

Calculate the largest resistance between a pair of opposite faces.

Solution 3.4 (a)

Method: Which faces does the current use? — p.10

Worked steps

the largest resistance uses the longest edge and the smallest faces: $L = 0.050$ m and $A = 0.010 \times 0.020 = 2.0 \times 10^{-4}$ m^2 M1 M1

$$R = \frac{(3.5 \times 10^{-5})(0.050)}{2.0 \times 10^{-4}} = \mathbf{8.8 \times 10^{-3}}\ \Omega$$ A1

Exam-style 3.4 (b) ▪ 2 marks

Calculate the smallest resistance between a pair of opposite faces.

Solution 3.4 (b)

Worked steps

the smallest uses $L = 0.010$ m and $A = 0.020 \times 0.050 = 1.0 \times 10^{-3}$ m^2 M1

$$R = \frac{(3.5 \times 10^{-5})(0.010)}{1.0 \times 10^{-3}} = \mathbf{3.5 \times 10^{-4}}\ \Omega$$ A1

Exam-style 3.4 (c) ▪ 1 mark

State the ratio of the largest resistance to the smallest, and explain why it has this value.

Solution 3.4 (c)

Worked steps

the ratio is $\mathbf{25}$: the length is 5 times larger and the area is 5 times smaller, and $R = \rho L/A$ B1

Exam-style 3.5 ▪ 8 marks

A filament lamp is marked "12 V, 24 W".

Exam-style 3.5 (a) ▪ 3 marks

Sketch the I–V characteristic of a semiconductor diode, and state what it shows about the resistance of the diode.

Solution 3.5 (a)

Method: The three characteristics — p.5

Worked steps

almost no current in the reverse direction **B1**

- in the forward direction almost no current until about 0.6 V, then a steep rise **B1**
- so the resistance is very large in reverse and below the switch-on p.d., and small once the diode conducts **B1**

Exam-style 3.5 (b) ▪ 2 marks

At its normal operating p.d. of 12 V the current in the lamp is 2.0 A. Calculate the resistance of the lamp.

Solution 3.5 (b)

Worked steps

$R = V/I$ **M1**

$$R = \frac{12}{2.0} = \mathbf{6.0}\ \Omega$$ **A1**

Exam-style 3.5 (c) ▪ 3 marks

At a p.d. of 3.0 V the current in the same lamp is 0.80 A. Calculate the resistance and explain why it differs from your answer to (b).

Solution 3.5 (c)

Worked steps

$R = \frac{3.0}{0.80} = \mathbf{3.75}\ \Omega$ M1 A1

- at the lower p.d. the current is smaller, so the filament is cooler and its resistance is lower B1