Exercise walk-through

# 7.1.1

# Wave graphs, phase, the oscilloscope and intensity

A-Level Physics

# You must be able to

- describe what is meant by wave motion as illustrated by vibration in ropes, springs and ripple tanks
- understand and use the terms displacement, **amplitude 振幅**, phase difference, **period 周期**, **frequency 频率**, **wavelength 波长** and speed, and read them from graphs
- understand the use of the **time-base 时基** and **y-gain 垂直增益** of a cathode-ray oscilloscope (CRO) to determine frequency and amplitude
- derive, using the definitions of speed, frequency and wavelength, the wave equation $v = f\lambda$
- recall and use $v = f\lambda$, including frequencies in kHz, MHz and GHz
- understand that energy is transferred by a progressive wave
- recall and use intensity $=$ power/area and intensity $\propto$ (amplitude)$^2$ for a progressive wave

# 1 The method

# Method — Reading a wave from two graphs

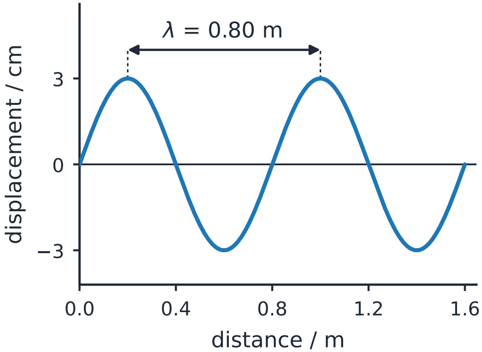
A **displacement–distance** graph is a snapshot of the whole wave at one instant. A **displacement–time** graph follows one particle as the wave passes.

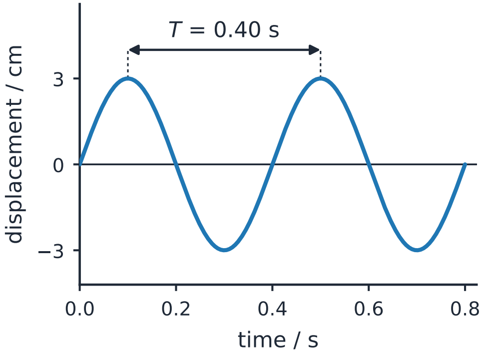
1. **Wavelength**, from the distance graph: the distance between neighbouring crests, $\lambda = 0.80$ m.
2. **Period**, from the time graph: the time between neighbouring crests, $T = 0.40$ s, so $f = 1/T = 2.5$ Hz.
3. **Amplitude**, from either graph: the greatest displacement from the rest position, $3.0$ cm, not the $6.0$ cm from crest to trough.
4. Speed: $v = f\lambda = 2.5 \times 0.80 = 2.0$ m s$^{-1}$.

- Check the label on the horizontal axis before you read a length from the graph: distance gives $\lambda$, time gives $T$.

# Method — Reading a wave from two graphs

A displacement–distance graph is a snapshot of the whole wave at one instant.



# Method — Phase difference

Phase difference says how far one oscillation is ahead of another, as a fraction of a cycle. One whole cycle is $360°$, or $2\pi$ rad.

$$\Delta\phi = \frac{x}{\lambda} \times 360° \qquad \text{or} \qquad \Delta\phi = \frac{t}{T} \times 360°$$

For the wave above, with $\lambda = 0.80$ m and $T = 0.40$ s:

1. Two particles $0.30$ m apart along the wave: $\Delta\phi = \dfrac{0.30}{0.80} \times 360° = 135°$, or $\frac{3}{4}\pi$ rad.
2. Two particles whose displacement–time graphs reach their crests $0.10$ s apart: $\Delta\phi = \dfrac{0.10}{0.40} \times 360° = 90°$.
3. Particles half a wavelength apart, $0.40$ m, are in **antiphase 反相**, $180°$. Particles one wavelength apart, $0.80$ m, are **in phase 同相**.

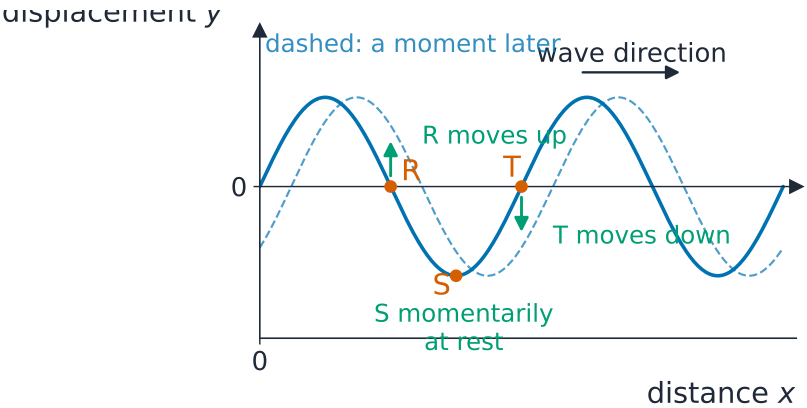
# Method — Which way does a particle move?

In a progressive wave each particle **oscillates 振动** about its own rest position, and the wave travels on past it. You can see this with a pulse on a rope, compressions moving along a spring, and ripples in a **ripple tank 水波槽**, where a floating cork bobs up and down but stays in place. The wave transfers **energy** from place to place, but not matter.

1. Draw the wave again a moment later, moved a little in the direction of travel (dashed).
2. Each particle moves straight up or down to reach the dashed wave: R moves **up** and T moves **down**.
3. A particle at a **crest 波峰** or a **trough 波谷**, such as S, is momentarily at rest.

# Method — Which way does a particle move?

In a progressive wave each particle oscillates 振动 about its own rest position, and the wave travels on past…



# Method — Reading an oscilloscope

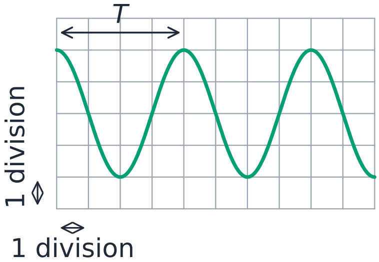
A cathode-ray oscilloscope shows how a voltage varies with time. The time-base is the time for each horizontal division, and the y-gain is the voltage for each vertical division. The time-base is $0.20$ ms per division and the y-gain is $5.0$ mV per division.

1. One cycle spans $4.0$ divisions, so $T = 4.0 \times 0.20$ ms $= 0.80$ ms.
2. $f = \dfrac{1}{T} = \dfrac{1}{0.80 \times 10^{-3}\ \text{s}} = 1250$ Hz.
3. From crest to trough is $4.0$ divisions, so the amplitude is $2.0$ divisions: $2.0 \times 5.0 = 10$ mV.

- For a smaller uncertainty, measure the width of several cycles and divide by the number of cycles.

# Method — Reading an oscilloscope

A cathode-ray oscilloscope shows how a voltage varies with time.




period $= n \times$ timebase · peak voltage $= n \times$ Y-gain

# Method — The wave equation, with awkward units

Speed $=$ distance $\div$ time. In one period $T$ the wave moves forward one wavelength $\lambda$, so

$$v = \frac{\lambda}{T} = f\lambda \qquad \text{because } f = \frac{1}{T}$$

1. A radio station broadcasts at $97.5$ MHz, and radio waves travel at $3.00 \times 10^8$ m s$^{-1}$: $\lambda = \dfrac{3.00 \times 10^8}{97.5 \times 10^6} = 3.08$ m.
2. Ultrasound of frequency $2.0$ MHz travels at $1540$ m s$^{-1}$ in soft tissue: $\lambda = \dfrac{1540}{2.0 \times 10^6} = 7.7 \times 10^{-4}$ m $= 0.77$ mm.
3. A sound wave has a period of $5.0$ ms and a wavelength of $1.7$ m: $v = \dfrac{1.7}{5.0 \times 10^{-3}} = 340$ m s$^{-1}$.

# Method — The wave equation, with awkward units

- Convert first: $1$ kHz $= 10^3$ Hz, $1$ MHz $= 10^6$ Hz, $1$ GHz $= 10^9$ Hz and $1$ ms $= 10^{-3}$ s.

# Method — Intensity

The intensity of a wave is the power passing through each unit area at right angles to the direction of travel: $I = P/A$, in W m$^{-2}$. For a progressive wave $I \propto x_0^2$, where $x_0$ is the amplitude.

A small lamp is a **point source 点源** that radiates $60$ W of light evenly in all directions. The light spreads over a sphere of area $4\pi r^2$.

1. At $2.0$ m: $I = \dfrac{60}{4\pi \times 2.0^2} = 1.19$ W m$^{-2}$.
2. At $4.0$ m the same power is spread over four times the area, so $I = 1.19/4 = 0.30$ W m$^{-2}$.
3. $I \propto x_0^2$, so a quarter of the intensity means half the amplitude.
4. The wave carries energy: a solar panel of area $0.50$ m$^2$ facing sunlight of intensity $800$ W m$^{-2}$ receives $800 \times 0.50 = 400$ W, which is $4000$ J in $10$ s.

# 2 Practice

# Practice 2.1 ▪ 2 marks

A wave on a spring has a wavelength of $24$ cm and a frequency of $12.5$ Hz. Calculate its speed.

# Solution 2.1

Method: The wave equation, with awkward units — p.11

**Worked steps**

- $v = f\lambda$ M1

$$v = 12.5 \times 0.24 = \mathbf{3.0}\ \text{m s}^{-1}$$ A1

# Practice 2.2 ▪ 2 marks

A radio station broadcasts at $1.2$ MHz. Radio waves travel at $3.0 \times 10^{8}$ m s$^{-1}$. Calculate the wavelength.

## Solution 2.2

Method: The wave equation, with awkward units — p.11

**Worked steps**

- $\lambda = v/f$ M1

$$\lambda = \frac{3.0 \times 10^8}{1.2 \times 10^6} = \mathbf{250}\ \mathrm{m}$$ A1

## Practice 2.3 ▪ 2 marks

Two points on a wave are $15$ cm apart, and the wavelength is $60$ cm. Calculate the phase difference between the oscillations of the two points.

# Solution 2.3

Method: Phase difference — p.6

## Worked steps

- $\Delta\phi = \dfrac{x}{\lambda} \times 360°$ **M1**
- $\Delta\phi = \dfrac{15}{60} \times 360° = \mathbf{90}°$ **A1**

## Practice 2.4 ▪ 2 marks

An oscilloscope shows exactly two cycles of a signal across $10$ divisions. The time-base is set to $0.20$ ms per division. Calculate the frequency of the signal.

# Solution 2.4

Method: Reading an oscilloscope — p.9

**Worked steps**

- one cycle spans $10/2 = 5.0$ divisions, so $T = 5.0 \times 0.20$ ms $= 1.0$ ms M1
- $f = 1/T = \mathbf{1.0 \times 10^3}$ Hz A1

## Practice 2.5 ▪ 2 marks

A laser beam of power $2.0$ mW has a cross-sectional area of $1.5$ mm$^2$. Calculate the intensity of the beam.

# Solution 2.5

Method: Intensity — p.13

## Worked steps

- $I = P/A$ with $A = 1.5 \times 10^{-6}$ m$^2$ **M1**

$$I = \frac{2.0 \times 10^{-3}}{1.5 \times 10^{-6}} = \mathbf{1.3} \times \mathbf{10^3}\ \mathrm{W\ m^{-2}}$$ **A1**

## Practice 2.6 ▪ 2 marks

A progressive wave passes along a stretched spring. State what the wave transfers along the spring, and describe the motion of one coil.

# Solution 2.6

Method: Which way does a particle move? — p.7

**Worked steps**

- the wave transfers energy along the spring **B1**
- each coil oscillates about its own rest position, and does not travel along with the wave **B1**

# 3 Exam-style

# Exam-style 3.1 ▪ 1 mark

Two points on a progressive wave are $0.12$ m apart. The phase difference between their oscillations is $60°$. What is the wavelength?

- **A** $0.020$ m
- **B** $0.36$ m
- **C** $0.72$ m
- **D** $7.2$ m

# Solution 3.1

Method: Phase difference — p.6

**A** $0.020$ m

**B** $0.36$ m

**C** $0.72$ m ✓

**D** $7.2$ m

## Worked steps

**C**

- $\lambda = \dfrac{360°}{\Delta\phi} \times x = 6 \times 0.12 = 0.72$ m **B1**

## Exam-style 3.2 ▪ 1 mark

The amplitude of a progressive wave falls to one third of its value, and the frequency does not change. What fraction of the original intensity remains?

**A** $\frac{1}{3}$

**B** $\frac{1}{6}$

**C** $\frac{1}{9}$

**D** $\frac{1}{27}$

# Solution 3.2

Method: Intensity — p.13

**A** $\frac{1}{3}$

**B** $\frac{1}{6}$

**C** $\frac{1}{9}$ ✓

**D** $\frac{1}{27}$

## Worked steps

**C**

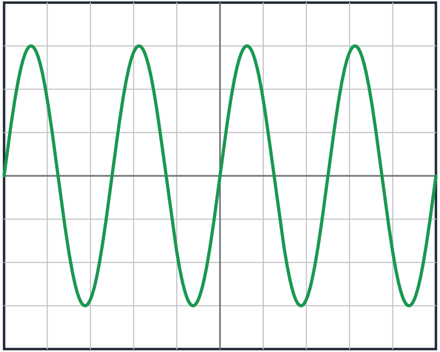
- intensity $\propto$ (amplitude)$^2$, so the intensity falls to $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ **B1**

# Exam-style 3.3 ▪ 8 marks

A signal generator drives a loudspeaker and is also connected to a cathode-ray oscilloscope. The trace is shown, and each square is one division. The time-base is set to $0.20$ ms per division and the y-gain to $10$ mV per division.

## Exam-style 3.3 ▪ 8 marks

The trace is shown, and each square is one division.

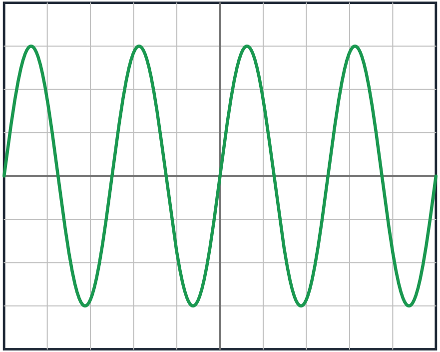


# Exam-style 3.3 (a) ▪ 2 marks

Determine the period of the signal.

# Exam-style 3.3 (a) ▪ 2 marks

Determine the period of the signal



# Solution 3.3 (a)

Method: Reading an oscilloscope — p.9

**Worked steps**

one cycle spans $2.5$ divisions **M1**

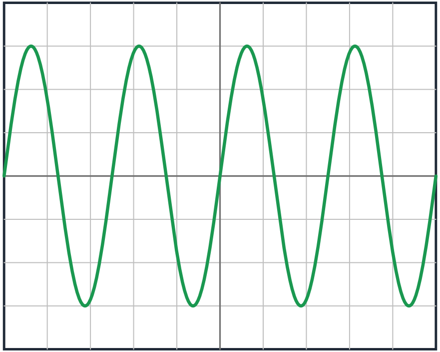
- $T = 2.5 \times 0.20$ ms $= \mathbf{0.50}$ ms **A1**

# Exam-style 3.3 (b) ▪ 1 mark

Calculate the frequency of the sound from the loudspeaker.

# Exam-style 3.3 (b) ▪ 1 mark

Calculate the frequency of the sound from the loudspeaker



# Solution 3.3 (b)

**Worked steps**

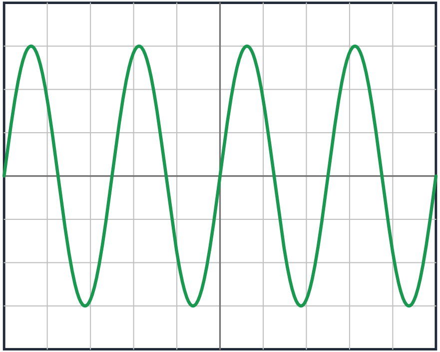
$f = 1/T = 1/(0.50 \times 10^{-3}) = \mathbf{2.0} \times \mathbf{10^{3}}$ Hz B1

## Exam-style 3.3 (c) ▪ 1 mark

Determine the amplitude of the signal.

# Exam-style 3.3 (c) ▪ 1 mark

Determine the amplitude of the signal



# Solution 3.3 (c)

**Worked steps**

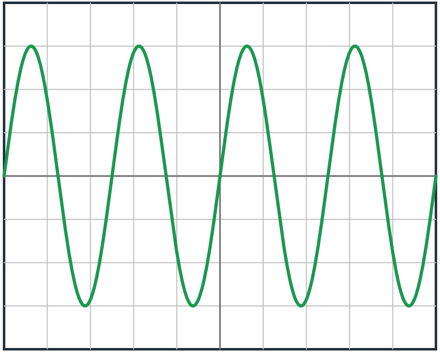
amplitude $= 3.0$ divisions $\times\ 10$ mV $= \mathbf{30}$ mV **B1**

## Exam-style 3.3 (d) ▪ 2 marks

The speed of sound is $340$ m s$^{-1}$. Calculate the wavelength of the sound.

## Exam-style 3.3 (d) ▪ 2 marks

Calculate the wavelength of the sound



# Solution 3.3 (d)

**Worked steps**

$\lambda = v/f$ **M1**

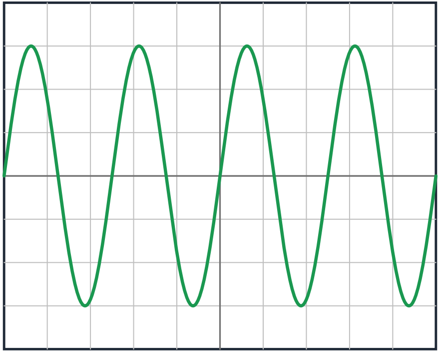
$$\lambda = \frac{340}{2000} = \mathbf{0.17} \text{ m}$$ **A1**

# Exam-style 3.3 (e) ▪ 2 marks

The output of the signal generator is turned down at the same frequency, and the amplitude of the trace falls to $2.0$ divisions. The amplitude of the sound is proportional to the amplitude of the signal. Calculate $\dfrac{\text{new intensity}}{\text{original intensity}}$ for the sound.

# Exam-style 3.3 (e) ▪ 2 marks

Calculate … for the sound



# Solution 3.3 (e)

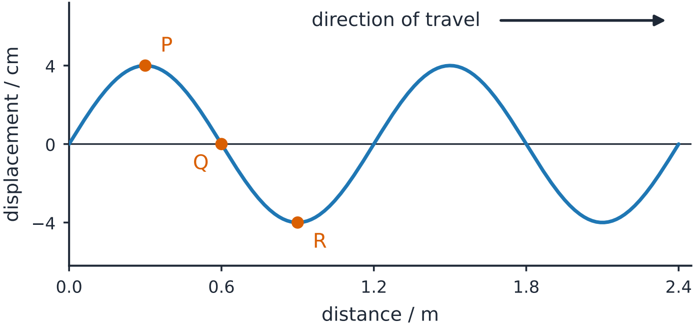
**Worked steps**

intensity $\propto$ (amplitude)$^2$ **M1**

$$\frac{I_{\text{new}}}{I_{\text{original}}} = \left(\frac{2.0}{3.0}\right)^2 = \mathbf{0.44}$$ **A1**

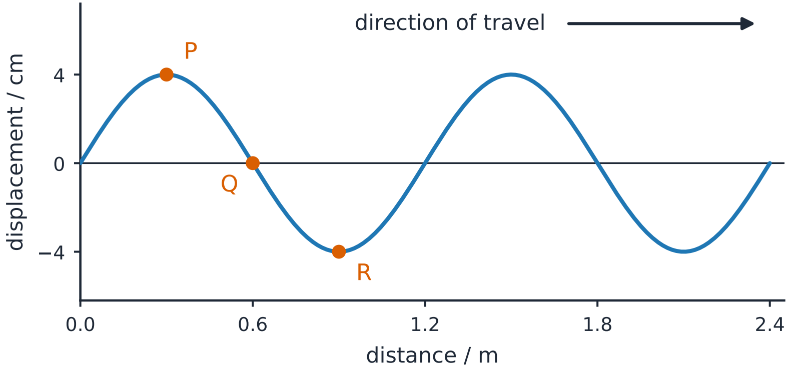
# Exam-style 3.4 ▪ 7 marks

A transverse wave travels to the right along a rope at $3.0$ m s$^{-1}$. The graph shows the displacement of the rope against distance at one instant. P is at a crest, Q has zero displacement and R is at a trough.



# Exam-style 3.4 (a) ▪ 2 marks

Calculate the frequency of the wave.



# Solution 3.4 (a)

Method: Reading a wave from two graphs — p.4

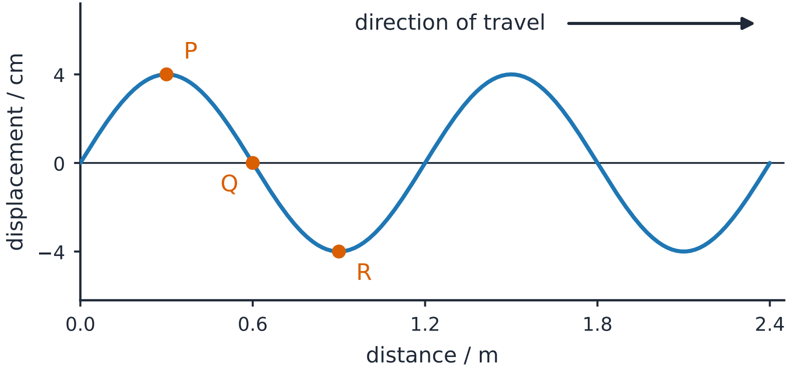
**Worked steps**

from the graph $\lambda = 1.2$ m **M1**

$$f = \frac{v}{\lambda} = \frac{3.0}{1.2} = \mathbf{2.5}\ \text{Hz}$$ **A1**

## Exam-style 3.4 (b) ▪ 1 mark

State the phase difference between the oscillations of P and R.



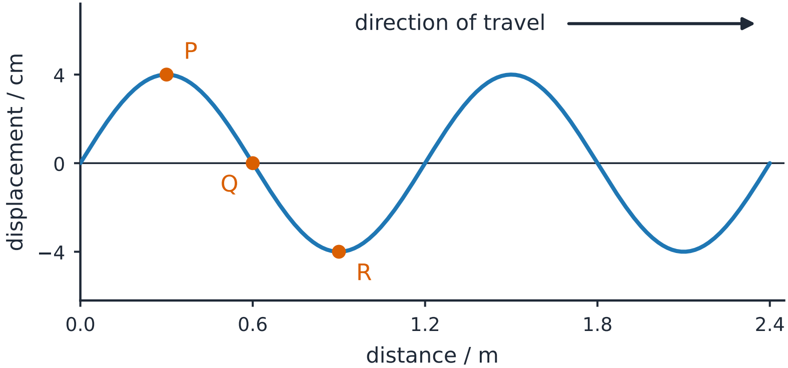
# Solution 3.4 (b)

**Worked steps**

$\mathbf{180^\circ}$, or $\pi$ rad: P and R are half a wavelength apart **B1**

# Exam-style 3.4 (c) ▪ 2 marks

State the direction in which Q is moving at this instant, and explain how you know.



# Solution 3.4 (c)

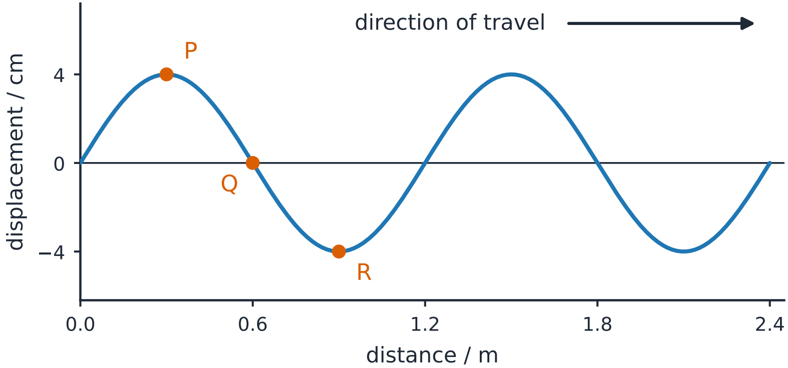
**Worked steps**

**upwards** **B1**

- the wave moves to the right, so the crest now at P moves towards Q; drawing the wave a moment later shows the rope at Q above the axis **B1**

# Exam-style 3.4 (d) ▪ 1 mark

State the displacement of P a quarter of a period later.



# Solution 3.4 (d)

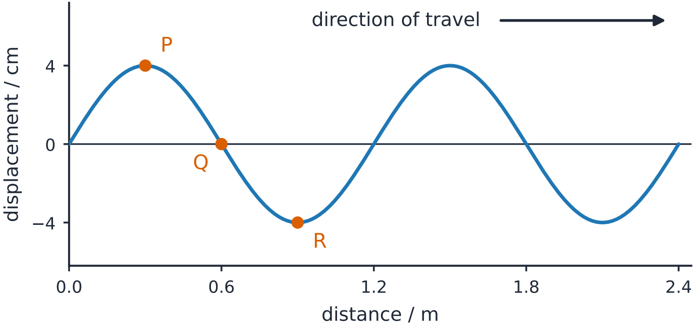
**Worked steps**

**zero**: in a quarter of a period P moves from the crest back to its rest position

**B1**

# Exam-style 3.4 (e) ▪ 1 mark

The amplitude of the wave is increased to $6.0$ cm at the same frequency. Calculate $\dfrac{\text{new intensity}}{\text{original intensity}}$.



# Solution 3.4 (e)

**Worked steps**

intensity $\propto$ (amplitude)$^2$: $\left(\dfrac{6.0}{4.0}\right)^2 = \mathbf{2.25}$ B1

## Exam-style 3.5 ▪ 6 marks

A small loudspeaker radiates sound evenly in all directions. The sound power is $0.80$ W.

## Exam-style 3.5 (a) ▪ 1 mark

Define intensity.

# Solution 3.5 (a)

Method: Intensity — p.13

**Worked steps**

intensity is the power per unit area, where the area is at right angles to the direction of travel of the wave **B1**

## Exam-style 3.5 (b) ▪ 2 marks

Show that the intensity of the sound $5.0$ m from the loudspeaker is about $2.5 \times 10^{-3}$ W m$^{-2}$.

# Solution 3.5 (b)

**Worked steps**

the power spreads over a sphere of area $4\pi r^2$ M1

$$I = \frac{0.80}{4\pi \times 5.0^2} = \mathbf{2.5 \times 10^{-3}}\ \text{W m}^{-2}$$ A1

## Exam-style 3.5 (c) ▪ 1 mark

State the distance from the loudspeaker at which the intensity is one quarter of this value.

# Solution 3.5 (c)

**Worked steps**

**10** m: twice the distance spreads the power over four times the area **B1**

# Exam-style 3.5 (d) ▪ 2 marks

The amplitude of the sound wave $5.0$ m from the loudspeaker is *a*.
Determine the amplitude $10$ m from the loudspeaker, in terms of *a*.

# Solution 3.5 (d)

**Worked steps**

intensity $\propto$ (amplitude)$^2$, so one quarter of the intensity means half the amplitude **M1**

- amplitude $= \mathbf{a/2}$ **A1**

## Exam-style 3.6 ▪ 2 marks

Using the definitions of speed, frequency and wavelength, derive the wave equation $v = f\lambda$.

# Solution 3.6

Method: The wave equation, with awkward units — p.11

**Worked steps**

- speed $=$ distance $\div$ time, and in one period $T$ the wave moves one wavelength $\lambda$, so $v = \lambda/T$ **M1**
- frequency is the number of oscillations per unit time, so $f = 1/T$ and $v = f\lambda$ **A1**