Exercise walk-through

6.1.1

Deformation, the force-extension graph and measuring the Young modulus

A-Level Physics

You must be able to

- understand that deformation is caused by tensile or compressive forces
- use the terms **load 负载**, **extension 伸长**, compression and **limit of proportionality 比例极限**
- recall and use **Hooke's law** and the **spring constant 弹簧常数** $k = F/x$
- define and use stress, strain and the **Young modulus 杨氏模量**
- describe an experiment to determine the Young modulus of a metal in the form of a wire

1 The method

Method — Two kinds of deformation, and the words for them

- A **tensile** force **stretches** an object: the forces at the two ends pull **outwards**, and the object gets **longer**. The increase in length is the **extension**.
- A **compressive** force **squashes** it: the forces push **inwards**, and the object gets **shorter**. The decrease is the **compression**.
- Either way, the forces come in a **pair**. A single force acting on a free object accelerates it; it takes two opposing forces to deform it.
- The **load** is the force applied, usually the weight of the masses hung on the specimen. Do not confuse it with the mass: $W = mg$.

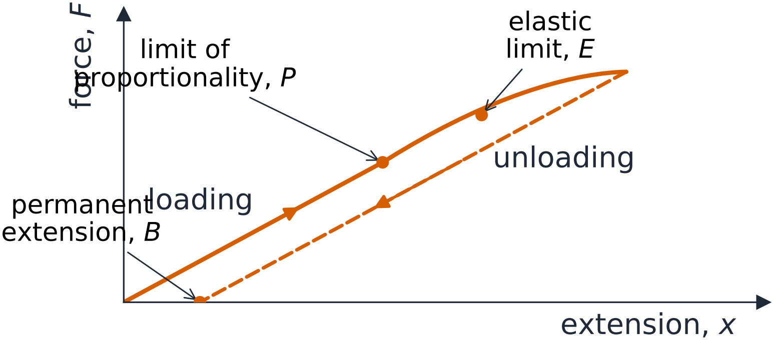
Method — Every named point on a force-extension graph

point	what it means
the **straight** region	extension is **proportional to** load; the specimen **obeys Hooke's law**
limit of proportionality	the point beyond which the graph is **no longer straight**; $F \propto x$ fails here
elastic limit	the point beyond which the specimen does **not** return to its original length when the load is removed; it is just past the limit of proportionality
plastic region	the specimen is **permanently deformed**; removing the load leaves an extension behind
breaking point	where it snaps

Method — Every named point on a force-extension graph

- The two limits are **different points** and the exam distinguishes them. Proportionality is about the **shape of the graph**; elasticity is about **what happens when you let go**.
- In the straight region the **gradient is the spring constant** k, so $k = \frac{F}{x}$. A **stiffer** specimen gives a **steeper** line.

Method — Every named point on a force-extension graph



Hooke up to limit of proportionality · plastic beyond yield

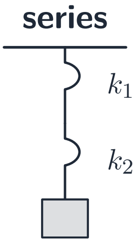
Method — Springs joined together

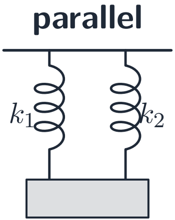
Do not memorise formulae for these – work them out from the two questions that matter:

joined	**what force does each spring carry?**	**what extension does the pair have?**
in series (end to end)	the **whole** load F, each of them	each extends F/k, and the extensions **add**: total $= 2F/k$, so the pair is **less stiff**
in parallel (side by side)	the load is **shared**, so $F/2$ each	each extends $F/2k$, and that **is** the extension of the pair: **stiffer**

Method — Springs joined together

Do not memorise formulae for these – work them out from the two questions that matter: center





$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k = k_1 + k_2$$

Method — Stress, strain, and why we bother

$$\text{stress } \sigma = \frac{F}{A} \quad \text{(Pa)} \qquad \text{strain } \varepsilon = \frac{x}{L} \quad \text{(no unit)} \qquad E = \frac{\sigma}{\varepsilon} \quad \text{(Pa)}$$

- A **force-extension** graph describes **this specimen**: a thicker or shorter wire of the same metal gives a different line.
- Dividing the force by the **area** and the extension by the **original length** removes the specimen's dimensions, so a **stress-strain** graph describes the **material**. Every copper wire gives the same one.
- In the straight region the gradient of a stress-strain graph is the **Young modulus**, and it is a property of the material alone.
- Strain has **no unit**, so the Young modulus has the same unit as stress: **pascals**. Typical metals are around 10^{11} Pa; an answer of 10^{9} usually means a unit was missed somewhere.

Method — Measuring the Young modulus of a wire

$$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{x/L} = \frac{FL}{Ax}$$

so plotting **load against extension** gives a straight line of gradient $\frac{EA}{L}$, and

$$E = \text{gradient} \times \frac{L}{A}$$

What is measured, and with what:

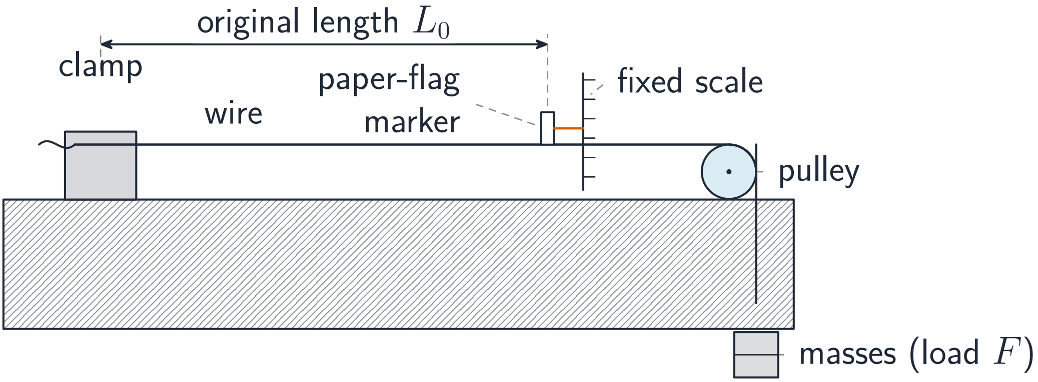
quantity	instrument	note
original length L	metre rule	measured from the clamp to the marker; make it **long** so the extension is measurable
diameter d	**micrometer**, at several points and in different directions	$A = \pi d^2/4$ – the wire may not be perfectly round
extension x	marker against a **millimetre scale**, or a travelling microscope	take a reading **before** loading, as the zero
load F	known masses, $F = mg$	add **and then remove** them, checking the wire returns to its original length

Method — Measuring the Young modulus of a wire

- △ **The diameter carries the largest percentage uncertainty**, because it is the smallest quantity measured – and it is **squared** in the area, so its percentage uncertainty is **doubled** in A and therefore in E. Measuring it carefully matters more than anything else in the experiment.
- Use a **long, thin** wire: a larger L and a smaller A both make the extension bigger and easier to measure.

Method — Measuring the Young modulus of a wire

[E = = F/A x/L = FL Ax] so plotting load against extension gives a straight line of gradient ... , and [E =...



2 Practice

Practice 2.1 ▪ 2 marks

State what is meant by a **tensile** force and by a **compressive** force.

Solution 2.1

Method: Two kinds of deformation, and the words for them — p.4

Worked steps

- a **tensile** force is a pair of forces pulling **outwards** on the ends of an object, stretching it so that it becomes longer **B1**
- a **compressive** force is a pair pushing **inwards**, squashing it so that it becomes shorter **B1**

Practice 2.2 ▪ 3 marks

Explain what is meant by the **limit of proportionality**, and state how it differs from the **elastic limit**.

Solution 2.2

Method: Every named point on a force-extension graph — p.5

Worked steps

- the **limit of proportionality** is the point beyond which the extension is **no longer proportional** to the load, so the graph stops being a straight line **B1**
- the **elastic limit** is the point beyond which the specimen no longer returns to its **original length** when the load is removed **B1**
- they are different points: the elastic limit lies just beyond the limit of proportionality **B1**

Practice 2.3 ▪ 3 marks

Two identical springs, each of spring constant 25 N m^{-1}, are joined end to end and a load of 3.0 N is hung from the pair. Calculate the total extension.

Solution 2.3

Method: Springs joined together — p.8

Worked steps

- springs in series each carry the **whole** load M1

$$x_{\text{each}} = \frac{F}{k} = \frac{3.0\ \text{N}}{25\ \text{N m}^{-1}} = 0.12\ \text{m} \qquad \text{M1}$$

- the extensions add

$$x_{\text{total}} = 2 \times 0.12\ \text{m} = \mathbf{0.24}\ \text{m} \qquad \text{A1}$$

Practice 2.4 ▪ 1 mark

State what the gradient of a **stress-strain** graph represents in its straight region.

Solution 2.4

Method: Stress, strain, and why we bother — p.10

Worked steps

- the **Young modulus** of the material B1

Practice 2.5 ▪ 2 marks

Explain why a stress-strain graph describes the **material** while a force-extension graph describes only that **specimen**.

Solution 2.5

Method: Stress, strain, and why we bother — p.10

Worked steps

- a force-extension graph changes if the wire is made **thicker or shorter**, even for the same metal, so it describes that specimen **B1**
- dividing force by **area** and extension by **original length** removes the specimen's dimensions, so every specimen of the same material gives the **same** stress-strain graph **B1**

Practice 2.6 ▪ 3 marks

A wire of diameter 0.40 mm carries a load of 25 N. Calculate the stress in the wire.

Solution 2.6

Method: Measuring the Young modulus of a wire — p.11

Worked steps

- the radius is half the diameter, so **M1**

$$A = \pi r^2 = \pi (0.20 \times 10^{-3}\ \mathrm{m})^2 = 1.26 \times 10^{-7}\ \mathrm{m}^2 \qquad \textbf{M1}$$

$$\sigma = \frac{F}{A} = \frac{25\ \mathrm{N}}{1.26 \times 10^{-7}\ \mathrm{m}^2} = \mathbf{2.0 \times 10^{8}}\ \mathrm{Pa} \qquad \textbf{A1}$$

3 Exam-style

Exam-style 3.1 ▪ 1 mark

A wire obeys Hooke's law. Which quantity is given by the gradient of a graph of stress against strain?

A the spring constant of the wire

B the Young modulus of the material

C the stiffness of that particular wire

D the breaking stress of the material

Solution 3.1

Method: Stress, strain, and why we bother — p.10

A the spring constant of the wire

B the Young modulus of the material ✓

C the stiffness of that particular wire

D the breaking stress of the material

Worked steps

B

- the gradient of stress against strain is $\frac{\sigma}{\varepsilon}$, which is the **Young modulus** **B1**

Exam-style 3.2 ▪ 6 marks

A spring obeys Hooke's law up to a load of 8.0 N, at which its extension is 0.16 m.

Exam-style 3.2 (a) ▪ 2 marks

Calculate the spring constant of the spring.

Solution 3.2 (a)

Method: Every named point on a force-extension graph — p.5

Worked steps

- use $k = F/x$ M1

$$k = \frac{8.0\ \text{N}}{0.16\ \text{m}} = \mathbf{50}\ \text{N m}^{-1}$$ A1

Exam-style 3.2 (b) ▪ 2 marks

Two such springs are connected **in parallel** and the same 8.0 N load is hung from the pair. Calculate the extension.

Solution 3.2 (b)

Worked steps

- in parallel the load is **shared**, so each spring carries 4.0 N **M1**

$$x = \frac{4.0 \text{ N}}{50 \text{ N m}^{-1}} = \mathbf{0.080} \text{ m}$$ **A1**

- the extension of the pair is the extension of each one, not the sum

Exam-style 3.2 (c) ▪ 2 marks

A single spring is loaded beyond its elastic limit and the load is then removed. Describe what has happened to the spring.

Solution 3.2 (c)

Worked steps

- the spring has been **permanently deformed** B1
- it does not return to its original length: some extension remains after the load is removed B1

Exam-style 3.3 ▪ 10 marks

A student determined the Young modulus of a metal using a wire of original length 2.50 m and diameter 0.38 mm. A graph of load against extension was a straight line of gradient 9.0×10^3 N m^{-1} up to the limit of proportionality.

Exam-style 3.3 (a) ▪ 2 marks

Calculate the cross-sectional area of the wire.

Solution 3.3 (a)

Method: Measuring the Young modulus of a wire — p.11

Worked steps

- the radius is half the diameter M1

$$A = \pi \left(\frac{0.38 \times 10^{-3}\ \mathrm{m}}{2} \right)^2 = \mathbf{1.13 \times 10^{-7}\ m^2}$$ A1

Exam-style 3.3 (b) ▪ 2 marks

Show that the Young modulus is given by $E = \text{gradient} \times \dfrac{L}{A}$.

Solution 3.3 (b)

Worked steps

- start from the definitions **M1**

$$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{x/L} = \frac{F}{x} \times \frac{L}{A}$$

- and $\frac{F}{x}$ is the **gradient** of the load-extension graph, so $E = \text{gradient} \times \frac{L}{A}$ **A1**

Exam-style 3.3 (c) ▪ 2 marks

Calculate the Young modulus of the metal.

Solution 3.3 (c)

Worked steps

- substitute M1

$$E = 9.0 \times 10^3\ \mathrm{N\ m^{-1}} \times \frac{2.50\ \mathrm{m}}{1.13 \times 10^{-7}\ \mathrm{m^2}} = \mathbf{2.0 \times 10^{11}}\ \mathrm{Pa}$$ A1

Exam-style 3.3 (d) ▪ 2 marks

State which measured quantity contributes the largest percentage uncertainty to the value of E, and explain why.

Solution 3.3 (d)

Worked steps

- the **diameter** of the wire **B1**
- it is the smallest quantity measured, so its percentage uncertainty is the largest, and it is **squared** in the area, which **doubles** that percentage uncertainty in A and therefore in E **B1**

Exam-style 3.3 (e) ▪ 2 marks

Suggest **two** changes that would improve the accuracy of the experiment.

Solution 3.3 (e)

Worked steps

- any **two** of: measure the diameter at **several points** along the wire and in different directions, and take a mean; use a **longer** wire so the extension is larger; use a **travelling microscope** or a vernier scale to read the extension; check the wire returns to its original length as the loads are removed; take readings for both increasing and decreasing loads **B1** **B1**