Exercise walk-through

Gravitational potential energy and kinetic energy ▪ 5.2

eXercise

You must be able to

- use $\Delta E_{\mathrm{P}} = mg\Delta h$ and derive it from $W = Fs$
- use $E_{\mathrm{K}} = \frac{1}{2}mv^2$ and derive it from the equations of motion
- solve problems where GPE and KE exchange

Method — GPE change

Lifting mass m slowly through height Δh needs an upward force mg, so the work done ($=$ GPE gained) is

$$\Delta E_{\mathrm{P}} = W = (mg)(\Delta h) = mg\Delta h.$$

Method — Kinetic energy

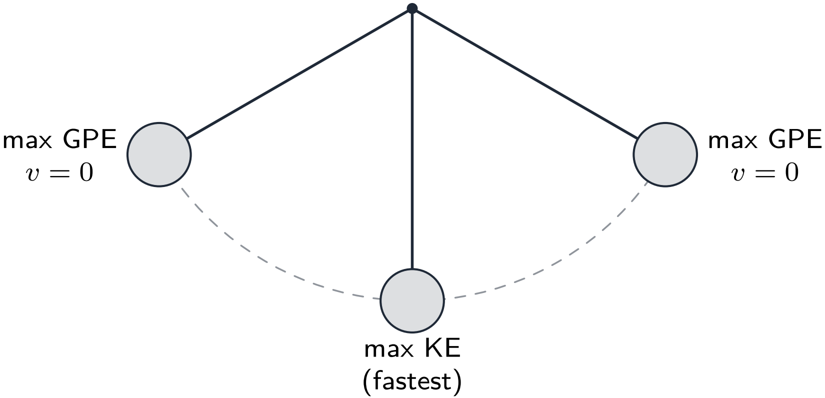
A force F accelerates a mass from rest to speed v over distance s. With $u = 0$, $v^2 = 2as$, so $s = v^2/2a$, and

$$E_K = W = Fs = (ma)\frac{v^2}{2a} = \tfrac{1}{2}mv^2.$$

Method — Energy exchange (pendulum)

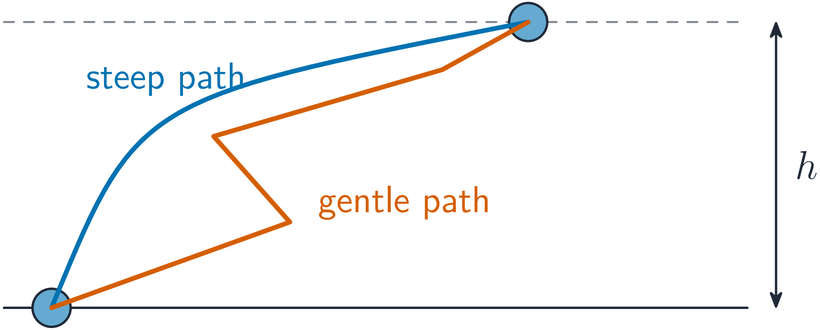
A pendulum bob trades GPE and KE: **max GPE** (and $v = 0$) at the ends, **max KE** at the bottom:
Energy is conserved: $mgh = \frac{1}{2}mv^2$, so the bob's speed at the bottom is $v = \sqrt{2gh}$.

Method — Energy exchange (pendulum)



Method — Energy exchange (pendulum)

same $\Delta E_P = mgh$ – only the height h matters



GPE depends only on height, not the path

Method — Energy accounting in a chain of transfers

The hardest energy questions are not one formula: they follow the energy through **several** stores and ask you to keep a running total. A falling block that lands on a spring is the standard case:

$$E_{\text{elastic, max}} = E_{\text{K on contact}} + \Delta E_{\text{P during compression}}$$

The second term is easy to forget — the block keeps falling *while* it compresses the spring, so it loses **more** potential energy after contact.

Two more results the exam builds on top:

- the **area under a force-compression graph** is the elastic potential energy, so for a spring obeying Hooke's law $E = \frac{1}{2}F_0 x_0$, which gives $F_0 = \dfrac{2E}{x_0}$;

Method — Energy accounting in a chain of transfers

- at maximum compression the spring pushes up with F_0 while the weight still pulls down, so the **resultant** force is $F_0 - mg$ and the acceleration is $\dfrac{F_0 - mg}{m}$ — large, upward, and the moment the block starts back up.

Practice 2.1 ▪ 2 marks

Write the formula for **GPE change** and for **kinetic energy**.

Solution 2.1

Worked steps

$\Delta E_{\mathsf{P}} = mg\Delta h$; $E_{\mathsf{K}} = \frac{1}{2}mv^2$ **B1** **B1**.

Practice 2.2 ▪ 2 marks

Find the **kinetic energy** of a 1200 kg car moving at 15 m s^{-1}.

Solution 2.2

Method: Kinetic energy

Worked steps

$E_K = \frac{1}{2}(1200)(15)^2 = 1.35 \times 10^5$ J M1 A1.

Practice 2.3 ▪ 2 marks

Find the **GPE gained** by a 2.0 kg book lifted 1.5 m ($g = 9.81$ m s^{-2}).

Solution 2.3

Method: GPE change

Worked steps

$\Delta E_{\mathrm{P}} = mg\Delta h = 2.0 \times 9.81 \times 1.5 = 29\ \mathrm{J}$ M1 A1.

Practice 2.4 ▪ 1 mark

At the **bottom** of a pendulum swing, which energy is at a maximum?

Solution 2.4

Method: Energy exchange (pendulum)

Worked steps

Kinetic energy B1.

Exam-style 3.1 ▪ 1 mark

An object's speed **doubles**. Its kinetic energy:

- **A** doubles
- **B** halves
- **C** becomes four times larger
- **D** stays the same

Solution 3.1

Method: Kinetic energy

A doubles

B halves

C becomes four times larger ✓

D stays the same

Worked steps

C — KE $\propto v^2$, so doubling v makes KE four times larger.

Exam-style 3.2 ▪ 3 marks

Derive $E_K = \frac{1}{2}mv^2$ using the equations of motion.

Solution 3.2

Worked steps

Work done $W = Fs = mas$ **M1**; with $u = 0$, $v^2 = 2as \Rightarrow s = v^2/2a$ **M1**;
$W = ma \cdot \dfrac{v^2}{2a} = \frac{1}{2}mv^2 = E_K$ **A1**.

Exam-style 3.3 ▪ 3 marks

A 0.50 kg ball is dropped from a height of 1.8 m. Calculate its speed just before it hits the ground (ignore air resistance).

Solution 3.3

Worked steps

$\frac{1}{2}mv^2 = mgh$ **M1**; $v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1.8}$ **M1** $= 5.9\ \mathrm{m\ s^{-1}}$ **A1**.

Exam-style 3.4 ▪ 2 marks

A pendulum bob is released from a point 0.20 m above its lowest point. Calculate its **maximum speed**.

Solution 3.4

Method: Energy exchange (pendulum)

Worked steps

$v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.20}$ **M1** $= 2.0\ \mathrm{m\ s^{-1}}$ **A1**.

Exam-style 3.5 ▪ 3 marks

A 0.20 kg ball is dropped from a height of 1.6 m and rebounds to a height of 1.2 m. Calculate the energy **transferred to other forms** during the bounce.

Solution 3.5

Method: Energy accounting in a chain of transfers

Worked steps

GPE before $= mgh_1 = 0.20 \times 9.81 \times 1.6 = 3.1$ J; GPE after $= mgh_2 = 0.20 \times 9.81 \times 1.2 = 2.4$ J **M1** **M1**; energy transferred $= 3.1 - 2.4 = 0.79$ J **A1**.

Exam-style 3.6 ▪ 2 marks

A vertical spring stands on a horizontal surface with its lower end fixed. Its mass is negligible. A block of mass 4.0 kg falls onto the spring and is brought to rest as the spring compresses. The block has kinetic energy 72 J at the instant it touches the spring.

(a) **Calculate** the speed of the block as it touches the spring.

(b) While the spring compresses, the gravitational potential energy of the block falls by a further 12 J. **Show that** the maximum compression x_0 of the spring is about 0.31 m.

(c) All the energy lost by the block becomes elastic potential energy in the spring. **Determine** the maximum elastic potential energy stored.

(d) The spring obeys Hooke's law, so the force it exerts is proportional to the compression. **Show that** the maximum force F_0 the spring exerts on the block is about 540 N.

(e) At maximum compression, **determine**

Exam-style 3.6 ▪ 2 marks

(i) the resultant force on the block,
(ii) the acceleration of the block.
(f) **Sketch** a graph to show how the **kinetic energy** of the block varies with the compression of the spring, from the instant of contact to maximum compression. **Label** both axes.
(g) The same block is dropped from a greater height onto the same spring. **Explain** why the maximum compression is greater, and why the maximum acceleration is greater too.

Solution 3.6

Method: Energy accounting in a chain of transfers

Worked steps

(a) $E_K = \frac{1}{2}mv^2$, so $72 = \frac{1}{2}(4.0)v^2$ **M1**; $v = \sqrt{36} = 6.0$ m s^{-1} **A1**.
(b) The block falls a further x_0 while compressing the spring, so $\Delta E_P = mgx_0$ **M1**;
$x_0 = \dfrac{12}{4.0 \times 9.81} = 0.31$ m **A1**.
(c) All of it goes into the spring: $72 + 12 = 84$ J **B1**.
(d) The area under the force-compression graph is the energy stored, and for a straight line that area is a triangle: $E = \frac{1}{2}F_0x_0$ **M1**; $F_0 = \dfrac{2 \times 84}{0.306} = 5.5 \times 10^2$ N **A1**.
(e)(i) The spring pushes **up** with F_0 and the weight pulls **down**: weight

Solution 3.6

$= 4.0 \times 9.81 = 39$ N **M1**; resultant $= 549 - 39 = 5.1 \times 10^2$ N, upwards **A1**.

(e)(ii) $a = \dfrac{F}{m} = \dfrac{510}{4.0}$ **M1** $= 1.3 \times 10^2$ m s^{-2}, upwards — about 13 times g **A1**.

(f) Axes labelled *kinetic energy / J* against *compression / m* **B1**; a curve starting at 72 J when the compression is zero and falling to zero at $x_0 = 0.31$ m, getting steeper as the spring force grows **B1**.

(g) A greater drop height gives the block **more kinetic energy** at contact, so more energy must be stored in the spring; since $E = \frac{1}{2}kx^2$, a larger E needs a larger compression x_0 **B1**. The spring force is proportional to the compression, so F_0 is larger too, and $a = \dfrac{F_0 - mg}{m}$ therefore rises with it **B1**.