Exercise walk-through

5.2.1

Potential energy, kinetic energy and energy transfers

A-Level Physics

You must be able to

- derive, using $W = Fs$, the formula $\Delta E_{\mathrm{P}} = mg\Delta h$ for gravitational potential energy changes in a uniform gravitational field
- recall and use the formula $\Delta E_{\mathrm{P}} = mg\Delta h$ for gravitational potential energy changes in a uniform gravitational field, whatever path the object takes
- derive, using the equations of motion, the formula for kinetic energy $E_{\mathrm{K}} = \frac{1}{2}mv^2$, and the change in kinetic energy when a force acts
- recall and use $E_{\mathrm{K}} = \frac{1}{2}mv^2$, including ratios of kinetic energies and energy changes with and without air resistance

1 The method

Method — Deriving the change in gravitational potential energy

To lift an object of mass m at a steady speed, an upward force equal to its weight, $F = mg$, must act on it. As the object rises through a height Δh, this force does work

$$W = Fs = mg\Delta h$$

and this work is stored as gravitational potential energy: $\Delta E_{\mathrm{P}} = mg\Delta h$.

- The derivation needs a **uniform** gravitational field, where g is the same at every height. This is true near the Earth's surface, but not for a rocket that rises hundreds of kilometres.
- Only the vertical height matters, not the path taken.

A bag of mass 12 kg is raised through 1.5 m.

1. Lifted straight up: $\Delta E_{\mathrm{P}} = 12 \times 9.81 \times 1.5 = 177$ J.

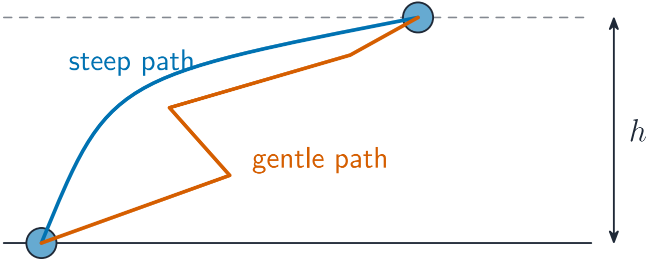
Method — Deriving the change in gravitational potential energy

2. Pushed without friction up a ramp 4.0 m long that rises 1.5 m: the force needed is the component of the weight along the ramp, $12 \times 9.81 \times \dfrac{1.5}{4.0} = 44.1$ N.
3. The work done on the ramp is $44.1 \times 4.0 = 177$ J: a smaller force over a longer distance gives the same energy.

Method — Deriving the change in gravitational potential energy

To lift an object of mass … at a steady speed, an upward force equal to its weight, … , must act on it.

same $\Delta E_P = mgh$ – only the height h matters



Method — Deriving the kinetic energy

A resultant force F acts on a mass m over a displacement s, and its speed increases from u to v. Using $F = ma$ and the equation of motion $v^2 = u^2 + 2as$:

$$Fs = ma \times \frac{v^2 - u^2}{2a} = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

- Starting from rest $(u = 0)$, all the work done becomes kinetic energy: $E_\mathrm{K} = \frac{1}{2}mv^2$.
- The work done by the resultant force equals the **change** in kinetic energy.

A car of mass 900 kg speeds up from 12 m s^{-1} to 20 m s^{-1} over a distance of 80 m.

1. $\Delta E_\mathrm{K} = \frac{1}{2} \times 900 \times (20^2 - 12^2) = 1.152 \times 10^5$ J.
2. The resultant force is $F = \dfrac{\Delta E_\mathrm{K}}{s} = \dfrac{1.152 \times 10^5}{80} = 1440$ N.

Method — Deriving the kinetic energy

3. Check with the equations of motion: $a = \dfrac{20^2 - 12^2}{2 \times 80} = 1.6$ m s^{-2}, and $F = 900 \times 1.6 = 1440$ N.

- Take care: $\frac{1}{2}m(v^2 - u^2)$ is **not** the same as $\frac{1}{2}m(v - u)^2$.

Method — Comparing kinetic energies

- $E_K \propto v^2$: tripling the speed multiplies the kinetic energy by 9.
- At the same speed, $E_K \propto m$.
- Using momentum $p = mv$ (topic 3): $E_K = \frac{1}{2}mv^2 = \dfrac{p^2}{2m}$.

A car of mass 1000 kg moves at 20 m s^{-1}. A truck of mass 4000 kg has the same momentum.

1. The car has $p = 1000 \times 20 = 2.0 \times 10^4$ kg m s^{-1} and $E_K = \frac{1}{2} \times 1000 \times 20^2 = 2.0 \times 10^5$ J.
2. The truck moves at $v = 2.0 \times 10^4/4000 = 5.0$ m s^{-1}, so $E_K = \frac{1}{2} \times 4000 \times 5.0^2 = 5.0 \times 10^4$ J.
3. The same momentum with four times the mass gives one quarter of the kinetic energy, as $E_K = p^2/2m$ predicts.

Method — Finding a speed from energy

A stone is thrown horizontally at 8.0 m s^{-1} from the top of a cliff 20 m high. Air resistance is negligible.

1. Energy is conserved: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mg\Delta h$. The mass cancels.
2. $v = \sqrt{u^2 + 2g\Delta h} = \sqrt{8.0^2 + 2 \times 9.81 \times 20} = 21.4$ m s^{-1}.

- Energy gives the **speed** at the bottom whatever the direction of the throw. For the direction of the velocity, or the time taken, use the equations of motion instead.

Method — Energy against height, and air resistance

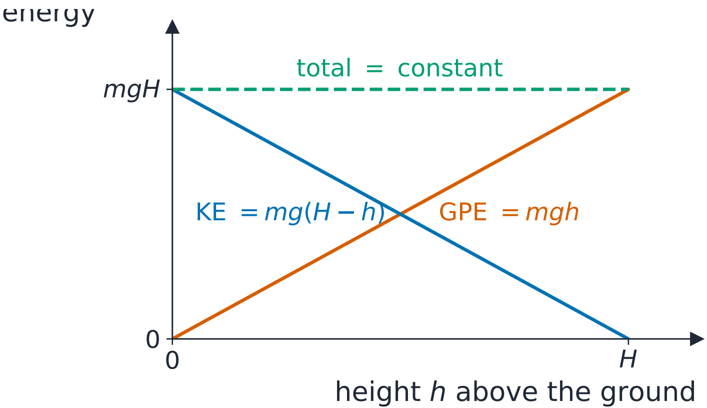
When a ball falls with no air resistance, it loses gravitational potential energy and gains the same amount of kinetic energy, so the total stays constant:
A ball of mass 0.20 kg is dropped from a height of 5.0 m and reaches the ground with 8.0 J of kinetic energy.

1. Its gravitational potential energy at the start is $0.20 \times 9.81 \times 5.0 = 9.81$ J.
2. With no air resistance it would land with 9.81 J of kinetic energy, so $9.81 - 8.0 = 1.8$ J was transferred to thermal energy.
3. The average air resistance was $\dfrac{1.81}{5.0} = 0.36$ N.

- With air resistance the total energy of the ball is not constant: the lower the ball, the less total energy it has left.

Method — Energy against height, and air resistance

When a ball falls with no air resistance, it loses gravitational potential energy and gains the same amount...



2 Practice

Practice 2.1 ▪ 2 marks

A cable car of total mass 1200 kg climbs through a vertical height of 18 m. Calculate the gain in its gravitational potential energy.

Solution 2.1

Method: Deriving the change in gravitational potential energy — p.4

Worked steps

- $\Delta E_{\mathrm{P}} = mg\Delta h$ M1

$$\Delta E_{\mathrm{P}} = 1200 \times 9.81 \times 18 = \mathbf{2.12} \times \mathbf{10^{5}}\ \mathrm{J}$$ A1

Practice 2.2 ▪ 2 marks

A tennis ball of mass 0.058 kg is served at 45 m s^{-1}. Calculate its kinetic energy.

Solution 2.2

Method: Energy against height, and air resistance — p.11

Worked steps

- $E_K = \frac{1}{2}mv^2$ **M1**

$$E_K = \frac{1}{2} \times 0.058 \times 45^2 = \mathbf{59}\ \text{J}$$ **A1**

Practice 2.3 ▪ 3 marks

A car of mass 1100 kg speeds up from 20 m s^{-1} to 30 m s^{-1}.

Practice 2.3 (a) ▪ 1 mark

State the factor by which its kinetic energy increases.

Solution 2.3 (a)

Method: Deriving the kinetic energy — p.7

Worked steps

$E_K \propto v^2$, so the factor is $(30/20)^2 = \mathbf{2.25}$ **B1**

Practice 2.3 (b) ▪ 2 marks

Calculate the increase in its kinetic energy.

Solution 2.3 (b)

Worked steps

$\Delta E_{\mathrm{K}} = \frac{1}{2} \times 1100 \times (30^2 - 20^2)$ **M1**

- $\Delta E_{\mathrm{K}} = \mathbf{2.75 \times 10^5}$ J **A1**

Practice 2.4 ▪ 2 marks

Water moves very slowly over the top of a waterfall and falls through 32 m. Ignoring air resistance, calculate its speed at the bottom.

Solution 2.4

Worked steps

- $\frac{1}{2}mv^2 = mg\Delta h$ **M1**

$$v = \sqrt{2 \times 9.81 \times 32} = \mathbf{25}\ \text{m s}^{-1}$$ **A1**

Practice 2.5 ▪ 1 mark

State the condition for the formula $\Delta E_{\text{P}} = mg\Delta h$ to be used.

Solution 2.5

Worked steps

- the gravitational field must be uniform: g does not change with height, as near the Earth's surface for small changes in height **B1**

3 Exam-style

Exam-style 3.1 ▪ 1 mark

Two objects X and Y have the same momentum. The mass of X is twice the mass of Y. What is the ratio $\dfrac{\text{kinetic energy of X}}{\text{kinetic energy of Y}}$?

A $\frac{1}{4}$

B $\frac{1}{2}$

C 2

D 4

Solution 3.1

Method: Comparing kinetic energies — p.9

A $\frac{1}{4}$

B $\frac{1}{2}$ ✓

C 2

D 4

Worked steps

B

- $E_K = p^2/2m$, so with the same momentum $E_K \propto 1/m$ and X, with twice the mass, has half the kinetic energy **B1**

Exam-style 3.2 ▪ 1 mark

A ball dropped from rest from a height h reaches a speed v. Air resistance is negligible. From what height must the ball be dropped to reach a speed $2v$?

A $\sqrt{2}\,h$

B $2h$

C $4h$

D $8h$

Solution 3.2

Method: Energy against height, and air resistance — p.11

A $\sqrt{2}\,h$

B $2h$

C $4h$ ✓

D $8h$

Worked steps

C

- $\frac{1}{2}mv^2 = mgh$, so $h \propto v^2$ and doubling v needs four times the height **B1**

Exam-style 3.3 ▪ 8 marks

A footballer kicks a ball of mass 0.45 kg from rest. The ball leaves the foot at 26 m s^{-1}.

Exam-style 3.3 (a) ▪ 3 marks

Use the equations of motion to show that the kinetic energy of an object of mass m, accelerated from rest to a speed v, is $\frac{1}{2}mv^2$.

Solution 3.3 (a)

Method: Deriving the kinetic energy — p.7

Worked steps

the work done by the force is $W = Fs$ with $F = ma$ **M1**

- from rest $v^2 = 2as$, so $s = v^2/2a$ **M1**
- $W = ma \times \dfrac{v^2}{2a} = \frac{1}{2}mv^2$, and this work becomes the kinetic energy **A1**

Exam-style 3.3 (b) ▪ 2 marks

The foot pushes on the ball over a distance of 0.20 m. Calculate the average force on the ball.

Solution 3.3 (b)

Worked steps

$E_K = \frac{1}{2} \times 0.45 \times 26^2 = 152$ J **M1**

- $F = \frac{E_K}{s} = \frac{152}{0.20} = \mathbf{760}$ N **A1**

Exam-style 3.3 (c) ▪ 3 marks

At the highest point of its path the ball moves horizontally at 15 m s^{-1}. Ignoring air resistance, calculate the height of this point above the point where the ball was kicked.

Solution 3.3 (c)

Worked steps

kinetic energy at the top $= \frac{1}{2} \times 0.45 \times 15^2 = 50.6$ J, so the gain in potential energy is $152.1 - 50.6 = 101.5$ J **M1**

- $mg\Delta h = 101.5$ **M1**

$$\Delta h = \frac{101.5}{0.45 \times 9.81} = \mathbf{23} \text{ m}$$ **A1**

Exam-style 3.4 ▪ 8 marks

A pole vaulter of mass 70 kg sprints along the runway at 9.0 m s^{-1}, then uses the pole to change her kinetic energy into gravitational potential energy.

Exam-style 3.4 (a) ▪ 2 marks

Calculate her kinetic energy just before she plants the pole.

Solution 3.4 (a)

Method: Deriving the change in gravitational potential energy — p.4

Worked steps

$E_K = \frac{1}{2} \times 70 \times 9.0^2$ **M1**

- $E_K = \mathbf{2.8 \times 10^3}$ J **A1**

Exam-style 3.4 (b) ▪ 2 marks

Assuming that all of this energy becomes gravitational potential energy, calculate how far her centre of gravity could rise.

Solution 3.4 (b)

Worked steps

$mg\Delta h = 2835$ J M1

$$\Delta h = \frac{2835}{70 \times 9.81} = \mathbf{4.1} \text{ m}$$ A1

Exam-style 3.4 (c) ▪ 2 marks

Her centre of gravity starts 1.0 m above the ground, and she only just clears a bar at 4.8 m. Suggest two reasons why she does not reach the height predicted by (b).

Solution 3.4 (c)

Worked steps

she still has kinetic energy at the top, because she must keep moving horizontally over the bar **B1**

- some energy is transferred to thermal energy and sound in the pole and in her body, and by air resistance **B1**

Exam-style 3.4 (d) ▪ 2 marks

After clearing the bar, she falls from rest at 4.8 m onto a landing mat whose top surface is 0.80 m above the ground. Treating her as a point mass, calculate her speed as she reaches the mat.

Solution 3.4 (d)

Worked steps

$\frac{1}{2}mv^2 = mg\Delta h$ with $\Delta h = 4.8 - 0.80 = 4.0$ m **M1**

$$v = \sqrt{2 \times 9.81 \times 4.0} = \mathbf{8.9}\ \text{m s}^{-1}$$ **A1**

Exam-style 3.5 ▪ 7 marks

A hiker of mass 60 kg walks 3.2 km along a mountain path, from a height of 1200 m to a height of 1650 m.

Exam-style 3.5 (a) ▪ 2 marks

Starting from the work done by a force, derive the formula for the change in gravitational potential energy near the Earth's surface.

Solution 3.5 (a)

Method: Deriving the change in gravitational potential energy — p.4

Worked steps

to raise the mass at a steady speed, a force equal to its weight mg acts through the vertical distance Δh **M1**

- the work done $W = Fs = mg\Delta h$ is stored as gravitational potential energy, so $\Delta E_P = mg\Delta h$ **A1**

Exam-style 3.5 (b) ▪ 1 mark

Explain why this formula cannot be used for a rocket that rises 2000 km above the Earth.

Solution 3.5 (b)

Worked steps

g decreases as the height increases, so the field is not uniform over 2000 km

B1

Exam-style 3.5 (c) ▪ 2 marks

Calculate the gain in gravitational potential energy of the hiker.

Solution 3.5 (c)

Worked steps

$\Delta h = 1650 - 1200 = 450$ m M1

$$\Delta E_{\mathrm{P}} = 60 \times 9.81 \times 450 = \mathbf{2.6 \times 10^5}\ \mathrm{J}$$ A1

Exam-style 3.5 (d) ▪ 2 marks

The hiker's muscles transfer food energy into useful work with an efficiency of 25%. Calculate the food energy used to gain this potential energy.

Solution 3.5 (d)

Worked steps

food energy = useful work ÷ efficiency $= 2.65 \times 10^5/0.25$ M1

- food energy $= \mathbf{1.1} \times \mathbf{10^6}$ J A1