Exercise walk-through

Energy conservation ▪ 5.1

eXercise

You must be able to

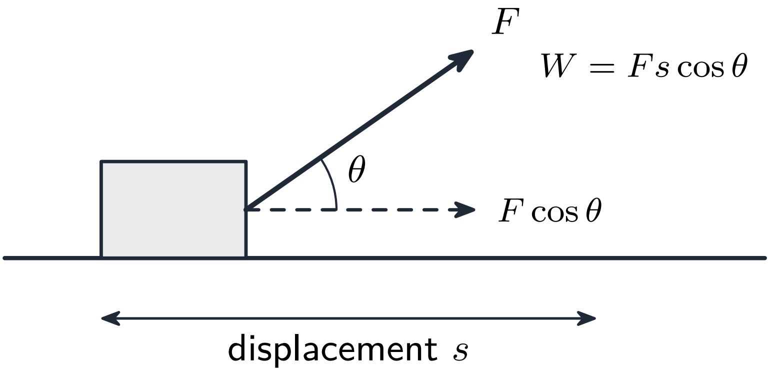
- use **work done** $= Fs\cos\theta$
- apply **conservation of energy** (e.g. GPE $\rightarrow$ KE)
- use **efficiency** and **power** ($P = W/t$ and $P = Fv$)

Method — Work done

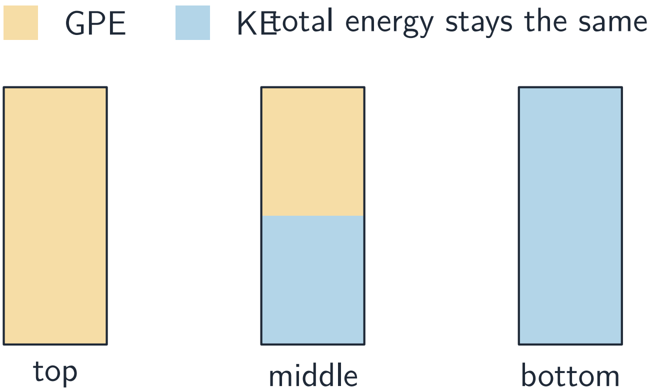
Work is done when a force moves along its own line: $W = Fs\cos\theta$. Only the component **along** the displacement does work:

- $\theta = 0$: $W = Fs$ (max); $\theta = 90°$: $W = 0$ (e.g. the normal force on a flat road); $\theta = 180°$: $W = -Fs$ (e.g. friction).

Method — Work done



Method — Work done



Energy is conserved as it transfers

Method — Conservation of energy

Energy is never made or destroyed. On a **smooth** ramp, GPE $\rightarrow$ KE:

$$mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}.$$

Method — Efficiency and power

$$\text{efficiency} = \frac{\text{useful energy out}}{\text{total energy in}}, \qquad P = \frac{W}{t}, \qquad P = Fv.$$

$P = Fv$ comes from $P = W/t = Fs/t = F(s/t) = Fv$.

Method — Efficiency, and where the wasted energy goes

Efficiency compares what you **want** with what you **paid for**, as energies or as powers:

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} = \frac{\text{useful power output}}{\text{total power input}}$$

It has **no unit**, and it can be written as a decimal or a percentage — but never above 1. Two habits win the marks:

- **Work out the useful output first**, from what the machine actually achieves (mgh for a lift, Fs for a drag, $\frac{1}{2}mv^2$ for a launch).
- **The wasted energy is input minus useful output**, and the exam wants to know *where it went*: thermal energy in the motor windings and bearings, sound, and in a lift the kinetic energy left over if it does not stop.

Method — Efficiency, and where the wasted energy goes

Worked example. A pump raises 600 kg of water through 12 m in 30 s, drawing 3.0 kW.

$$P_{\text{useful}} = \frac{mgh}{t} = \frac{600 \times 9.81 \times 12}{30} = 2354 \text{ W}, \qquad \text{efficiency} = \frac{2354}{3000} = 0.78$$

Practice 2.1 ▪ 1 mark

Define the **work done** by a force.

Solution 2.1

Method: Work done

Worked steps

Work $=$ force $\times$ displacement **in the direction of the force** $(W = Fs\cos\theta)$ **B1**.

Practice 2.2 ▪ 2 marks

A child pulls a sledge 5.0 m with a 20 N force at $60°$ to the ground. Find the **work done**.

Solution 2.2

Method: Work done

Worked steps

$W = Fs\cos\theta = 20 \times 5.0 \times \cos 60^\circ = 50$ J **M1** **A1**.

Practice 2.3 ▪ 1 mark

Define **power** and give its unit.

Solution 2.3

Worked steps

Power is the **work done (energy transferred) per unit time**, unit **watt** ($W = J\,s^{-1}$) **B1**.

Practice 2.4 ▪ 1 mark

State the **principle of conservation of energy**.

Solution 2.4

Worked steps

Energy cannot be **created or destroyed**, only **transferred** from one form to another; the total energy of a closed system is constant **B1**.

Exam-style 3.1 ▪ 1 mark

A force acts at 90° to the direction of motion. The work it does is:

A maximum

B Fs

C zero

D negative

Solution 3.1

Method: Work done

A maximum

B Fs

C zero ✓

D negative

Worked steps

C — zero.

Exam-style 3.2 ▪ 3 marks

A ball is released from rest at the top of a **smooth** ramp 1.2 m high. Calculate its speed at the bottom. ($g = 9.81$ m s^{-2}.)

Solution 3.2

Method: Conservation of energy

Worked steps

$mgh = \frac{1}{2}mv^2$ **M1**; $v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1.2}$ **M1** $= \sqrt{23.5} = 4.9\ \text{m s}^{-1}$ **A1**.

Exam-style 3.3 ▪ 2 marks

A motor lifts a 50 kg load through 8.0 m in 10 s.
(a) Calculate the **useful power** output.
(b) If the input power is 500 W, calculate the **efficiency**.

Solution 3.3

Method: Efficiency, and where the wasted energy goes

Worked steps

(a) useful work $= mgh = 50 \times 9.81 \times 8.0 = 3924$ J; $P = W/t = 3924/10 = 392$ W M1 A1. (b) efficiency $= 392/500 = 0.78 = 78\%$ M1 A1.

Exam-style 3.4 ▪ 2 marks

Derive $P = Fv$ from $P = W/t$.

Solution 3.4

Worked steps

$P = W/t = Fs/t$ **M1**; since $s/t = v$, $P = Fv$ **A1**.

Exam-style 3.5 ▪ 1 mark

A crane is used on a building site.

(a) **Define** the **work done** by a force.

(b) A block of mass m is raised vertically at constant speed through a height Δh. **Derive** an expression, in terms of m and Δh, for the change in gravitational potential energy ΔE_P of the block. State the meaning of any other symbol you use.

(c) The crane raises a steel beam of mass 450 kg at constant speed through a vertical height of 24 m, taking 40 s.

(i) **Calculate** the work done on the beam.

(ii) **Calculate** the useful output power of the crane.

(iii) The input power to the crane is 3.6 kW. **Calculate** its efficiency.

(iv) **Determine** the energy wasted during the 40 s, and **state** one form it takes.

Solution 3.5

Method: Efficiency, and where the wasted energy goes

Worked steps

(a) The product of the force and the **displacement in the direction of the force** **B1**.

(b) Raising the block at constant speed needs an upward force equal to its weight, mg, where g is the acceleration of free fall **B1**; the gain in potential energy is the work done by that force, so $\Delta E_\mathrm{P} = mg\Delta h$ **B1**.

(c)(i) $W = mg\Delta h = 450 \times 9.81 \times 24$ **M1** $= 1.1 \times 10^5$ J **A1**.

(c)(ii) $P = \dfrac{W}{t} = \dfrac{1.06 \times 10^5}{40}$ **M1** $= 2.6 \times 10^3$ W **A1**.

Solution 3.5

(c)(iii) efficiency $= \dfrac{P_{\text{useful}}}{P_{\text{input}}} = \dfrac{2650}{3600}$ **M1** $= 0.74$, i.e. 74% **A1**.

(c)(iv) input energy $= 3600 \times 40 = 1.44 \times 10^5$ J, so wasted $= 1.44 \times 10^5 - 1.06 \times 10^5 = 3.8 \times 10^4$ J **M1**; it becomes **thermal energy** in the motor windings, gearbox and cable (also some sound) **A1**.