Exercise walk-through

4.3.1

Pressure in fluids, upthrust and floating

A-Level Physics

You must be able to

- define and use **density 密度** as mass per unit volume, $\rho = m/V$
- define and use pressure as force per unit area, $p = F/A$, including the largest and smallest pressure a block can exert
- derive, from the definitions of pressure and density, the equation for **hydrostatic pressure 流体静压强** $\Delta p = \rho g \Delta h$, also for two liquids in layers
- use the equation $\Delta p = \rho g \Delta h$ for the total pressure at a depth, for liquids in layers and for the levels in a U-tube
- understand that the upthrust acting on an object in a fluid is due to a difference in hydrostatic pressure between its bottom and top surfaces
- calculate the upthrust acting on an object in a fluid using the equation $F = \rho g V$ (**Archimedes' principle 阿基米德原理**), from the readings of a **newton meter 弹簧测力计** and for floating objects

1 The method

Method — Pressure under a solid

Pressure is the force acting at right angles to a surface divided by the area: $p = F/A$, in pascals, where 1 Pa $= 1$ N m^{-2}.

A woman of mass 60 kg stands on the ground. Her weight is $W = mg = 60 \times 9.81 = 588.6$ N.

1. Convert the area to m^2. In flat shoes each sole has an area of 150 cm^2, so the two soles together have 300 cm$^2 = 300 \times 10^{-4}$ m$^2 = 0.0300$ m^2.
2. Divide the force by the area:

$$p = \frac{588.6 \text{ N}}{0.0300 \text{ m}^2} = 1.96 \times 10^4 \text{ Pa}$$

3. If she stands with all her weight on two heels of area 1.0 cm^2 each, the area is 2.0×10^{-4} m^2 and $p = 588.6/(2.0 \times 10^{-4}) = 2.9 \times 10^6$ Pa: 150 times larger, for the same weight.

Method — Pressure under a solid

A block standing on one face. Density is mass per unit volume, $\rho = m/V$. A uniform block of density ρ and height h above the face it stands on has weight $W = mg = \rho Vg = \rho Ahg$, so

$$p = \frac{W}{A} = \frac{\rho Ahg}{A} = \rho gh$$

- The area cancels. The block presses hardest when it stands on its **smallest** face, because then its height h is largest.
- A brick of density 1900 kg m^{-3} measuring 20 cm $\times$ 10 cm $\times$ 6.5 cm gives $1900 \times 9.81 \times 0.20 = 3.7$ kPa standing on its end, and $1900 \times 9.81 \times 0.065 = 1.2$ kPa lying flat.

Method — Pressure at a depth, and total pressure

The same argument for a column of liquid of depth Δh gives the hydrostatic pressure $\Delta p = \rho g \Delta h$. It depends only on the depth and on the density of the liquid.
A diver is 12 m below the surface of the sea. The density of sea water is 1030 kg m^{-3}, and the **atmospheric pressure 大气压强** at the surface is 1.01×10^5 Pa.

1. The pressure due to the water is $\Delta p = 1030 \times 9.81 \times 12 = 1.21 \times 10^5$ Pa.
2. The air presses on the surface and the water passes this pressure on, so the **total pressure** is

$$p = 1.01 \times 10^5 + 1.21 \times 10^5 = 2.22 \times 10^5 \text{ Pa}$$

3. Read the question carefully: "the pressure due to the water" means step 1 only, while "the pressure at this depth" means the total.

Method — Two liquids in layers

A tank holds 0.30 m of oil on top of 0.50 m of water. The oil floats on the water because it is less dense.

1. Take a column of area A down to the bottom. Its weight is the weight of the oil plus the weight of the water:

$$W = \rho_{\text{oil}} A h_{\text{oil}} g + \rho_{\text{water}} A h_{\text{water}} g$$

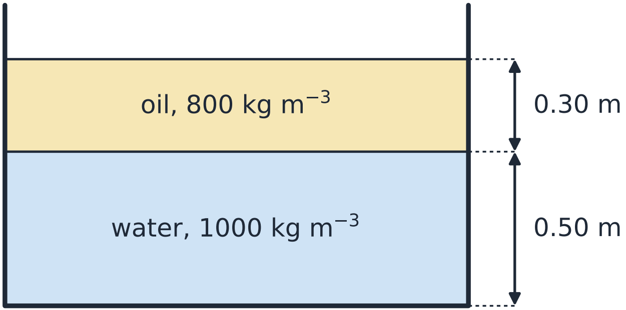
2. Divide by A. The pressures of the two layers **add**:

$$\Delta p = \rho_{\text{oil}}\, g\, h_{\text{oil}} + \rho_{\text{water}}\, g\, h_{\text{water}} = 800 \times 9.81 \times 0.30 + 1000 \times 9.81 \times 0.50$$

3. $\Delta p = 2354 + 4905 = 7.26 \times 10^3$ Pa.

Method — Two liquids in layers

A tank holds … of oil on top of … of water.



Method — Two liquids in a U-tube

Oil is poured into one arm of a U-tube that contains water. The two liquids do not mix.

1. Choose the level of the **oil–water boundary**. Below this level both arms hold only water, so the pressure at this level is the **same in both arms**.
2. Above this level the left arm holds 9.6 cm of water and the right arm holds 12.0 cm of oil. Both tops are open to the air, so atmospheric pressure cancels:

$$\rho_{\text{water}}\, g\, h_{\text{water}} = \rho_{\text{oil}}\, g\, h_{\text{oil}}$$

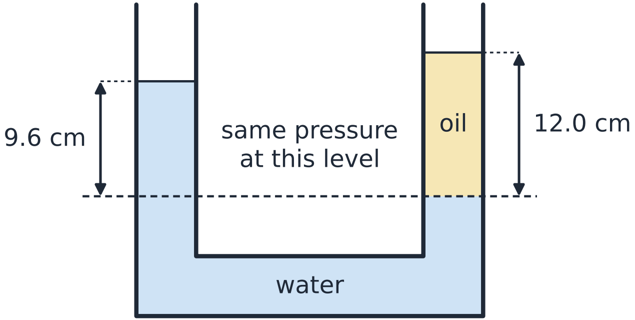
3. g cancels, and the heights can stay in cm because they appear on both sides:

$$\rho_{\text{oil}} = \frac{1000 \times 9.6}{12.0} = 800 \text{ kg m}^{-3}$$

- "Same level, same pressure" only works when the two points are joined by one liquid. A level inside the oil column does not work, because at that level the left arm holds water.

Method — Two liquids in a U-tube

Oil is poured into one arm of a U-tube that contains water.



Method — Where upthrust comes from

A cuboid is held under water. Its top and bottom faces each have an area $A = 0.020$ m^2. The top face is 0.20 m deep and the bottom face is 0.35 m deep.

1. Down on the top face: $p_1 = 1000 \times 9.81 \times 0.20 = 1962$ Pa, so $F_1 = p_1 A = 39.2$ N.
2. Up on the bottom face, which is deeper: $p_2 = 1000 \times 9.81 \times 0.35 = 3434$ Pa, so $F_2 = p_2 A = 68.7$ N.
3. The forces on the sides are equal and opposite, so they cancel. The resultant force of the water is **upwards**:

$$F = F_2 - F_1 = 68.7 - 39.2 = 29.4 \text{ N}$$

This is the upthrust. It exists because the pressure on the bottom face is larger than the pressure on the top face.

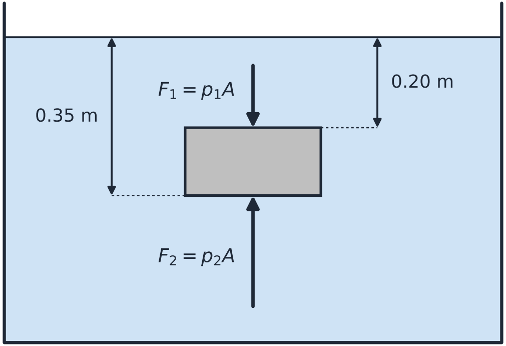
Method — Where upthrust comes from

4 In symbols, $F = (p_2 - p_1)A = \rho g(h_2 - h_1)A = \rho g V$, where $V = 0.020 \times 0.15 = 3.0 \times 10^{-3}$ m^3 is the volume of water **displaced 排开**.
Check: $1000 \times 9.81 \times 3.0 \times 10^{-3} = 29.4$ N.

- Atmospheric pressure adds the same amount to p_1 and to p_2, so it does not change the upthrust.
- The upthrust depends only on the fluid and on the volume displaced: not on the depth, and not on what the cuboid is made of.

Method — Where upthrust comes from

A cuboid is held under water.



Method — Weighing an object in a liquid

A metal block hangs from a newton meter. The meter reads 5.4 N in air and 3.4 N when the block is fully **submerged 浸没** in water.

1. In the water three forces act on the block: the pull T of the meter and the upthrust F upwards, and the weight W downwards. The block is in **equilibrium 平衡**, so $T + F = W$.
2. The upthrust is the drop in the reading: $F = 5.4 - 3.4 = 2.0$ N.
3. The volume of the block comes from $F = \rho g V$ for the water:

$$V = \frac{2.0}{1000 \times 9.81} = 2.04 \times 10^{-4}\ \mathrm{m}^3$$

4. The mass is $m = 5.4/9.81 = 0.550$ kg, so the density of the metal is

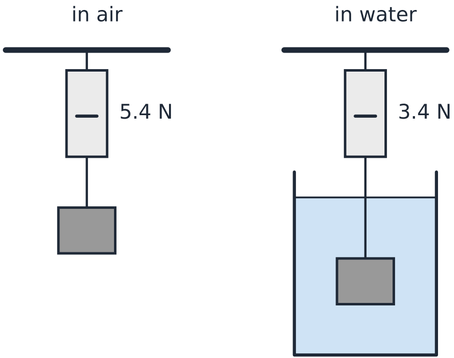
$$\rho = \frac{0.550}{2.04 \times 10^{-4}} = 2700\ \mathrm{kg\ m}^{-3}$$

Method — Weighing an object in a liquid

- Shortcut: the block and the water it displaces have the same volume, so $\rho_{\text{metal}}/\rho_{\text{water}} = W/F = 5.4/2.0 = 2.7$.

Method — Weighing an object in a liquid

A metal block hangs from a newton meter.



Method — Floating with a load

An object floats at the depth where the upthrust equals its weight. A raft is made of foam of density 40 kg m^{-3}. It is 0.20 m thick with a top area of 3.0 m^2, so its volume is 0.60 m^3.

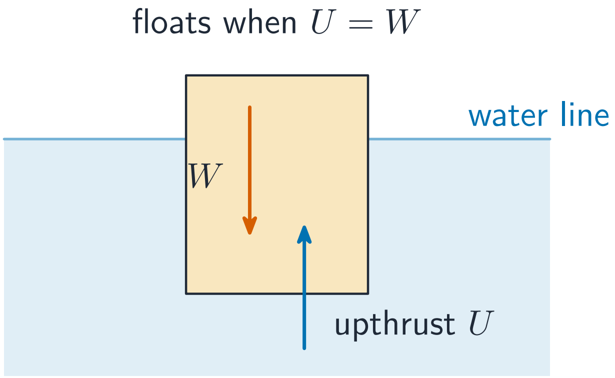
1. The mass of the raft is $40 \times 0.60 = 24$ kg. When it is **floating 漂浮**, the water displaced has the same mass, 24 kg, so its volume is $24/1000 = 0.024$ m^3.
2. The raft sinks to a depth of $0.024/3.0 = 0.0080$ m, only 8.0 mm.
3. The upthrust is largest when the raft is just fully under the water:

$$F = 1000 \times 9.81 \times 0.60 = 5886 \text{ N}$$

4. The raft's own weight is $24 \times 9.81 = 235$ N, so the largest load it can carry is $5886 - 235 = 5651$ N, a mass of $5651/9.81 = 576$ kg.

Method — Floating with a load

An object floats at the depth where the upthrust equals its weight.



2

Practice

Practice 2.1 ▪ 2 marks

A gas cylinder of weight 540 N stands on its base, which has an area of 150 cm^2. Calculate the pressure it exerts on the floor.

Solution 2.1

Method: Pressure under a solid — p.4

Worked steps

- area $= 150 \times 10^{-4} = 0.0150\ \mathrm{m}^2$ M1

$$p = \frac{540\ \mathrm{N}}{0.0150\ \mathrm{m}^2} = \mathbf{3.6 \times 10^4}\ \mathrm{Pa}$$ A1

Practice 2.2 ▪ 3 marks

A diver's gauge shows a total pressure of 3.00×10^5 Pa. The atmospheric pressure is 1.01×10^5 Pa and the density of the sea water is 1030 kg m^{-3}. Calculate the depth of the diver.

Solution 2.2

Method: Pressure at a depth, and total pressure — p.6

Worked steps

- pressure due to the water $= 3.00 \times 10^5 - 1.01 \times 10^5 = 1.99 \times 10^5$ Pa M1
- $\Delta p = \rho g h$ M1

$$h = \frac{1.99 \times 10^5}{1030 \times 9.81} = \mathbf{19.7} \text{ m}$$ A1

Practice 2.3 ▪ 2 marks

A jar holds a 5.0 cm layer of syrup of density 1400 kg m^{-3}, with a 3.0 cm layer of oil of density 920 kg m^{-3} floating on it. Calculate the pressure due to the two liquids at the bottom of the jar.

Solution 2.3

Method: Two liquids in layers — p.7

Worked steps

- the pressures of the layers add:
  $\Delta p = 1400 \times 9.81 \times 0.050 + 920 \times 9.81 \times 0.030$ M1
- $\Delta p = 687 + 271 = \mathbf{960}$ Pa A1

Practice 2.4 ▪ 2 marks

A difference in pressure holds up a column of water 25.0 cm high. Calculate the height of the column of mercury, of density $13\,600$ kg m^{-3}, that the same difference in pressure would hold up.

Solution 2.4

Method: Two liquids in a U-tube — p.9

Worked steps

- the same Δp means $\rho_{\text{water}}\, h_{\text{water}} = \rho_{\text{mercury}}\, h_{\text{mercury}}$ **M1**

$$h = \frac{1000 \times 25.0}{13\,600} = \mathbf{1.84}\ \text{cm}$$ **A1**

Practice 2.5 ▪ 3 marks

A stone hangs from a newton meter, which reads 12.0 N in air and 7.5 N when the stone is fully under water.

Practice 2.5 (a) ▪ 1 mark

State the upthrust on the stone.

Solution 2.5 (a)

Method: Weighing an object in a liquid — p.14

Worked steps

upthrust $= 12.0 - 7.5 = \mathbf{4.5}$ N B1

Practice 2.5 (b) ▪ 2 marks

Calculate the volume of the stone.

Solution 2.5 (b)

Worked steps

$F = \rho g V$ M1

$$V = \frac{4.5}{1000 \times 9.81} = \mathbf{4.6 \times 10^{-4}\ m^3}$$ A1

Practice 2.6 ▪ 2 marks

Explain why the upthrust on the stone in **2.5** does not change when the stone is lowered deeper into the water.

Solution 2.6

Method: Where upthrust comes from — p.11

Worked steps

- the pressure on the top and the pressure on the bottom of the stone both increase by the same amount **B1**
- so the difference in pressure, and so the upthrust, stays the same; the volume of water displaced does not change either **B1**

3 Exam-style

Exam-style 3.1 ▪ 1 mark

A metal cube is held at rest under water. The atmospheric pressure above the water increases. What happens to the upthrust on the cube?

A It increases, because the pressure on the bottom face increases.

B It decreases, because the pressure on the top face increases.

C It stays the same, because the pressure on every face increases by the same amount.

D It stays the same, because atmospheric pressure acts only on the surface of the water.

Solution 3.1

Method: Where upthrust comes from — p.11

A It increases, because the pressure on the bottom face increases.

B It decreases, because the pressure on the top face increases.

C It stays the same, because the pressure on every face increases by the same amount. ✓

D It stays the same, because atmospheric pressure acts only on the surface of the water.

Worked steps

C

- the extra atmospheric pressure is passed on through the water to every face, so p_1 and p_2 rise by the same amount and their difference does not change **B1**

Exam-style 3.2 ▪ 1 mark

A boat sails from a river into the sea. Sea water is denser than river water, and the mass of the boat does not change. Which statement is correct?

A The upthrust increases and the boat floats higher in the water.

B The upthrust stays the same and the boat floats higher in the water.

C The upthrust stays the same and the boat floats lower in the water.

D The upthrust decreases and the boat floats lower in the water.

Solution 3.2

Method: Where upthrust comes from — p.11

A The upthrust increases and the boat floats higher in the water.

B The upthrust stays the same and the boat floats higher in the water. ✓

C The upthrust stays the same and the boat floats lower in the water.

D The upthrust decreases and the boat floats lower in the water.

Worked steps

B

- a floating boat has upthrust equal to its weight, and the weight does not change, so the upthrust stays the same **B1**
- $F = \rho g V$ with a larger ρ needs a smaller volume V under the water, so the boat floats **higher**

Exam-style 3.3 ▪ 9 marks

A cylindrical buoy of radius 0.50 m and mass 380 kg floats upright in sea water of density 1030 kg m^{-3}. The buoy is 1.2 m tall.

Exam-style 3.3 (a) ▪ 3 marks

Show that the bottom face of the buoy is 0.47 m below the surface of the water.

Solution 3.3 (a)

Worked steps

floating: upthrust = weight, so $\rho g A d = mg$ **M1**

- $A = \pi r^2 = \pi \times 0.50^2 = 0.785\ \mathrm{m}^2$ **M1**

$$d = \frac{380}{1030 \times 0.785} = \mathbf{0.47}\ \mathrm{m}$$ **A1**

Exam-style 3.3 (b) ▪ 2 marks

Calculate the pressure due to the sea water at the bottom face of the buoy.

Solution 3.3 (b)

Worked steps

$\Delta p = \rho g h$ **M1**

$$\Delta p = 1030 \times 9.81 \times 0.47 = \mathbf{4.7 \times 10^3} \text{ Pa}$$ **A1**

Exam-style 3.3 (c) ▪ 2 marks

Use your answer to (b) to calculate the upward force of the water on the bottom face. Compare it with the weight of the buoy, and explain why the two are equal.

Solution 3.3 (c)

Worked steps

$F = pA = 4.75 \times 10^3 \times 0.785 = 3.73 \times 10^3$ N, and the weight is $380 \times 9.81 = 3.73 \times 10^3$ N, the same **B1**

- the top of the buoy is in the air, so this upward push on the bottom face is the whole upthrust, and a floating buoy is in equilibrium **B1**

Exam-style 3.3 (d) ▪ 2 marks

An instrument of mass 120 kg is fixed to the top of the buoy.
Calculate the new depth of the bottom face.

Solution 3.3 (d)

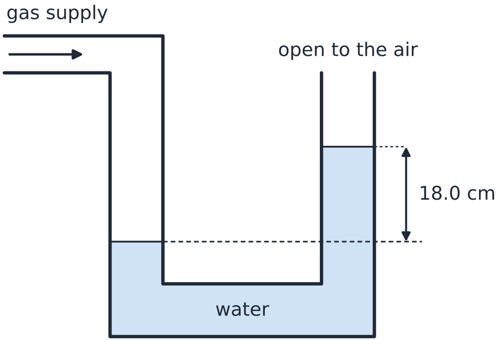
Worked steps

the total mass is 500 kg, so $d = \dfrac{500}{1030 \times 0.785}$ **M1**

- $d = \mathbf{0.62}$ m **A1**

Exam-style 3.4 ▪ 7 marks

A U-tube containing water is used to measure the pressure of a gas supply.

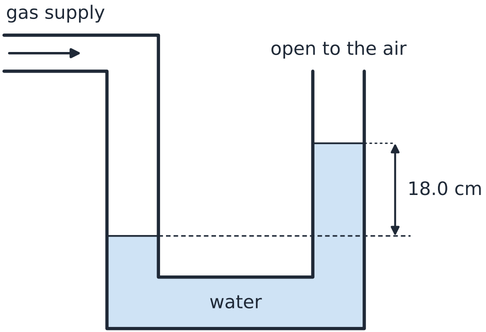


Exam-style 3.4 (a) ▪ 3 marks

Starting from the definitions of density and pressure, show that a column of water of height h exerts a pressure $\rho g h$ on the area below it.

Exam-style 3.4 (a) ▪ 3 marks

Starting from the definitions of density and pressure, show that a column of water of height ... exerts a...



Solution 3.4 (a)

Method: Two liquids in a U-tube — p.9

Worked steps

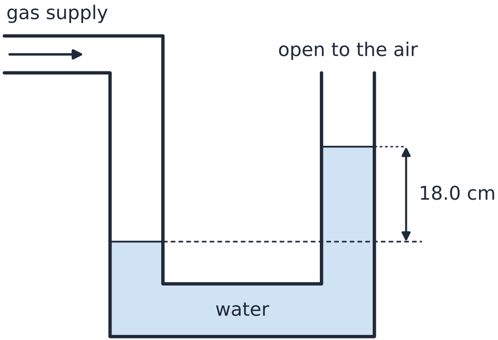
the mass of the column is $m = \rho V = \rho A h$ **M1**

- its weight $W = mg = \rho A h g$ acts on the area A **M1**

$$p = \frac{W}{A} = \frac{\rho A h g}{A} = \rho g h$$ **A1**

Exam-style 3.4 (b) ▪ 2 marks

The atmospheric pressure is 1.01×10^5 Pa. Calculate the pressure of the gas.



Solution 3.4 (b)

Worked steps

the gas pressure is larger than atmospheric pressure by the pressure of the 18.0 cm column: $\Delta p = 1000 \times 9.81 \times 0.180 = 1.77 \times 10^3$ Pa M1

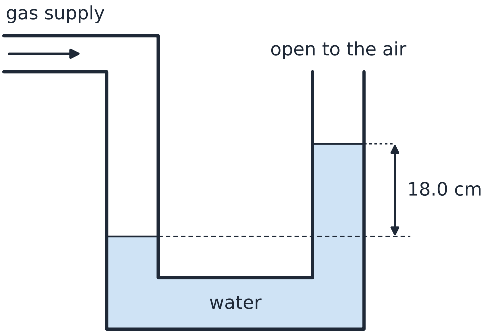
- $p = 1.01 \times 10^5 + 1.77 \times 10^3 = \mathbf{1.03 \times 10^5}$ Pa A1

Exam-style 3.4 (c) ▪ 2 marks

The water is replaced by oil of density 800 kg m^{-3}, joined to the same gas supply. Calculate the new difference between the two levels.

Exam-style 3.4 (c) ▪ 2 marks

Calculate the new difference between the two levels



Solution 3.4 (c)

Worked steps

the same Δp needs $h = \Delta p/(\rho g)$ M1

$$h = \frac{1.77 \times 10^3}{800 \times 9.81} = 0.225 \text{ m} = \mathbf{22.5} \text{ cm}$$ A1

Exam-style 3.5 ▪ 6 marks

A helium balloon of volume 0.012 m^3 is tied to the ground by a string. The mass of the balloon and the helium inside it is 4.0 g. The density of the air is 1.20 kg m^{-3}. Ignore the mass of the string.

Exam-style 3.5 (a) ▪ 2 marks

Calculate the upthrust of the air on the balloon.

Solution 3.5 (a)

Worked steps

upthrust $= \rho_{\text{air}}\, g\, V$ **M1**

$$F = 1.20 \times 9.81 \times 0.012 = \mathbf{0.14}\ \text{N}$$ **A1**

Exam-style 3.5 (b) ▪ 2 marks

Calculate the **tension 张力** in the string.

Solution 3.5 (b)

Worked steps

weight $= 0.0040 \times 9.81 = 0.039$ N **M1**

- the balloon is in equilibrium: $T = F - W = 0.141 - 0.039 = \mathbf{0.10}$ N **A1**

Exam-style 3.5 (c) ▪ 2 marks

On a mountain the density of the air is 0.90 kg m^{-3}. Assuming that the volume of the balloon does not change, calculate the tension in the string there.

Solution 3.5 (c)

Worked steps

$F = 0.90 \times 9.81 \times 0.012 = 0.106$ N M1

- $T = 0.106 - 0.039 = \mathbf{0.067}$ N A1