Exercise walk-through

4.1.1

Centre of gravity, moments at an angle, and couples

A-Level Physics

You must be able to

- understand that the **weight** of an object may be taken as acting at a single point, its centre of gravity
- define and apply the **moment of a force**
- understand that a couple is a pair of forces that produces **rotation only**
- define and apply the **torque of a couple**

1 The method

Method — The centre of gravity, and how to find it

The **centre of gravity** is the single point at which the **whole weight** of an object may be taken to act. For a **uniform** object it is at the geometric centre – the middle of a uniform rod, the centre of a uniform disc.

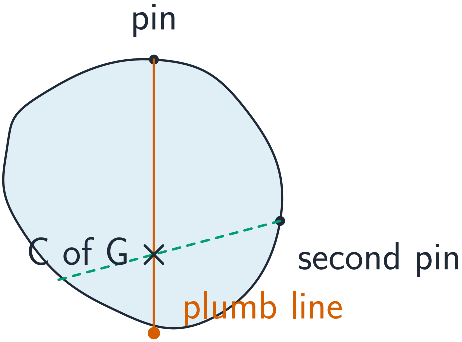
To find it for an irregular lamina:

1. Hang the lamina from a pin through a hole near its edge, so it can swing freely.
2. Hang a **plumb line** from the same pin and draw the vertical line on the lamina.
3. Repeat from a **second** hole well away from the first.
4. The centre of gravity is where the two lines **cross**. A third hole checks the result.

- It works because the object hangs in equilibrium only when its centre of gravity is **directly below the point of suspension** – otherwise the weight has a moment about the pin and it swings.

Method — The centre of gravity, and how to find it

To find it for an irregular lamina



hanging freely, the sheet settles with its centre of gravity vertically below the pin

hang it from a second point: the two plumb lines cross at the centre of gravity

Method — A moment when the force is not perpendicular

*The **moment of a force** about a point is the **force** multiplied by the **perpendicular distance from the point to the line of action of the force**.*

Every word of that definition is a marking point, and the phrase that is usually dropped is *line of action*. Two routes, both fully acceptable:

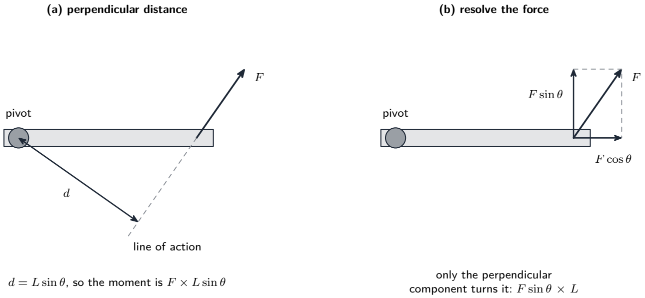
route	what to do
(a) perpendicular distance	extend the **line of action** of F, drop a perpendicular from the pivot onto it, and use $d = L\sin\theta$: moment $= FL\sin\theta$
(b) resolve	split F into $F\sin\theta$ **perpendicular** to the bar and $F\cos\theta$ **along** it; only the perpendicular part turns the bar: moment $= F\sin\theta \times L$

Method — A moment when the force is not perpendicular

- The component **along** the bar has a line of action **through the pivot**, so its perpendicular distance is zero and it contributes **nothing**. That is the whole reason $\cos\theta$ never appears in the answer.
- A moment is measured in **newton metres, N m**. It is not a joule, although the units look the same: a moment is a force times a perpendicular distance, not a force times a distance moved.

Method — A moment when the force is not perpendicular

quote The moment of a force about a point is the force multiplied by the perpendicular distance from the...



The two routes give the same answer, and either earns full marks. The component *along* the bar, $F\cos\theta$, has a line of action passing **through the pivot**, so its perpendicular distance is zero and it produces no turning effect at all.

Method — A couple, and its torque

A **couple** is a pair of forces that satisfies **three** conditions, all of which are needed:

1. the forces are **equal in magnitude**;
2. they are **opposite in direction**;
3. their **lines of action are parallel but not the same line** – they are separated.

Because the forces are equal and opposite, the **resultant force is zero**: the object does **not** accelerate. It rotates, and only rotates. That is what makes a couple different from any other pair of forces.

> *The **torque of a couple** is one **force** multiplied by the **perpendicular distance between the lines of action** of the two forces.*

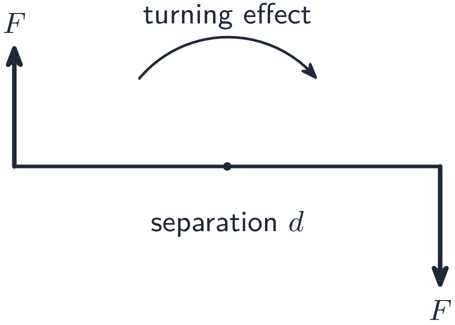
$$\tau = F \times d$$

Method — A couple, and its torque

- Use **one** of the forces, not both, and not their sum. Doubling because "there are two forces" is the standard error.
- d is the separation of the two lines, which for a wheel turned at opposite ends of a diameter is the **diameter**, not the radius.

Method — A couple, and its torque

A couple is a pair of forces that satisfies three conditions, all of which are needed: Because the forces are...



2 Practice

Practice 2.1 ▪ 3 marks

Describe how you would find the centre of gravity of an irregularly shaped flat sheet of card.

Solution 2.1

Method: The centre of gravity, and how to find it — p.4

Worked steps

- hang the card from a pin through a hole near its edge, so that it can swing **freely**, and hang a **plumb line** from the same pin **B1**
- mark the vertical line on the card, then repeat from a **second** hole well away from the first **B1**
- the centre of gravity is where the two lines **cross**; a third hole can be used as a check **B1**

Practice 2.2 ▪ 3 marks

A force of 40 N acts at the end of a spanner of length 0.25 m, at $60°$ to the spanner. Calculate its moment about the nut, using the **perpendicular distance**.

Solution 2.2

Method: A moment when the force is not perpendicular — p.6

Worked steps

- the perpendicular distance from the pivot to the line of action is $d = L\sin\theta$ M1

$$d = 0.25 \text{ m} \times \sin 60^\circ = 0.217 \text{ m}$$ M1

$$\text{moment} = Fd = 40 \text{ N} \times 0.217 \text{ m} = \mathbf{8.7} \text{ N m}$$ A1

Practice 2.3 ▪ 2 marks

Obtain the same answer by **resolving** the force, and state what this shows.

Solution 2.3

Worked steps

- the component of F perpendicular to the spanner is $F \sin\theta = 40\ \text{N} \times \sin 60^\circ = 34.6\ \text{N}$ **M1**

$$\text{moment} = 34.6\ \text{N} \times 0.25\ \text{m} = \mathbf{8.7}\ \text{N m}$$

- the two methods give the **same answer**, because both amount to $FL \sin\theta$ **A1**

Practice 2.4 ▪ 2 marks

Explain why the component of a force **along** a bar has no moment about a pivot at the end of that bar.

Solution 2.4

Method: A moment when the force is not perpendicular — p.6

Worked steps

- its **line of action passes through the pivot** **B1**
- so the perpendicular distance from the pivot to that line is **zero**, and the moment Fd is zero **B1**

Practice 2.5 ▪ 2 marks

Define the **torque of a couple**, and state its unit.

Solution 2.5

Method: A couple, and its torque — p.9

Worked steps

- the torque of a couple is **one of the forces** multiplied by the **perpendicular distance between the lines of action** of the two forces B1
- its unit is the **newton metre, N m** B1

3 Exam-style

Exam-style 3.1 ▪ 1 mark

A force F acts at a point a distance L from a pivot, at an angle θ to the line joining the pivot to that point. The moment of the force about the pivot is

A FL

B $FL\sin\theta$

C $FL\cos\theta$

D $FL\tan\theta$

Solution 3.1

Method: A moment when the force is not perpendicular — p.6

A FL

B $FL\sin\theta$ ✓

C $FL\cos\theta$

D $FL\tan\theta$

Worked steps

B

- the moment is the force times the **perpendicular distance to the line of action**, which is $L\sin\theta$ **B1**

Exam-style 3.2 ▪ 6 marks

A steering wheel of diameter 36 cm is turned by two forces of 15 N, applied tangentially at opposite ends of a diameter and in opposite directions.

Exam-style 3.2 (a) ▪ 2 marks

Explain why these two forces form a couple.

Solution 3.2 (a)

Method: A couple, and its torque — p.9

Worked steps

- the two forces are **equal in magnitude** (15 N each) and **opposite in direction** **B1**
- their lines of action are **parallel but separated**, so they produce a turning effect with no resultant force **B1**

Exam-style 3.2 (b) ▪ 2 marks

Calculate the torque of the couple.

Solution 3.2 (b)

Worked steps

- use one force and the separation of the lines of action, which here is the **diameter** M1

$$\tau = Fd = 15 \text{ N} \times 0.36 \text{ m} = \mathbf{5.4} \text{ N m}$$ A1

Exam-style 3.2 (c) ▪ 2 marks

State the resultant force on the wheel, and explain what this means for its motion.

Solution 3.2 (c)

Worked steps

- the resultant force is **zero**, because the forces are equal and opposite B1
- so the centre of the wheel does not accelerate: the wheel **rotates only**, without moving off its axis B1

Exam-style 3.3 ▪ 10 marks

A uniform plank of weight 220 N and length 3.0 m rests on a single pivot placed 1.2 m from its left-hand end. A child of weight 300 N stands on the left-hand end.

Exam-style 3.3 (a) ▪ 2 marks

State where the weight of the plank acts, and explain why.

Solution 3.3 (a)

Method: The centre of gravity, and how to find it — p.4

Worked steps

- the weight acts at the **centre of gravity**, which for a **uniform** plank is at its **midpoint**, 1.5 m from either end **B1**
- the plank is uniform, so its mass is evenly distributed and the whole weight may be taken to act at that single point **B1**

Exam-style 3.3 (b) ▪ 2 marks

Calculate the moment of the child's weight about the pivot.

Solution 3.3 (b)

Worked steps

- the child is at the left end, 1.2 m from the pivot **M1**

$$\text{moment} = 300\ \text{N} \times 1.2\ \text{m} = \mathbf{360}\ \text{N m}$$

, anticlockwise **A1**

Exam-style 3.3 (c) ▪ 2 marks

Calculate the moment of the plank's weight about the pivot.

Solution 3.3 (c)

Worked steps

- the centre of gravity is 1.5 m from the left end, so it is $1.5 - 1.2 = 0.3$ m to the **right** of the pivot **M1**

$$\text{moment} = 220 \text{ N} \times 0.3 \text{ m} = \mathbf{66} \text{ N m}$$

, clockwise **A1**

Exam-style 3.3 (d) ▪ 2 marks

State whether the plank turns, and in which direction, explaining your answer.

Solution 3.3 (d)

Worked steps

- the plank **turns anticlockwise**, so the left-hand end goes down **B1**
- the child's moment, 360 N m, is much larger than the plank's 66 N m in the opposite sense, so the moments are not balanced **B1**

Exam-style 3.3 (e) ▪ 2 marks

A second child stands on the right-hand end. Calculate the smallest weight this child can have if the plank is to stay horizontal.

Solution 3.3 (e)

Worked steps

- the second child is at the right end, $3.0 - 1.2 = 1.8$ m from the pivot, and must supply the missing clockwise moment **M1**

$$W \times 1.8 \text{ m} + 66 \text{ N m} = 360 \text{ N m} \quad \Longrightarrow \quad W = \frac{294 \text{ N m}}{1.8 \text{ m}} = \mathbf{160} \text{ N}$$ **A1**