Exercise walk-through

3.3.1

Momentum in two dimensions, and testing a glancing collision for elasticity

A-Level Physics

You must be able to

- apply the principle of conservation of momentum to interactions in **two dimensions**
- resolve a momentum into **perpendicular 垂直 components 分量**, and apply conservation along each axis on its own
- combine two perpendicular momenta into a magnitude and a direction
- decide whether a two-dimensional collision is **elastic 弹性碰撞**, using the **full** speed of each body

1 The method

Method — One rule, applied twice

Momentum is a **vector**. So conservation of momentum is not one equation but **two**: the total momentum is conserved along each of two perpendicular directions, **independently**.

The method never changes:

1. **Choose the axes.** Take x along the motion of the incoming object and y across it. That makes the initial y-momentum zero, which is one equation almost solved before you start.
2. **Resolve every velocity.** A momentum p at angle θ to the x-axis has components $p\cos\theta$ along x and $p\sin\theta$ along y.
3. **Write the two equations**, "total before = total after", once for x and once for y.
4. **Recombine** if the question wants a speed and a direction: $p = \sqrt{p_x^2 + p_y^2}$ and $\theta = \tan^{-1}(p_y/p_x)$.

Method — One rule, applied twice

Worked example. *Ball A has momentum* 4.0 *N s due east and ball B has momentum* 3.0 *N s due north. They collide and stick together. Find the momentum of the combined object.*

Nothing outside acts, so each axis keeps what it had: 4.0 N s east and 3.0 N s north.

$$p = \sqrt{4.0^2 + 3.0^2} = 5.0\ \text{N s}, \qquad \theta = \tan^{-1}\left(\frac{3.0}{4.0}\right) = 37^\circ$$

so 5.0 N s at 37° north of east.

- A multiple-choice question showing two momentum arrows is asking exactly this: add them tip to tail.

Method — A glancing collision

Worked example. *A* 0.20 *kg ball moving east at* 5.0 *m s*$^{-1}$ *strikes a stationary* 0.30 *kg ball. Afterwards the* 0.20 *kg ball moves at* 3.0 *m s*$^{-1}$ *at* $40°$ *north of east. Find the velocity of the* 0.30 *kg ball.*

Along x (east positive):

$$(0.20)(5.0) = (0.20)(3.0)\cos 40° + (0.30)\,v_{2x}$$

$$1.00 = 0.460 + 0.30\,v_{2x} \quad \Rightarrow \quad v_{2x} = 1.80 \text{ m s}^{-1}$$

Along y (north positive). Before the collision there is **no** y-momentum at all:

$$0 = (0.20)(3.0)\sin 40° + (0.30)\,v_{2y}$$

Method — A glancing collision

$$0 = 0.386 + 0.30\, v_{2y} \quad \Rightarrow \quad v_{2y} = -1.29 \text{ m s}^{-1}$$

The minus sign means south, which it must: one ball went north, so the other has to go south.

Recombine:

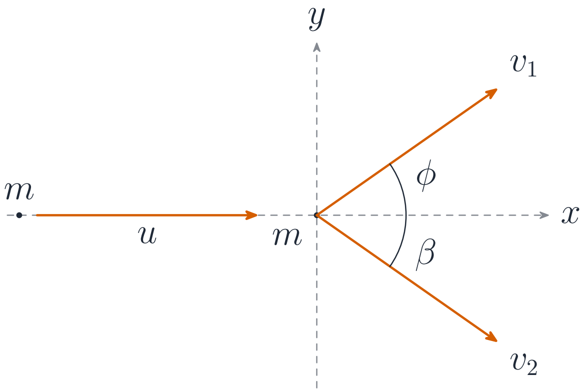
$$v_2 = \sqrt{1.80^2 + 1.29^2} = 2.2 \text{ m s}^{-1}, \qquad \theta = \tan^{-1}\left(\frac{1.29}{1.80}\right) = 36°$$

so 2.2 m s^{-1} at $36°$ **south** of east.

- Write the axis you are working on beside each equation. Almost every lost mark here is a $\sin$ where a $\cos$ belongs, and labelling the axis catches it.

Method — A glancing collision

Find the velocity of the ... ball.



Method — Testing whether it was elastic

Momentum is conserved in **every** collision; kinetic energy is not. Test the energy separately, and use the **full speed** of each body.
For the collision above:

$$E_{k,\text{before}} = \tfrac{1}{2}(0.20)(5.0)^2 = 2.5 \text{ J}$$

$$E_{k,\text{after}} = \tfrac{1}{2}(0.20)(3.0)^2 + \tfrac{1}{2}(0.30)(2.2)^2 = 0.90 + 0.73 = 1.6 \text{ J}$$

About 0.87 J, or 35%, has gone, so the collision is **inelastic**.

- **Kinetic energy uses the whole speed, never a component.** $\frac{1}{2}mv_x^2$ is not a quantity in this subject. Momentum resolves; energy does not, because it is a scalar.

Method — Equal masses, elastic, target at rest

A standard result worth recognising: when one object hits an equal mass at rest **elastically**, the two move off at 90° **to each other**.
Momentum gives $\vec{u} = \vec{v_1} + \vec{v_2}$, and kinetic energy gives $u^2 = v_1^2 + v_2^2$. Squaring the first,

$$u^2 = v_1^2 + 2\,\vec{v_1} \cdot \vec{v_2} + v_2^2$$

Comparing the two, $\vec{v_1} \cdot \vec{v_2} = 0$, so the two velocities are perpendicular.

2 Practice

Practice 2.1 ▪ 2 marks

A momentum of 6.0 kg m s^{-1} acts at $30°$ above the horizontal. Calculate its horizontal and vertical components.

Solution 2.1

Method: One rule, applied twice — p.4

Worked steps

- horizontal: $6.0\cos 30° = \mathbf{5.2}$ kg m s^{-1} **B1**
- vertical: $6.0\sin 30° = \mathbf{3.0}$ kg m s^{-1} **B1**

Practice 2.2 ▪ 3 marks

Two pieces of clay collide and stick together. Just before the collision one has momentum 4.0 N s due east and the other 3.0 N s due north.
Calculate the magnitude and direction of the momentum of the combined lump.

Solution 2.2

Method: One rule, applied twice — p.4

Worked steps

- momentum is conserved along each axis, so the lump has 4.0 N s east and 3.0 N s north **M1**
- $p = \sqrt{4.0^2 + 3.0^2} = \mathbf{5.0}$ N s **A1**
- $\theta = \tan^{-1}(3.0/4.0) = \mathbf{37}°$ north of east **A1**

Practice 2.3 ▪ 2 marks

State the **principle of conservation of momentum**, and state why it may be applied to each of two perpendicular directions separately.

Solution 2.3

Method: One rule, applied twice — p.4

Worked steps

- the **total momentum** of a system of interacting bodies stays constant, provided **no resultant external force** acts on it **B1**
- momentum is a **vector**, so its components along two perpendicular directions are independent, and the condition holds along each of them separately **B1**

Practice 2.4 ▪ 2 marks

After a collision a 0.50 kg ball has momentum components 1.2 N s east and 0.90 N s north.
Calculate its speed.

Solution 2.4

Method: One rule, applied twice — p.4

Worked steps

- $p = \sqrt{1.2^2 + 0.90^2} = 1.5$ N s **M1**
- $v = p/m = 1.5/0.50 = \mathbf{3.0}$ m s^{-1} **A1**

Practice 2.5 ▪ 1 mark

A student calculates the kinetic energy of a ball using only the horizontal component of its velocity. State why this is wrong.

Solution 2.5

Method: Testing whether it was elastic — p.9

Worked steps

- kinetic energy is a **scalar** and depends on the full speed, $\frac{1}{2}mv^2$; only vectors have components, so a component of the velocity gives an answer that is too small B1

Practice 2.6 ▪ 2 marks

State the **two** things that are true of an **elastic** collision: one about energy, and one about relative speeds.

Solution 2.6

Method: Testing whether it was elastic — p.9

Worked steps

- the total **kinetic energy** is conserved, as well as the momentum **B1**
- the **relative speed of approach** equals the **relative speed of separation** **B1**
- the relative-speed test is applied along the **line joining** the two bodies, so in two dimensions it is the components along that line that must satisfy it; testing the kinetic energy is the general method and is what question 3.2 uses

3 Exam-style

Exam-style 3.1 ▪ 5 marks

On a frictionless horizontal surface, a ball **A** of mass 0.20 kg moving east at 5.0 m s^{-1} strikes a stationary ball **B** of mass 0.30 kg.
After the collision **A** moves at 3.0 m s^{-1} at $40°$ north of east.
Calculate the magnitude and direction of the velocity of **B**.

Solution 3.1

Method: A glancing collision — p.6

Worked steps

- along east: $(0.20)(5.0) = (0.20)(3.0)\cos 40° + (0.30)v_{Bx}$ **M1**
- $1.00 = 0.460 + 0.30\,v_{Bx}$, so $v_{Bx} = 1.80$ m s^{-1} **A1**
- along north, the total before is **zero**: $0 = (0.20)(3.0)\sin 40° + (0.30)v_{By}$ **M1**
- $v_{By} = -1.29$ m s^{-1}, that is 1.29 m s^{-1} south **A1**
- $v_B = \sqrt{1.80^2 + 1.29^2} = \mathbf{2.2}$ m s^{-1} at $\tan^{-1}(1.29/1.80) = \mathbf{36°}$ **south** of east **A1**
- a direction with no sense named ("$36°$ to the east") does not earn the final mark: the question asks for a direction, and south is half of it

Exam-style 3.2 ▪ 3 marks

Determine whether the collision in 3.1 is elastic. Show your working.

Solution 3.2

Method: Testing whether it was elastic — p.9

Worked steps

- before: $E_k = \frac{1}{2}(0.20)(5.0)^2 = 2.5$ J M1
- after: $\frac{1}{2}(0.20)(3.0)^2 + \frac{1}{2}(0.30)(2.2)^2 = 0.90 + 0.73 = 1.6$ J A1
- the kinetic energy has fallen by about 0.9 J, so the collision is **inelastic** A1
- both speeds are the **full** speeds, not components

Exam-style 3.3 ▪ 5 marks

An object of mass 2.0 kg moving east at 6.0 m s^{-1} explodes into two fragments, each of mass 1.0 kg.
One fragment moves at 5.0 m s^{-1} at $53°$ north of east.
Calculate the magnitude and direction of the velocity of the other fragment.

Solution 3.3

> Method: A glancing collision — p.6

Worked steps

- along east: $(2.0)(6.0) = (1.0)(5.0)\cos 53° + (1.0)v_x$ **M1**
- $12.0 = 3.01 + v_x$, so $v_x = 9.0$ m s^{-1} **A1**
- along north: $0 = (1.0)(5.0)\sin 53° + (1.0)v_y$ **M1**
- $v_y = -3.99$ m s^{-1}, that is 4.0 m s^{-1} south **A1**
- $v = \sqrt{9.0^2 + 4.0^2} = \mathbf{9.8}$ m s^{-1} at $\tan^{-1}(4.0/9.0) = \mathbf{24°}$ **south** of east **A1**
- an explosion conserves momentum exactly as a collision does, because no external resultant force acts during it

Exam-style 3.4 ▪ 4 marks

A moving snooker ball strikes a second, identical ball that is at rest.

The collision is elastic.
Show that after the collision the two balls move off at 90° to each other.

Solution 3.4

Method: A glancing collision — p.6

Worked steps

- momentum: with the masses equal, $m\vec{u} = m\vec{v_1} + m\vec{v_2}$, so $\vec{u} = \vec{v_1} + \vec{v_2}$ **M1**
- elastic, so kinetic energy is conserved: $\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$, giving $u^2 = v_1^2 + v_2^2$ **M1**
- squaring the momentum equation: $u^2 = v_1^2 + 2\,\vec{v_1} \cdot \vec{v_2} + v_2^2$ **M1**
- comparing the two gives $\vec{v_1} \cdot \vec{v_2} = 0$, so the velocities are **perpendicular**: the balls move off at $90°$ to each other **A1**
- the same result follows from a vector triangle whose sides satisfy $u^2 = v_1^2 + v_2^2$, which is Pythagoras, so the angle between v_1 and v_2 is a right angle

Exam-style 3.5 ▪ 3 marks

In a two-dimensional collision the total kinetic energy after the collision is less than before, yet the total momentum is unchanged.
Explain how both statements can be true.

Solution 3.5

Method: Testing whether it was elastic — p.9

Worked steps

- momentum is conserved because there is **no resultant external force** on the system during the collision, and that is true whatever happens to the energy **B1**
- kinetic energy is not a conserved quantity: some of it is transferred to other forms – thermal energy, sound and the work done in deforming the objects **B1**
- the two are independent because momentum is a **vector** that only the external force can change, while kinetic energy is a **scalar** that internal forces can convert into other forms **B1**