Exercise walk-through

Momentum and Newton's laws of motion ▪ 3.1

eXercise

You must be able to

- recall and use $F = ma$ and **weight** $= mg$
- use **linear momentum** $p = mv$ and force as **rate of change of momentum**, $F = \Delta p / \Delta t$
- state and apply **Newton's three laws**
- read a **momentum–time graph**: the change, the gradient, and what the motion is doing

Method — Newton's laws and F = ma

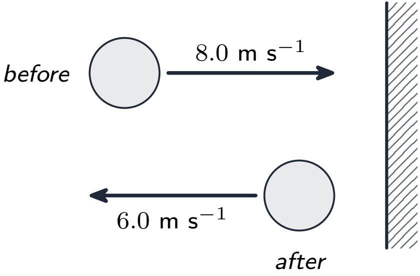
- **1st law:** with **zero resultant force**, an object stays at rest or moves at constant velocity.
- **2nd law:** resultant force = rate of change of momentum; for constant mass, $F = ma$.
- **3rd law:** if A pushes B, B pushes A with an **equal, opposite** force (a different object, same type).

Weight is the effect of a **gravitational field 引力场** on a mass: $W = mg$, where g is both the **gravitational field strength** (N kg^{-1}) and the **acceleration of free fall** (m s^{-2}) — the same number, 9.81 near the Earth's surface, because a mass in free fall has only its weight acting on it, so $mg = ma$ gives $a = g$. **Mass** is the property that resists a change in motion (in kg) and is the **same everywhere**; **weight** is a force and **changes** with the field — an object weighs less on the Moon (smaller g) even though its mass is unchanged.

Method — Momentum and impulse

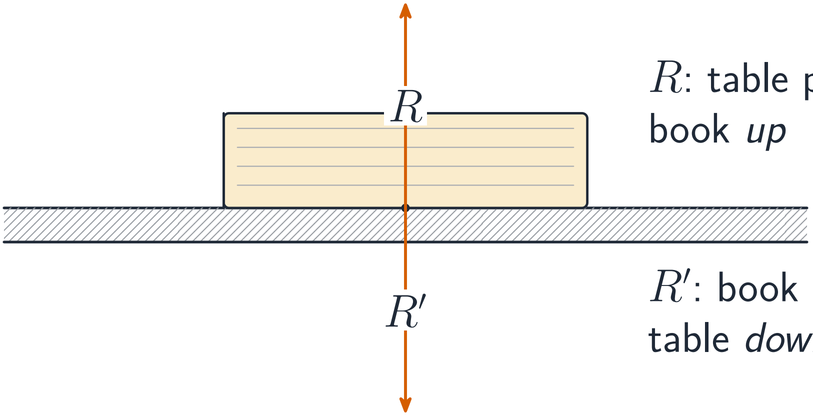
Linear momentum $p = mv$ (a vector, unit kg m s^{-1} = N s). In a collision the average force is $F = \dfrac{\Delta p}{\Delta t}$ — assign each direction a sign.

Worked example. A 0.20 kg ball hits a wall at 8.0 m s^{-1} and rebounds at 6.0 m s^{-1} in 0.050 s. Take the rebound direction as positive ($u = -8.0$, $v = +6.0$):



$$\Delta p = m(v - u) = 0.20\,(6.0 - (-8.0)) = 2.8 \text{ kg m s}^{-1}, \qquad F = \frac{2.8}{0.050} = 56 \text{ N}.$$

Method — Momentum and impulse



R: table pushes the book *up*

R': book pushes the table *down*

Newton third-law force pairs

Method — Reading a momentum–time graph

Because $F = \dfrac{\Delta p}{\Delta t}$, the **gradient of a momentum–time graph is the resultant force**. Read one off in four steps:

1. **A straight line means a constant resultant force.** A curve means the force is changing.
2. **The gradient is the force** — take it over the whole line, not from a single point.
3. **Divide by the mass to get the velocity** at any instant: $v = p/m$.
4. **Where the line crosses $p = 0$ the object is momentarily at rest**, and after that it is moving the other way.

A resistive force such as air resistance **cannot** give a straight line: it gets smaller as the object slows, and it is **zero** when the object is at rest — so a graph whose gradient is unchanged at the crossing point rules it out.

Practice 2.1 ▪ 1 mark

State **Newton's first law** of motion.

Solution 2.1

Method: Newton's laws and $F = ma$

Worked steps

An object stays at **rest** or moves with **constant velocity** unless a **resultant (external) force** acts on it **B1**.

Practice 2.2 ▪ 2 marks

Calculate the **momentum** of a 1500 kg car travelling at 20 m s^{-1}.

Solution 2.2

Worked steps

$p = mv = 1500 \times 20 = 3.0 \times 10^4$ kg m s^{-1} M1 A1.

Practice 2.3 ▪ 2 marks

A resultant force of 5.0 N acts on a 2.0 kg mass. Find its **acceleration**.

Solution 2.3

Method: Newton's laws and F = ma

Worked steps

$a = F/m = 5.0/2.0 = 2.5$ m s^{-2} **M1** **A1**.

Practice 2.4 ▪ 1 mark

Calculate the **weight** of a 60 kg person ($g = 9.81$ m s^{-2}).

Solution 2.4

Method: Newton's laws and F $=$ ma

Worked steps

$W = mg = 60 \times 9.81 = 590$ N **B1**.

Exam-style 3.1 ▪ 1 mark

The unit kg m s^{-1} is the unit of which quantity?

A force

B momentum

C energy

D power

Solution 3.1

- **A** force
- **B** momentum ✓
- **C** energy
- **D** power

Worked steps

B — momentum.

Exam-style 3.2 ▪ 2 marks

State **Newton's third law**, and give **one** example of a force pair.

Solution 3.2

Method: Momentum and impulse

Worked steps

When object A exerts a force on B, B exerts an **equal and opposite** force on A (same type, different bodies) **B1**; example: a rocket pushes gas down, the gas pushes the rocket up (or foot/ground, etc.) **B1**.

Exam-style 3.3 ▪ 4 marks

A 0.20 kg ball hits a wall at 8.0 m s^{-1} and rebounds straight back at 6.0 m s^{-1}. The contact lasts 0.050 s. **Show that** the magnitude of the average force on the ball is about 56 N.

Solution 3.3

Method: Momentum and impulse

Worked steps

Take rebound as positive: $u = -8.0$, $v = +6.0$ M1; $\Delta p = 0.20\,(6.0 - (-8.0)) = 2.8$ kg m s^{-1} M1 A1; $F = \Delta p/\Delta t = 2.8/0.050 = 56$ N A1.

Exam-style 3.4 ▪ 2 marks

Using Newton's second law, explain why a **longer** contact time gives a **smaller** average force for the same change in momentum.

Solution 3.4

Method: Newton's laws and F = ma

Worked steps

$F = \Delta p/\Delta t$ **B1**; for the same Δp, a larger Δt gives a smaller F (the force is inversely proportional to the contact time) **B1**.

Exam-style 3.5 ▪ 2 marks

An object is taken from the Earth to the Moon, where g is smaller. State what happens to its **mass** and to its **weight**.

Solution 3.5

Method: Newton's laws and $F = ma$

Worked steps

The **mass stays the same** B1; the **weight decreases**, because g is smaller on the Moon B1.

Exam-style 3.6 ▪ 1 mark

A trolley of mass 1.2 kg moves in a straight line along a horizontal track. The graph shows how the momentum p of the trolley varies with time t.

(a) **Define** momentum.

(b) **Determine** the speed of the trolley at time $t = 0$.

(c) Calculate the change in momentum of the trolley between $t = 0$ and $t = 10$ s.

(d) **Determine** the magnitude of the resultant force acting on the trolley.

(e) **Describe** the motion of the trolley between $t = 0$ and $t = 6.0$ s.

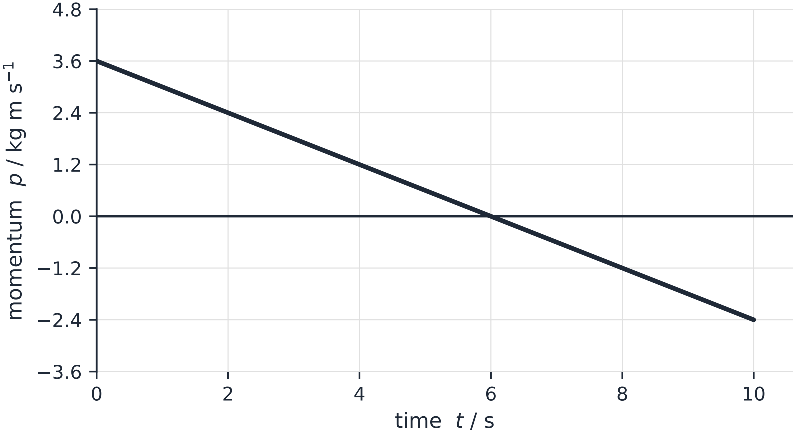
(f) By reference to the graph, **explain** why the resultant force acting on the trolley cannot be air resistance.

(g) At time $t = 0$ the displacement of the trolley is zero. In the space below, **sketch** the variation with time of its displacement from $t = 0$ to $t = 10$ s. Numerical values are not required, but **label** both axes.

Exam-style 3.6 ▪ 1 mark

(h) **Compare** the momentum of the trolley at $t = 2.0$ s with its momentum at $t = 8.0$ s.

Exam-style 3.6 ▪ 1 mark



Solution 3.6

Method: Reading a momentum–time graph

Worked steps

(a) Momentum is the **product of mass and velocity** **B1**.

(b) $v = \dfrac{p}{m} = \dfrac{3.6}{1.2}$ **M1** $= 3.0$ m s^{-1} **A1**.

(c) $\Delta p = (-2.4) - (+3.6) = -6.0$ kg m s^{-1}, i.e. a change of magnitude 6.0 kg m s^{-1} **B1**.

(d) $F = \dfrac{\Delta p}{\Delta t} =$ the gradient $= \dfrac{6.0}{10}$ **M1** $= 0.60$ N **A1**.

(e) The trolley slows down **B1** **uniformly** — the line is straight, so the force and therefore the deceleration are constant — coming momentarily to rest at $t = 6.0$ s

Solution 3.6

B1.

(f) The line is **straight**, so the resultant force is constant, whereas air resistance would get **smaller** as the trolley slows B1; and at $t = 6.0$ s the trolley is momentarily at rest, where air resistance would be **zero**, yet the gradient there is unchanged B1.

(g) Axes labelled *displacement* and *time / s* B1; from the origin the line rises with a **decreasing** positive gradient, reaching a maximum at $t = 6.0$ s B1; after 6.0 s the displacement **falls**, with a gradient of increasing magnitude, but is still positive at $t = 10$ s B1.

(h) At $t = 2.0$ s, $p = +2.4$ kg m s^{-1}; at $t = 8.0$ s, $p = -1.2$ kg m s^{-1} B1. The magnitude has halved and the **direction has reversed** — momentum is a vector, so the two are not simply "smaller" and "bigger" B1.