Exercise walk-through

3.1.1

Newton's laws, resultant force and $F = ma$

A-Level Physics

You must be able to

- explain that **mass 质量** is the property of an object that resists change in motion, and use weight $W = mg$
- draw the forces on one object and add them to find the resultant force
- use $F = ma$ to solve problems, knowing that the acceleration and the resultant force are always in the same direction
- state and apply each of Newton's laws, including to an object moving at constant velocity
- find the normal contact force on a person in a lift
- find the acceleration of two joined objects, then the force between them
- name the other force in a Newton's third-law pair
- define and use **linear momentum 动量** as the product of mass and velocity, and use force as the **rate of change of momentum 动量变化率** for a stream of water or air

1 The method

Method — Mass and weight

- **Mass** is the property of an object that resists a change in its motion. It is measured in kg, and it is the **same everywhere**.
- **Weight** is the effect of a **gravitational field 重力场** on a mass. It is the product of the mass and the **acceleration of free fall 自由落体加速度**: $W = mg$.
- Near the Earth's surface $g = 9.81$ N kg^{-1}, the **gravitational field strength 重力场强度**, which is the same as 9.81 m s^{-2}.
- A 2.0 kg bag weighs $2.0 \times 9.81 = 19.6$ N on the Earth. On Mars, where $g = 3.7$ N kg^{-1}, it weighs $2.0 \times 3.7 = 7.4$ N, but its mass is still 2.0 kg.

Method — Finding the resultant force

A box of mass 6.0 kg is pulled to the left along a rough floor. The rope pulls with a force $P = 40$ N at 20° above the horizontal, and the **friction 摩擦力** force is $F = 15$ N.

1. Draw a **free-body diagram 受力图**: every force on the box, and only the forces on the box.
2. Resolve the force at an angle (Topic 1). The horizontal component of the pull is $40\cos 20^\circ = 37.6$ N and the vertical component is $40\sin 20^\circ = 13.7$ N.
3. Add the forces along the direction of motion, giving each direction a sign:

$$F_{\text{resultant}} = 37.6\text{ N} - 15\text{ N} = 22.6\text{ N}$$

4. Use $F = ma$. The acceleration is in the direction of the resultant force, to the left:

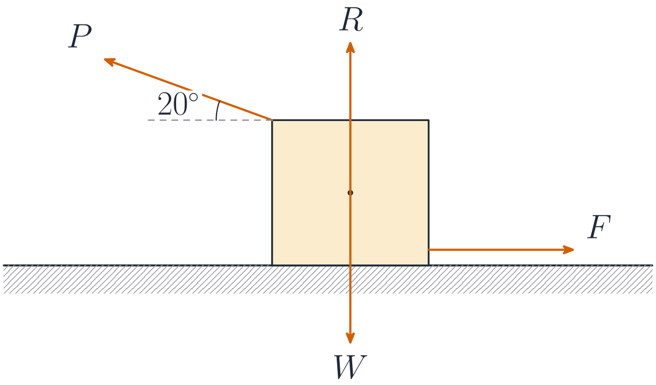
$$a = \frac{22.6\text{ N}}{6.0\text{ kg}} = 3.8\text{ m s}^{-2}$$

Method — Finding the resultant force

5. The box does not move up or down, so the vertical forces balance:
$R + 13.7$ N $= W = 58.9$ N, so $R = 45$ N.

Method — Finding the resultant force

A box of mass … is pulled to the left along a rough floor.



Method — Constant velocity, speeding up and slowing down

- **Newton's first law:** if the resultant force is zero, an object stays at rest or keeps moving at **constant velocity**. A lorry moving at a steady speed has a driving force **equal** to the total resistive force, for example 3200 N each.
- If the driving force rises to 4000 N, the resultant force is 800 N forwards, and the lorry of mass $16\,000$ kg speeds up: $a = 800/16\,000 = 0.050$ m s^{-2}.
- When a car brakes, the resultant force points **backwards**, so the acceleration points backwards too, even though the car still moves forwards. For a 1200 kg car with a braking force of 6000 N and air resistance of 600 N:

$$a = \frac{-(6000 + 600)\ \text{N}}{1200\ \text{kg}} = -5.5\ \text{m s}^{-2}$$

- From 20 m s^{-1} the car then stops in $s = \dfrac{0 - 20^2}{2(-5.5)} = 36$ m (Topic 2).

Method — The normal contact force in a lift

A person of mass 60 kg stands in a lift. Two forces act on the person: the weight $W = 60 \times 9.81 = 589$ N downwards, and the **normal contact force 支持力** R from the floor upwards. Take up as positive, so $R - W = ma$ and $R = m(g + a)$.

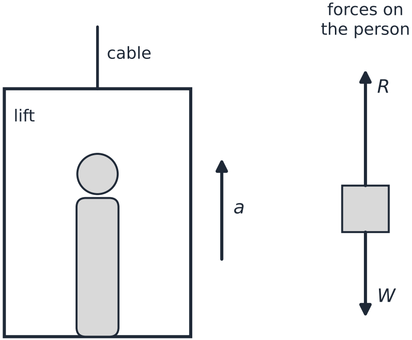
motion of the lift	**acceleration**	$R = m(g + a)$
going up and speeding up	$+1.5$ m s^{-2}	$60(9.81 + 1.5) = 680$ N
steady speed, up or down	0	589 N
going up and slowing down	-1.5 m s^{-2}	$60(9.81 - 1.5) = 500$ N

Method — The normal contact force in a lift

- A lift going **down** and speeding up also has an acceleration of -1.5 m s^{-2}, so again $R = 500$ N and the person feels lighter.
- Bathroom scales in the lift read R, not the weight.

Method — The normal contact force in a lift

A person of mass … stands in a lift.



Method — Two objects joined together

A car of mass 1000 kg tows a trailer of mass 500 kg. The driving force is 3000 N. The resistive forces are 400 N on the car and 200 N on the trailer.

1. **Treat both as one object** to find the acceleration. The **tension 张力** in the tow bar is inside this object, so it cancels:

$$a = \frac{(3000 - 400 - 200)\ \text{N}}{(1000 + 500)\ \text{kg}} = 1.6\ \text{m s}^{-2}$$

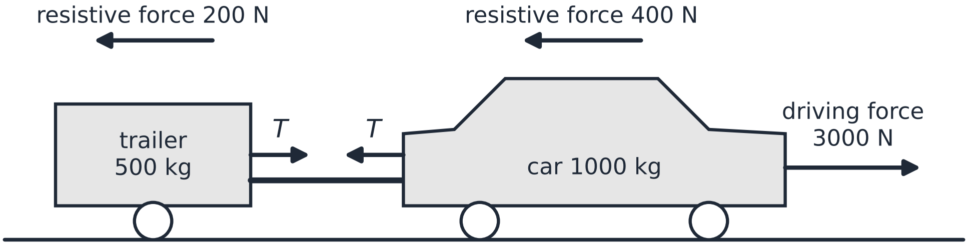
2. **Then take one object on its own** to find the force between them. On the trailer, T pulls forwards and 200 N acts backwards:

$$T - 200\ \text{N} = 500\ \text{kg} \times 1.6\ \text{m s}^{-2} \quad \Rightarrow \quad T = 1000\ \text{N}$$

3. Check with the car: $3000 - 400 - 1000 = 1600$ N, and $1000\ \text{kg} \times 1.6\ \text{m s}^{-2} = 1600$ N.

Method — Two objects joined together

A car of mass ... tows a trailer of mass ... .



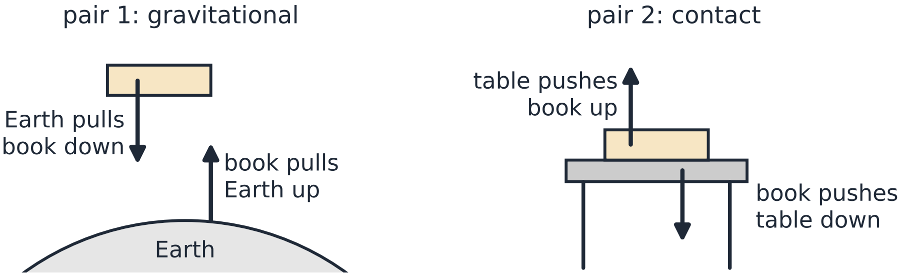
Method — Newton's third-law pairs

Newton's third law: if object A exerts a force on object B, then B exerts a force on A that is **equal in size, opposite in direction and of the same type**. The two forces act on **different** objects.

- To find the partner of a force, **swap the two objects** in its description: "the Earth pulls the book" becomes "the book pulls the Earth".
- The weight of the book and the push of the table are equal in size, but they are **not** a pair. Both act on the book, and they are different types of force. They are equal because the book is in equilibrium (first law), not because of the third law.

Method — Newton's third-law pairs

Newton's third law: if object A exerts a force on object B, then B exerts a force on A that is equal in size,...



Method — A force from a stream of water or air

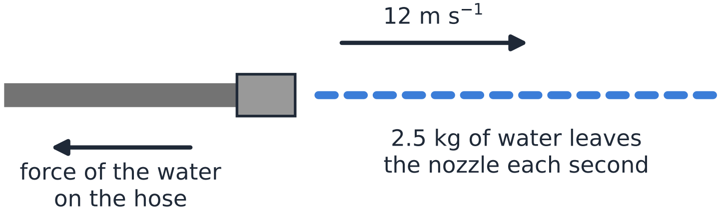
Linear momentum is the product of mass and velocity, $p = mv$, measured in kg m s^{-1}. Newton's second law says that the resultant force is the rate of change of momentum. For a stream, find the momentum given to the fluid **each second**:

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta m}{\Delta t} \times \Delta v$$

- Each second, 2.5 kg of water speeds up from rest to 12 m s^{-1} at the **nozzle 喷嘴**: $F = 2.5$ kg $s^{-1} \times 12$ m $s^{-1} = 30$ N on the water, forwards.
- By the third law, the water pushes the hose with 30 N **backwards**, so the person holding the hose must push forwards with 30 N.

Method — A force from a stream of water or air

Linear momentum is the product of mass and velocity, ... , measured in ... .



2 Practice

Practice 2.1 ▪ 2 marks

A rock sample has a weight of 12 N on a planet where $g = 4.0$ N kg^{-1}.

Practice 2.1 (a) ▪ 1 mark

Calculate the mass of the rock.

Solution 2.1 (a)

Method: Mass and weight — p.4

Worked steps

$m = W/g = 12\ \mathrm{N}/4.0\ \mathrm{N\ kg^{-1}} = \mathbf{3.0}\ \mathrm{kg}$ **B1**

Practice 2.1 (b) ▪ 1 mark

Calculate the weight of the rock on the Earth.

Solution 2.1 (b)

Worked steps

the mass does not change, so $W = mg = 3.0$ kg $\times\ 9.81$ N kg^{-1} $=$ **29** N B1

Practice 2.2 ▪ 3 marks

A sledge of mass 12 kg is pulled across flat snow by a rope at 25° above the horizontal. The tension in the rope is 50 N, and the friction force on the sledge is 20 N.

Practice 2.2 (a) ▪ 1 mark

Calculate the horizontal component of the tension.

Solution 2.2 (a)

Method: Finding the resultant force — p.5

Worked steps

$T_{\mathrm{H}} = 50\ \mathrm{N} \times \cos 25^\circ = \mathbf{45}\ \mathrm{N}$ **B1**

Practice 2.2 (b) ▪ 2 marks

Calculate the acceleration of the sledge.

Solution 2.2 (b)

Worked steps

resultant force $= 45.3$ N $- 20$ N $= 25.3$ N **M1**, so $a = 25.3$ N$/12$ kg $= \mathbf{2.1}$ m s$^{\mathbf{-2}}$ **A1**

Practice 2.3 ▪ 3 marks

A lift and its passengers have a total mass of 800 kg. The lift is pulled up by a cable. Calculate the tension in the cable when the lift is:

Practice 2.3 (a) ▪ 2 marks

accelerating upwards at 0.60 m s^{-2}

Solution 2.3 (a)

Method: Constant velocity, speeding up and slowing down — p.8

Worked steps

up positive: $T - mg = ma$ M1

$$T = 800 \text{ kg} \times (9.81 + 0.60) \text{ m s}^{-2} = \mathbf{8330} \text{ N}$$ A1

Practice 2.3 (b) ▪ 1 mark

moving upwards at a constant speed.

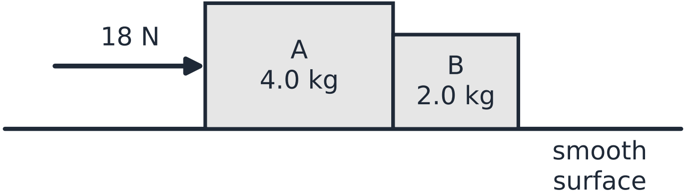
Solution 2.3 (b)

Worked steps

constant speed, so the resultant force is zero: $T = mg = 800\ \text{kg} \times 9.81\ \text{m s}^{-2} = \mathbf{7850}\ \text{N}$ B1

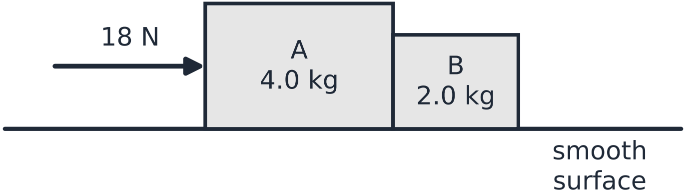
Practice 2.4 ▪ 3 marks

Two blocks, A and B, are pushed along a smooth horizontal surface by a force of 18 N, as shown.



Practice 2.4 (a) ▪ 1 mark

Calculate the acceleration of the blocks.



Solution 2.4 (a)

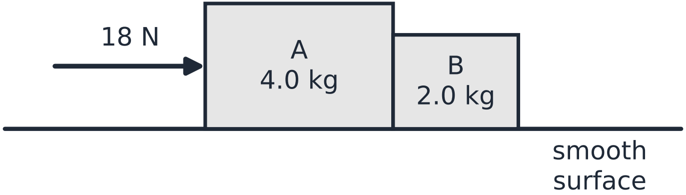
Method: Finding the resultant force — p.5

Worked steps

both blocks together: $a = \dfrac{18\ \text{N}}{(4.0 + 2.0)\ \text{kg}} = \mathbf{3.0}\ \text{m s}^{\mathbf{-2}}$ B1

Practice 2.4 (b) ▪ 2 marks

Calculate the force that block A exerts on block B.



Solution 2.4 (b)

Worked steps

the only horizontal force on B is the push of A **M1**: $F = 2.0\ \text{kg} \times 3.0\ \text{m s}^{-2} = \mathbf{6.0}\ \text{N}$ **A1**

Practice 2.5 ▪ 2 marks

A leaf blower takes in still air and blows out 0.30 kg of air each second at 40 m s^{-1}.

Practice 2.5 (a) ▪ 1 mark

Calculate the force on the air.

Solution 2.5 (a)

Worked steps

$$F = \frac{\Delta m}{\Delta t}\Delta v = 0.30 \text{ kg s}^{-1} \times 40 \text{ m s}^{-1} = \mathbf{12} \text{ N}$$ **B1**

Practice 2.5 (b) ▪ 1 mark

State the size and the direction of the force of the air on the leaf blower.

Solution 2.5 (b)

Worked steps

12 N, **opposite** to the direction in which the air leaves the blower (Newton's third law) B1

3 Exam-style

Exam-style 3.1 ▪ 1 mark

A lift is moving upwards and slowing down. T is the tension in the cable and W is the weight of the lift. Which statement is correct?

A T is greater than W.

B T is equal to W.

C T is less than W.

D T is zero.

Solution 3.1

Method: The normal contact force in a lift — p.9

A T is greater than W.

B T is equal to W.

C T is less than W. ✓

D T is zero.

Worked steps

C

- the lift slows down while it moves up, so its acceleration and the resultant force are **downwards**: $W > T$ **B1**

Exam-style 3.2 ▪ 1 mark

A horse pulls a cart forwards along a level road. Which force forms a Newton's third-law pair with the force of the horse on the cart?

A the force of the road on the cart

B the force of the cart on the horse

C the weight of the cart

D the force of the road on the horse

Solution 3.2

Method: Newton's third-law pairs — p.14

A the force of the road on the cart

B the force of the cart on the horse ✓

C the weight of the cart

D the force of the road on the horse

Worked steps

B

- swap the two objects: the horse pulls the cart, so the partner is the cart pulling the horse **B1**
- **A**, **C** and **D** involve other objects or other types of force

Exam-style 3.3 ▪ 1 mark

Two forces act on an object of mass 2.5 kg: 6.0 N to the east and 8.0 N to the north. No other forces act. What is the magnitude of the acceleration of the object?

A 0.80 m s^{-2}

B 2.4 m s^{-2}

C 4.0 m s^{-2}

D 5.6 m s^{-2}

Solution 3.3

Method: Two objects joined together — p.12

A 0.80 m s^{-2}

B 2.4 m s^{-2}

C 4.0 m s^{-2} ✓

D 5.6 m s^{-2}

Worked steps

C

- the forces are at right angles: resultant $= \sqrt{6.0^2 + 8.0^2} = 10$ N, so $a = 10$ N$/2.5$ kg $= 4.0$ m s^{-2} **B1**

Exam-style 3.4 ▪ 6 marks

A water jet pack lifts a rider above a lake. The total mass of the rider and the pack is 90 kg. Water that starts from rest leaves the pack vertically downwards at 20 m s^{-1}.

Exam-style 3.4 (a) ▪ 2 marks

Explain how sending water downwards gives an upward force on the pack.

Solution 3.4 (a)

Method: A force from a stream of water or air — p.16

Worked steps

the pack pushes the water downwards, changing the momentum of the water **B1**

- by Newton's third law, the water pushes the pack **upwards** with an equal force **B1**

Exam-style 3.4 (b) ▪ 2 marks

The rider stays at the same height. Calculate the mass of water that leaves the pack each second.

Solution 3.4 (b)

Worked steps

at a steady height, upward force = weight = 90 kg $\times$ 9.81 m s^{-2} = 883 N **M1**, so $\Delta m/\Delta t = 883$ N$/20$ m s^{-1} = $\mathbf{44}$ **kg s$^{\mathbf{-1}}$** **A1**

Exam-style 3.4 (c) ▪ 2 marks

The rider now accelerates upwards at 1.5 m s^{-2}, and the water still leaves at 20 m s^{-1}. Calculate the mass of water that must leave the pack each second.

Solution 3.4 (c)

Worked steps

upward force $= m(g + a) = 90$ kg $\times (9.81 + 1.5)$ m s$^{-2} = 1018$ N M1, so $\Delta m/\Delta t = 1018$ N$/20$ m s^{-1} $= \mathbf{51}$ kg s^{-1} A1

Exam-style 3.5 ▪ 8 marks

A car of mass 1100 kg tows a caravan of mass 900 kg along a straight, level road. The driving force on the car is 2600 N. The resistive force is 350 N on the car and 450 N on the caravan.

Exam-style 3.5 (a) ▪ 1 mark

State Newton's second law of motion.

Solution 3.5 (a)

Worked steps

the resultant force on an object is equal to its **rate of change of momentum**

B1

Exam-style 3.5 (b) ▪ 2 marks

Calculate the acceleration of the car and caravan.

Solution 3.5 (b)

Worked steps

resultant force $= 2600 - 350 - 450 = 1800$ N **M1**, so $a = 1800$ N$/2000$ kg $=$ $\mathbf{0.90}$ m $\mathrm{s}^{\mathbf{-2}}$ **A1**

Exam-style 3.5 (c) ▪ 2 marks

Show that the force of the tow bar on the caravan is about 1300 N.

Solution 3.5 (c)

Worked steps

caravan on its own: $T - 450\ \text{N} = 900\ \text{kg} \times 0.90\ \text{m s}^{-2}$ M1, so $T = 810\ \text{N} + 450\ \text{N} = \mathbf{1260}\ \text{N} \approx 1300\ \text{N}$ A1

Exam-style 3.5 (d) ▪ 2 marks

State the size and the direction of the force of the caravan on the tow bar, and name the law you used.

Solution 3.5 (d)

Worked steps

1260 N **backwards** B1

- Newton's **third** law B1

Exam-style 3.5 (e) ▪ 1 mark

Later the car and caravan move at a constant speed, with the same resistive forces. State the driving force.

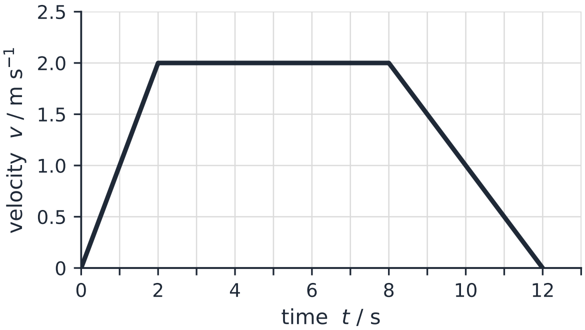
Solution 3.5 (e)

Worked steps

the resultant force is zero, so the driving force $= 350 + 450 = \mathbf{800}$ N B1

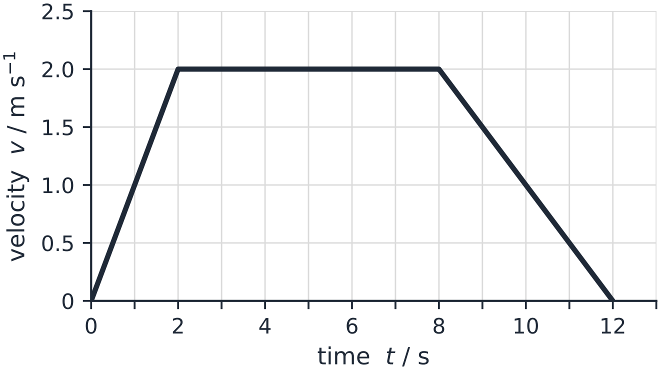
Exam-style 3.6 ▪ 8 marks

A student of mass 55 kg stands on bathroom scales in a lift. The graph shows how the velocity of the lift changes as it moves upwards from rest.



Exam-style 3.6 (a) ▪ 1 mark

Determine the acceleration of the lift between $t = 0$ and $t = 2.0$ s.



Solution 3.6 (a)

Method: The normal contact force in a lift — p.9

Worked steps

gradient of the graph: $a = \dfrac{2.0 \text{ m s}^{-1}}{2.0 \text{ s}} = \mathbf{1.0} \text{ m s}^{-2}$ **B1**

(b)(i) up positive: $R - mg = ma$ **M1**

$$R = 55 \text{ kg} \times (9.81 + 1.0) \text{ m s}^{-2} = \mathbf{595} \text{ N}$$ **A1**

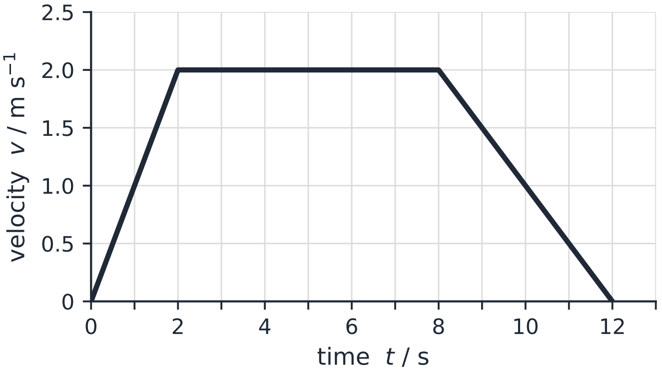
(ii) constant velocity: $R = mg = 55 \text{ kg} \times 9.81 \text{ m s}^{-2} = \mathbf{540} \text{ N}$ **B1**

(iii) $a = \dfrac{0 - 2.0 \text{ m s}^{-1}}{4.0 \text{ s}} = -0.50 \text{ m s}^{-2}$ **M1**

$$R = 55 \text{ kg} \times (9.81 - 0.50) \text{ m s}^{-2} = \mathbf{512} \text{ N}$$ **A1**

Exam-style 3.6 (b)

Calculate the reading of the scales, in newtons:

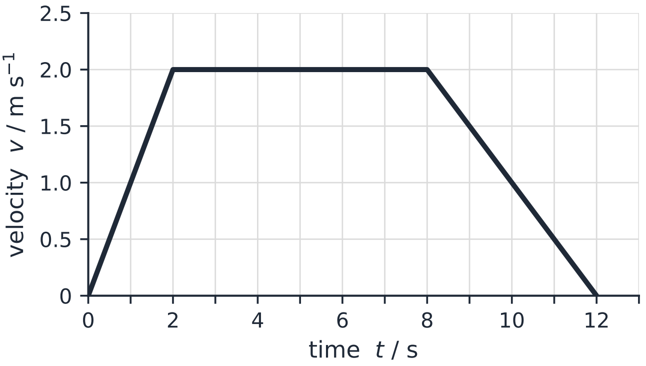


Exam-style 3.6 (i) ▪ 2 marks

between $t = 0$ and $t = 2.0$ s
(ii) between $t = 2.0$ s and $t = 8.0$ s
(iii) between $t = 8.0$ s and $t = 12.0$ s.

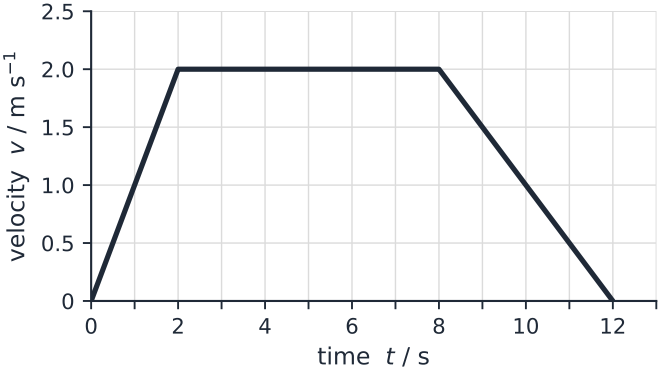
Exam-style 3.6 (i) ▪ 2 marks

between ... and ... (ii) between ... and ... (iii) between ... and



Exam-style 3.6 (c) ▪ 2 marks

Explain, using Newton's laws, why the reading between $t = 8.0$ s and $t = 12.0$ s is less than the weight of the student.



Solution 3.6 (c)

Worked steps

the lift slows down while it moves up, so the student's acceleration is **downwards** **B1**

- by the second law the resultant force on the student is downwards, so $W > R$ and the scales read less than the weight **B1**