Exercise walk-through

Stellar radii ▪ 25.2

eXercise

You must be able to

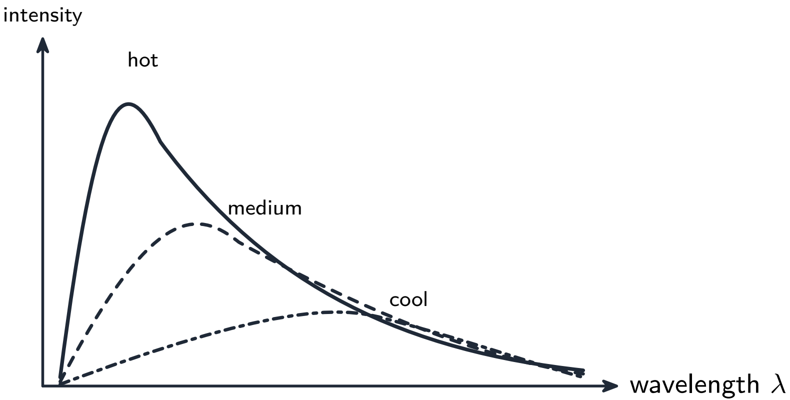
- use **Wien's law** $\lambda_{\max} T =$ constant
- use the **Stefan–Boltzmann law** $L = 4\pi\sigma r^2 T^4$
- combine them to estimate a star's **radius**

Method — Wien's law

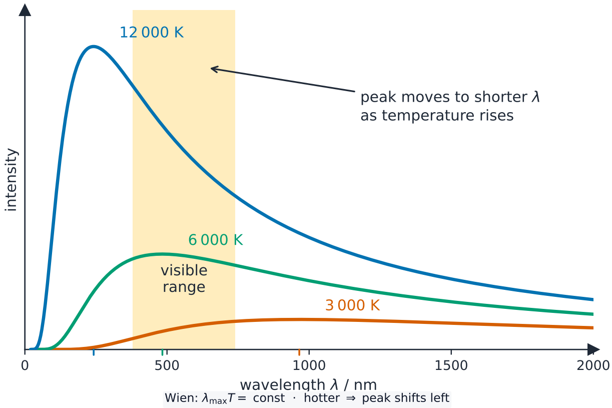
A star radiates a **black-body** spectrum; a **hotter** star radiates more and peaks at a **shorter** wavelength:

$$\lambda_{\max} T = b, \qquad b = 2.90 \times 10^{-3}\ \text{m K}.$$

Method — Wien's law



Method — Wien's law



A blackbody curve and Wien law

Method — Stefan–Boltzmann law and radius

$$L = 4\pi\sigma r^2 T^4, \qquad \sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\,K^{-4}}.$$

Find T from Wien's law, then $r = \sqrt{\dfrac{L}{4\pi\sigma T^4}}$.

Worked example. $\lambda_{\max} = 500$ nm: $T = \dfrac{2.90 \times 10^{-3}}{500 \times 10^{-9}} = 5800$ K.

Practice 2.1 ▪ 1 mark

Write **Wien's displacement law**.

Solution 2.1

Worked steps

$\lambda_{\max} T =$ constant (or $\lambda_{\max} \propto 1/T$) **B1**.

Practice 2.2 ▪ 2 marks

A star's spectrum peaks at 500 nm. Find its surface **temperature**.

Solution 2.2

Method: Wien's law

Worked steps

$$T = \frac{b}{\lambda_{\mathrm{max}}} = \frac{2.90 \times 10^{-3}}{500 \times 10^{-9}} = 5800 \text{ K} \quad \boxed{\textbf{M1}}\boxed{\textbf{A1}}.$$

Practice 2.3 ▪ 1 mark

Write the **Stefan–Boltzmann law**.

Solution 2.3

Method: Stefan–Boltzmann law and radius

Worked steps

$L = 4\pi\sigma r^2 T^4$ **B1**.

Practice 2.4 ▪ 1 mark

State how $\lambda_{\max}$ changes for a **hotter** star.

Solution 2.4

Method: Wien's law

Worked steps

It is **shorter** ($\lambda_{\max} \propto 1/T$) B1.

Exam-style 3.1 ▪ 1 mark

A **hotter** star has its peak emission at a:

A longer wavelength

B shorter wavelength

C the same wavelength

D no peak

Solution 3.1

Method: Wien's law

A longer wavelength

B shorter wavelength ✓

C the same wavelength

D no peak

Worked steps

B — shorter wavelength.

Exam-style 3.2 ▪ 2 marks

A star's black-body spectrum peaks at 290 nm. Calculate its surface **temperature**.

Solution 3.2

Method: Wien's law

Worked steps

$$T = \frac{2.90 \times 10^{-3}}{290 \times 10^{-9}} = 1.0 \times 10^{4}\ \text{K}$$ **M1** **A1**.

Exam-style 3.3 ▪ 2 marks

Star P has radius 7.0×10^8 m and surface temperature 5800 K. Star Q has the **same** surface temperature but **three times** the radius. ($\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}.)

(a) Calculate the luminosity of star P.

(b) Without further calculation, state and explain the luminosity of star Q.

Solution 3.3

Method: Stefan–Boltzmann law and radius

Worked steps

(a) $L = 4\pi r^2 \sigma T^4 = 4\pi(7.0 \times 10^8)^2(5.67 \times 10^{-8})(5800)^4$ **M1** $= 4.0 \times 10^{26}$ W **A1**.
(b) $L \propto r^2$ at fixed temperature **B1**, so tripling the radius makes the luminosity $3^2 = 9$ times larger: 3.6×10^{27} W **B1**.

Exam-style 3.4 ▪ 2 marks

Outline how a star's **radius** can be estimated from its spectrum.

Solution 3.4

Method: Wien's law

Worked steps

Find T from $\lambda_{\max}$ (Wien's law) and measure L **B1**; then use the Stefan–Boltzmann law $L = 4\pi\sigma r^2 T^4$ to solve for r **B1**.

Exam-style 3.5 ▪ 2 marks

The light from a distant star is analysed. Its intensity is greatest at a wavelength of 580 nm, and its radiant flux intensity at the Earth is 3.6×10^{-10} W m^{-2}. The star is 8.4×10^{17} m away. Take Wien's displacement constant as 2.90×10^{-3} m K and $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}.

(a) **State** Wien's displacement law.

(b) **Determine** the surface temperature of the star.

(c) **Determine** the luminosity of the star.

(d) **Determine** the radius of the star, and **compare** it with the Sun's radius of 7.0×10^{8} m.

(e) A second star has the **same** luminosity but a surface temperature twice as high.

Determine the ratio $\dfrac{\text{radius of the second star}}{\text{radius of the first}}$, and **state** what kind of star that suggests.

Solution 3.5

Worked steps

(a) The wavelength at which the emitted intensity is a **maximum** is **inversely proportional** to the thermodynamic temperature of the body **B1**: $\lambda_{\max} T =$ constant **B1**.

(b) $T = \dfrac{2.90 \times 10^{-3}}{580 \times 10^{-9}}$ **M1** $= 5.0 \times 10^{3}$ K **A1**.

(c) $L = 4\pi d^2 F = 4\pi(8.4 \times 10^{17})^2 \times 3.6 \times 10^{-10}$ **M1** $= 3.2 \times 10^{27}$ W **A1**.

(d) $r = \sqrt{\dfrac{L}{4\pi\sigma T^4}} = \sqrt{\dfrac{3.19 \times 10^{27}}{4\pi(5.67 \times 10^{-8})(5000)^4}}$ **M1** $= 7.6 \times 10^{8}$ m **A1**; that is about 1.1 times the Sun's radius, so it is a star of very similar size **A1**.

(e) At fixed luminosity $r^2 T^4$ is constant, so $r \propto \dfrac{1}{T^2}$ **M1**; doubling T gives a radius $\dfrac{1}{4}$

Solution 3.5

Worked steps

as large, i.e. a ratio of 0.25 A1. A star of the same luminosity packed into a quarter of the radius and much hotter suggests a **white dwarf** rather than a main-sequence star B1.