

Exercise walk-through

24.2.1

**Intensity against hardness,
attenuation through layers, and
sharpness against contrast**

A-Level Physics

You must be able to

- explain how X-rays are produced by electron bombardment, and calculate the minimum wavelength from the accelerating p.d.
- state which control sets the **intensity** of the beam and which sets its **hardness**, and why
- use $I = I_0 e^{-\mu x}$, including through **layers** of different materials, and use the half-value thickness
- distinguish **sharpness** from **contrast**, and state what improves each

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The method

Method — The tube has two controls, and they do different jobs

Electrons are released from a **heated filament**, accelerated through a large **p.d.**, and stopped suddenly by a metal **target**. Most of their kinetic energy becomes heat, which is why the target is cooled; a small fraction becomes X-ray photons.

Control	Sets	Why
filament current	the intensity	a hotter filament emits more electrons per second , so more photons per second
accelerating p.d.	the hardness (penetrating power)	a larger p.d. gives each electron more energy, so higher-energy, shorter-wavelength photons

Method — The tube has two controls, and they do different jobs

- Swapping these two round is the commonest error in the topic. The p.d. does **not** set the intensity, and the filament current does **not** set the hardness.
- A metal **filter** is placed in the beam to absorb the softest X-rays, which would be absorbed in the patient's skin without ever reaching the detector, adding dose but no information.

The spectrum stops abruptly at a **minimum wavelength**, which is the case where an electron gives up **all** its kinetic energy in a single collision:

$$eV = \frac{hc}{\lambda_{\min}} \quad \Rightarrow \quad \lambda_{\min} = \frac{hc}{eV}$$

Method — The tube has two controls, and they do different jobs

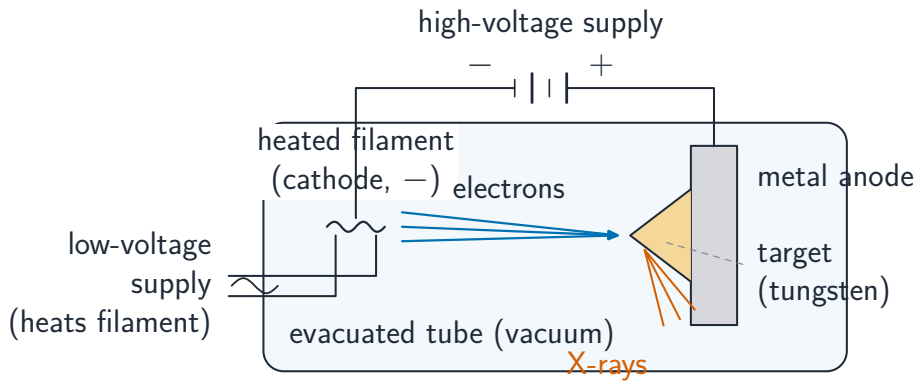
Worked example. Find $\lambda_{\min}$ for a tube at 50 kV.

$$\lambda_{\min} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{(1.60 \times 10^{-19})(50 \times 10^3)} = \mathbf{2.5 \times 10^{-11} \text{ m}}$$

- $\lambda_{\min}$ depends **only** on the p.d., not on the target material or the current. Doubling the p.d. halves it.

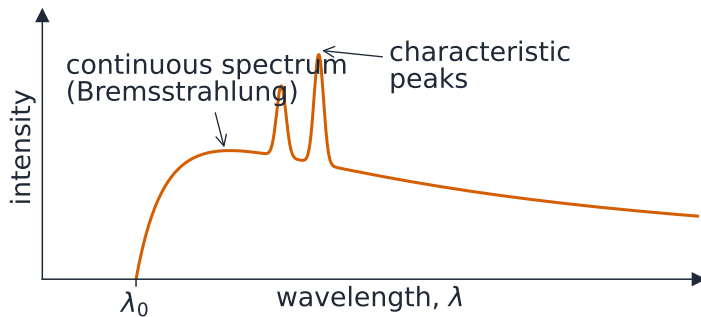
Method — The tube has two controls, and they do different jobs

Find ... for a tube at



Method — The tube has two controls, and they do different jobs

Find ... for a tube at



X-ray: continuous bremsstrahlung + characteristic peaks

Method — Attenuation, layers, and the half-value thickness

$$I = I_0 e^{-\mu x}$$

Worked example. *A beam passes through 2.8 cm of a material with $\mu = 1.4 \text{ cm}^{-1}$. What fraction is absorbed?*

$$\frac{I}{I_0} = e^{-1.4 \times 2.8} = e^{-3.92} = 0.020$$

So 2.0% is transmitted and **98%** is absorbed.

- Read the question twice: “absorbed” is $1 - I/I_0$, not I/I_0 .
- Keep μ and x in the **same** units. A μ in cm^{-1} needs x in cm.

Method — Attenuation, layers, and the half-value thickness

Through layers, the exponents **add**, because each layer attenuates what the previous one passed on:

$$\frac{I}{I_0} = e^{-(\mu_1 x_1 + \mu_2 x_2)}$$

The **half-value thickness** is the thickness that halves the intensity:

$$x_{1/2} = \frac{\ln 2}{\mu}$$

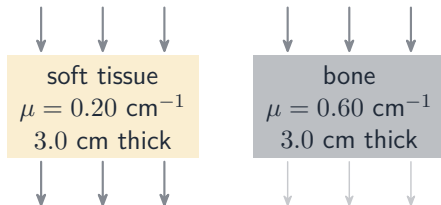
Here $x_{1/2} = 0.693/1.4 = 0.50$ cm, so 2.8 cm is 5.6 half-value thicknesses and $(1/2)^{5.6} = 0.020$ – the same answer, and a useful check.

Method — Sharpness and contrast are different things

- **Contrast** is the difference in **degree of blackening** between neighbouring regions. It comes from a difference in μ – bone attenuates far more than soft tissue, so it shows white against grey. Contrast is improved by choosing a p.d. that maximises the difference in attenuation, or by using a **contrast medium** such as barium, whose large μ makes a soft-tissue organ visible at all.
- **Sharpness** is how well **defined the edges** are. It is improved by a **small** X-ray source (a small spot on the target), a **stationary** patient, and the detector **close** to the patient.
- **The two are independent.** An image can be sharp with poor contrast, or high in contrast but blurred. A question naming one is not asking about the other.

Method — Sharpness and contrast are different things

X-ray beam, intensity I_0 everywhere



$$I/I_0 = e^{-0.60} = 0.55 \quad I/I_0 = e^{-1.80} = 0.17$$

detector well exposed: dark little exposure: white

contrast comes from the difference in μ : the two exposures differ by a factor of 3

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Practice

Practice 2.1 ■ 2 marks

State which control of an X-ray tube sets the **intensity** of the beam, and which sets its **hardness**.

Solution 2.1

Method: The tube has two controls, and they do different jobs — p.4

Worked steps

- the **filament current** sets the intensity: a hotter filament releases more electrons per second **B1**
- the **accelerating p.d.** sets the hardness: a larger p.d. gives each electron more energy and so produces higher-energy, more penetrating photons **B1**

Practice 2.2 ■ 3 marks

Calculate the minimum wavelength of the X-rays from a tube operating at 80 kV.

Solution 2.2

Method: The tube has two controls, and they do different jobs — p.4

Worked steps

- $\lambda_{\min} = hc/(eV)$ **M1**
- $= (6.63 \times 10^{-34})(3.00 \times 10^8) \div (1.60 \times 10^{-19} \times 80 \times 10^3)$ **M1**
- $= 1.6 \times 10^{-11} \text{ m}$ **A1**

Practice 2.3 ■ 2 marks

State what is meant by the **half-value thickness**, and write it in terms of μ .

Solution 2.3

Method: Attenuation, layers, and the half-value thickness — p.9

Worked steps

- the **thickness** of a material that reduces the intensity of the beam to **half** its incident value **B1**
- $x_{1/2} = \ln 2 / \mu$, or $0.693 / \mu$ **B1**

Practice 2.4 ■ 2 marks

State the difference between the **sharpness** and the **contrast** of an X-ray image.

Solution 2.4

Method: Sharpness and contrast are different things — p.11

Worked steps

- **sharpness** is how well **defined the edges** of structures are in the image B1
- **contrast** is the difference in **degree of blackening** between neighbouring regions, which comes from a difference in attenuation B1

Practice 2.5 ■ 2 marks

State why a metal filter is placed in the beam leaving an X-ray tube.

Solution 2.5

Method: The tube has two controls, and they do different jobs — p.4

Worked steps

- it absorbs the **softest** (lowest-energy, longest-wavelength) X-rays **B1**
- those would be absorbed in the patient's skin without reaching the detector, so they add to the radiation dose without contributing to the image **B1**

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Exam-style

Exam-style 3.1 ■ 6 marks

An X-ray beam passes through 2.8 cm of tissue for which $\mu = 1.4 \text{ cm}^{-1}$.

Exam-style 3.1 (a) ■ 3 marks

Calculate the percentage of the incident intensity that is **absorbed**.

Solution 3.1 (a)

Method: Attenuation, layers, and the half-value thickness — p.9

Worked steps

$$I/I_0 = e^{-\mu x} = e^{-1.4 \times 2.8} = e^{-3.92} \quad \text{M1}$$

- = 0.020, so 2.0% is transmitted A1
- the percentage **absorbed** is $100 - 2.0 = 98\%$ A1
- quoting 2.0% answers the opposite question

Exam-style 3.1 (b) ■ 3 marks

Calculate the half-value thickness of this tissue, and use it to check your answer to (a).

Solution 3.1 (b)

Worked steps

$$x_{1/2} = \ln 2 / \mu = 0.693 / 1.4 = \mathbf{0.50} \text{ cm} \quad \text{M1} \quad \text{A1}$$

- 2.8 cm is $2.8 / 0.50 = 5.6$ half-value thicknesses, and $(1/2)^{5.6} = 0.020$, which agrees with (a) B1

Exam-style 3.2 ■ 4 marks

A beam passes through 1.5 cm of bone ($\mu = 2.9 \text{ cm}^{-1}$) and then 4.0 cm of soft tissue ($\mu = 0.20 \text{ cm}^{-1}$).
Calculate the fraction of the incident intensity that emerges.

Solution 3.2

Method: Attenuation, layers, and the half-value thickness — p.9

Worked steps

- the exponents add, because each layer attenuates what the one before it transmitted **M1**
- bone: $\mu x = 2.9 \times 1.5 = 4.35$ **M1**
- tissue: $\mu x = 0.20 \times 4.0 = 0.80$, giving a total of 5.15 **M1**
- $I/I_0 = e^{-5.15} = 5.8 \times 10^{-3}$ **A1**
- multiplying the two thicknesses, or averaging the two μ values, is the error this question is set to catch

Exam-style 3.3 ■ 3 marks

Explain why bone appears white and soft tissue grey on an X-ray image, and state what is meant by the **contrast** of the image.

Solution 3.3

Method: Sharpness and contrast are different things — p.11

Worked steps

- bone has a much **larger attenuation coefficient** than soft tissue, so far less of the beam passes through it (B1)
- less radiation reaches the detector behind bone, so that region is **less blackened** and appears white, while more passes through soft tissue and blackens the image there (B1)
- the **contrast** is the difference in degree of blackening between neighbouring regions of the image (B1)

Exam-style 3.4 ■ 4 marks

State **two** changes to the apparatus that would improve the **sharpness** of an X-ray image, and explain why each works.

Solution 3.4

Method: Sharpness and contrast are different things — p.11

Worked steps

any two, each with its reason:

- use a **smaller** X-ray source – a smaller spot on the target – so that the edge of a structure casts a narrower penumbra **B1** **B1**
- keep the **patient still**, since any movement during the exposure blurs every edge **B1** **B1**
- place the **detector close to the patient**, which reduces the geometric spreading of the beam between the structure and the image
- a **contrast medium** is not accepted here: it improves contrast, not sharpness

Exam-style 3.5 ■ 4 marks

Computed tomography (CT) scanning produces a 3D image of an **internal** structure: **multiple** X-ray images of the same section are taken from different angles and combined to **obtain** a 2D image of that section, and the process is repeated along an axis so that the 2D images of many sections can be combined.

Exam-style 3.5 (a) ■ 2 marks

State **two** advantages of a CT image over a single X-ray image of the same body part.

Solution 3.5 (a)

Worked steps

any two: a CT image shows a **cross-section**, so structures are not superimposed on one another as they are in a single X-ray **B1**

- it has much better **contrast between soft tissues**, because each voxel's attenuation is computed from many views rather than from one; and the image can be rotated and viewed as a three-dimensional structure or as any chosen slice **B1**

Exam-style 3.5 (b) ■ 2 marks

State **one** disadvantage, and explain it.

Solution 3.5 (b)

Worked steps

the **radiation dose** is much higher B1

- because many exposures are taken from many angles for each section, and the process is repeated for every section along the axis; the scan also takes longer and the equipment is far more expensive B1

Exam-style 3.6 ■ 3 marks

A radiographer increases the accelerating p.d. of an X-ray tube. State the effect on the minimum wavelength, on the penetrating power of the beam, and on the intensity.

Solution 3.6

Method: The tube has two controls, and they do different jobs — p.4

Worked steps

- the minimum wavelength **decreases**, since $\lambda_{\min} = hc/(eV)$ and V has increased **B1**
- the beam becomes **more penetrating**, because the photons have higher energy: it is harder **B1**
- the **intensity** also rises somewhat, because each electron delivers more energy, but the intensity is **controlled** by the filament current, not by the p.d. **B1**