Exercise walk-through

Radioactive decay ▪ 23.2

eXercise

You must be able to

- describe decay as **spontaneous** and **random**
- use **activity** $A = \lambda N$ and $\lambda = 0.693/t_{1/2}$
- use the exponential decay $x = x_0 e^{-\lambda t}$ and **half-life**

Method — Random and exponential

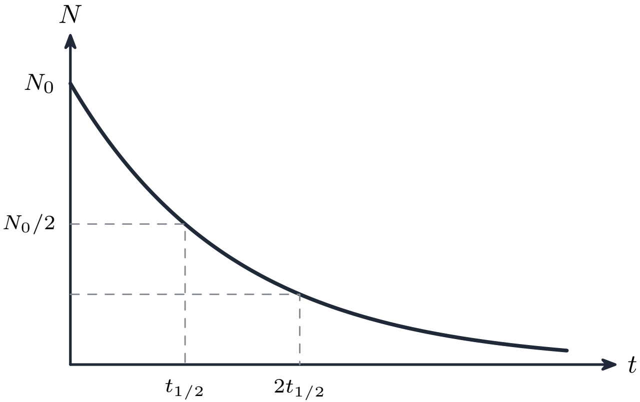
Decay is **spontaneous** (no trigger) and **random** (the count rate fluctuates). The number of undecayed nuclei falls **exponentially**, halving every **half-life**:

$$A = \lambda N, \qquad \lambda = \frac{0.693}{t_{1/2}}, \qquad N = N_0 e^{-\lambda t}.$$

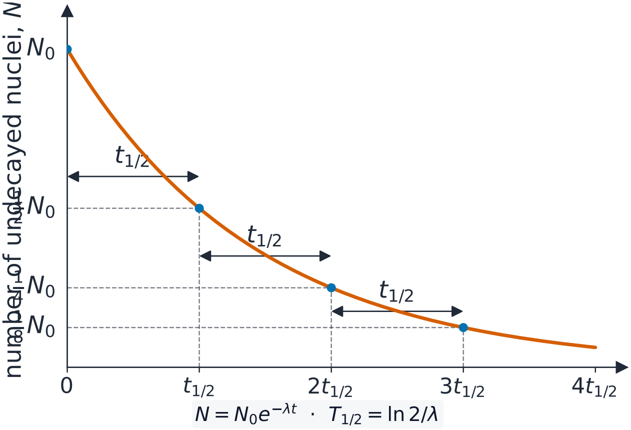
- **activity** A — decays per second (becquerel, Bq); **decay constant** λ — probability of decay per second.

The same exponential law applies to the **activity** and the **count rate**: $A = A_0 e^{-\lambda t}$.

Method — Random and exponential



Method — Random and exponential



The exponential radioactive decay curve

Method — Finding the time — take logs

For a **whole number** of half-lives, just halve repeatedly (after n half-lives a fraction $(\frac{1}{2})^n$ remains). For **any** time or fraction, take the natural log of $N = N_0 e^{-\lambda t}$:

$$\ln \frac{N}{N_0} = -\lambda t \quad \implies \quad t = -\frac{1}{\lambda} \ln \frac{N}{N_0}.$$

Worked example. A source has half-life 8.0 days, so $\lambda = \dfrac{0.693}{8.0} = 0.0866$ day^{-1}. The time for its activity to fall to 20% is $t = -\dfrac{1}{0.0866} \ln(0.20) = 18.6$ days.

Practice 2.1 ▪ 2 marks

State **two** features of radioactive decay.

Solution 2.1

Method: Random and exponential

Worked steps

It is **spontaneous** (unaffected by external conditions) and **random** (cannot predict which nucleus decays when) B1 B1.

Practice 2.2 ▪ 2 marks

Define **activity** and the **decay constant**.

Solution 2.2

Method: Random and exponential

Worked steps

Activity — the number of decays per unit time; **decay constant** — the probability per unit time that a nucleus decays **B1** **B1**.

Practice 2.3 ▪ 1 mark

Define **half-life**.

Solution 2.3

Worked steps

The time for **half** the undecayed nuclei (or the activity) to decay **B1**.

Practice 2.4 ▪ 1 mark

Write the equation linking **activity**, **decay constant** and **number** of nuclei.

Solution 2.4

Method: Random and exponential

Worked steps

$A = \lambda N$ **B1**.

Exam-style 3.1 ▪ 1 mark

The **half-life** is the time for:

A all the nuclei to decay

B half the (undecayed) nuclei to decay

C the activity to reach exactly zero

D the decay constant to halve

Solution 3.1

- **A** all the nuclei to decay
- **B** half the (undecayed) nuclei to decay ✓
- **C** the activity to reach exactly zero
- **D** the decay constant to halve

Worked steps

B — half the undecayed nuclei to decay.

Exam-style 3.2 ▪ 2 marks

A source has a decay constant of 0.040 s^{-1}. Calculate its **half-life**.

Solution 3.2

Worked steps

$$t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.040} = 17 \text{ s}$$ **M1** **A1**.

Exam-style 3.3 ▪ 2 marks

A sample has 2.0×10^{20} undecayed nuclei and a decay constant of 1.5×10^{-9} s^{-1}. Calculate its **activity**.

Solution 3.3

Method: Random and exponential

Worked steps

$A = \lambda N = (1.5 \times 10^{-9})(2.0 \times 10^{20}) = 3.0 \times 10^{11}$ Bq **M1** **A1**.

Exam-style 3.4 ▪ 2 marks

A radioactive isotope has a half-life of 6.0 hours. What **fraction** of the original nuclei remains after 18 hours?

Solution 3.4

Method: Finding the time — take logs

Worked steps

18 h $= 3$ half-lives **M1**; fraction $= (\frac{1}{2})^3 = \frac{1}{8}$ **A1**.

Exam-style 3.5 ▪ 4 marks

A radioactive source has a half-life of 8.0 days. Calculate the **time** for its activity to fall to 20% of its initial value.

Solution 3.5

Method: Finding the time — take logs

Worked steps

$\lambda = \dfrac{0.693}{t_{1/2}} = \dfrac{0.693}{8.0} = 0.0866\ \text{day}^{-1}$ **M1**; $t = -\dfrac{1}{\lambda}\ln\dfrac{A}{A_0} = -\dfrac{1}{0.0866}\ln(0.20)$ **M1** **M1** $= 18.6$ days **A1**.

Exam-style 3.6 ▪ 2 marks

Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(a) **State** what is meant by a **tracer**.

(b) The decay is ${}^{15}_{8}\text{O} \rightarrow {}^{P}_{Q}\text{N} + {}^{R}_{S}\text{Z}$. **State** the values of P, Q, R and S, and **name** the particle Z.

(c) **Define** the **activity** of a radioactive sample.

(d) **Determine** the decay constant of oxygen-15. Give a **unit** with your answer.

(e) **Determine** the initial activity of the sample.

(f) **Determine** the activity 10 minutes after preparation.

(g) **Suggest** why a tracer with a half-life of a few **days** would be unsuitable for this use.

Solution 3.6

Method: Random and exponential

Worked steps

(a) A radioactive substance introduced into a system **B1** whose progress can be followed from outside by detecting the radiation it emits **B1**.

(b) $P = 15$, $Q = 7$, $R = 0$, $S = +1$ **B1** **B1**; Z is a **positron** **B1**.

(c) The **rate of decay** — the number of nuclei decaying per unit time **B1**.

(d) $\lambda = \dfrac{\ln 2}{t_{1/2}} = \dfrac{0.693}{122}$ **M1** $= 5.7 \times 10^{-3}\ \text{s}^{-1}$ **A1**.

(e) $A = \lambda N = 5.68 \times 10^{-3} \times 4.8 \times 10^{14}$ **M1** $= 2.7 \times 10^{12}$ Bq **A1**.

(f) 10 minutes is 600 s **M1**; $A = A_0 e^{-\lambda t} = 2.73 \times 10^{12} \times e^{-5.68\times10^{-3}\times600}$ **M1**

Solution 3.6

$= 2.73 \times 10^{12} \times 0.0332 = 9.1 \times 10^{10}$ Bq **A1**. (That is about five half-lives, so roughly $1/32$ of the start.)

(g) The patient would keep receiving **ionising radiation** long after the scan is over, raising the dose and the risk of cell damage **B1**; the tracer should decay away soon after the measurement, which a half-life of minutes achieves and one of days does not **B1**.