Exercise walk-through

23.2.1

Random decay, the exponential law and the log graph

A-Level Physics

You must be able to

- understand that fluctuations in **count rate 计数率** are evidence for the random nature of **radioactive decay 放射性衰变**
- understand that radioactive decay is both spontaneous and random
- define **activity 放射性活度** and the **decay constant 衰变常数**, and recall and use $A = \lambda N$
- define **half-life 半衰期** and use $\lambda = \dfrac{0.693}{t_{1/2}}$
- understand the exponential nature of radioactive decay, and sketch and use $x = x_0 e^{-\lambda t}$, where x may be the activity, the number of undecayed nuclei, or a received count rate

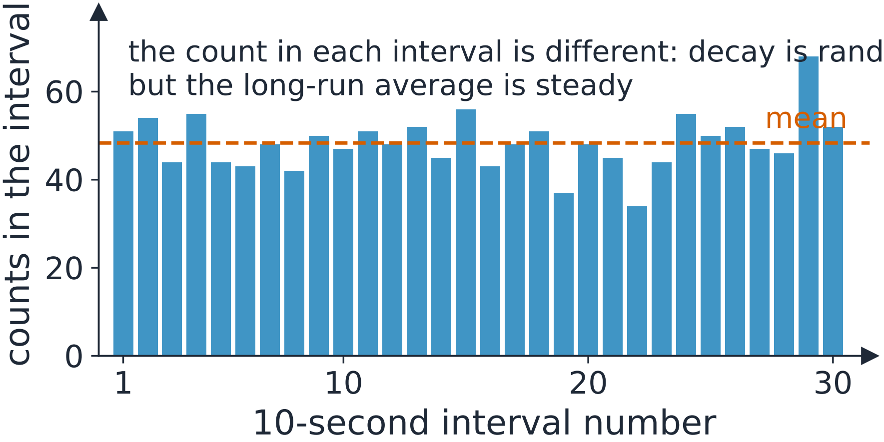
1 The method

Method — Two words, two separate marks

word	what it means	how you would answer
spontaneous	the decay is **not affected by anything outside the nucleus**	changing the temperature, the pressure or the chemical state of the sample does **not** change the rate
random	you **cannot predict which nucleus** will decay next, or when; every undecayed nucleus has the **same probability** of decaying in the next interval	repeated counts over equal intervals **fluctuate** about a mean instead of repeating exactly

- The two words are independent, and a question asking for both gets no credit for saying the same thing twice.
- The fluctuation evidence is the one experimental fact you are expected to quote: the count rate from a long-lived source is **not constant**, even though its activity is.

Method — Two words, two separate marks



Method — The exponential law, and the three things it describes

$$x = x_0 e^{-\lambda t}$$

x may be any of three quantities, and they all decay with the **same** λ:

x	**what it is**
N	the number of **undecayed nuclei** left in the sample
$A = \lambda N$	the **activity**: decays per second, in becquerel
C	the **received count rate** at a detector

Method — The exponential law, and the three things it describes

- The **received count rate is not the activity**. A detector collects only a small fraction of the radiation – it subtends a small solid angle, some radiation is absorbed on the way, and the detector is not 100% efficient – so C is proportional to A, not equal to it. Because it is *proportional*, it decays with the same λ and gives the same half-life.
- **Background radiation must be subtracted first.** A background count is a constant added to every reading; it does not decay, so leaving it in bends the graph.

Method — The log graph – the workhorse of this topic

Take natural logs of both sides:

$$\ln A = \ln A_0 - \lambda t$$

That is $y = c + mx$, so:

1. a **straight line** is the evidence that the decay really is **exponential**;
2. the **gradient** is $-\lambda$, so $\lambda = -$gradient, and it is positive;
3. the **intercept** on the $\ln A$ axis is $\ln A_0$, so $A_0 = e^{\text{intercept}}$.

Worked example. A graph of $\ln(C/s^{-1})$ *against t is a straight line through* $(0,\ 4.80)$ *and* $(600\ \text{s},\ 3.00)$. *Find the decay constant and the half-life.*

- the gradient is the change in $\ln C$ over the change in t

$$\text{gradient} = \frac{3.00 - 4.80}{600\ \text{s} - 0} = -3.0 \times 10^{-3}\ \text{s}^{-1}$$

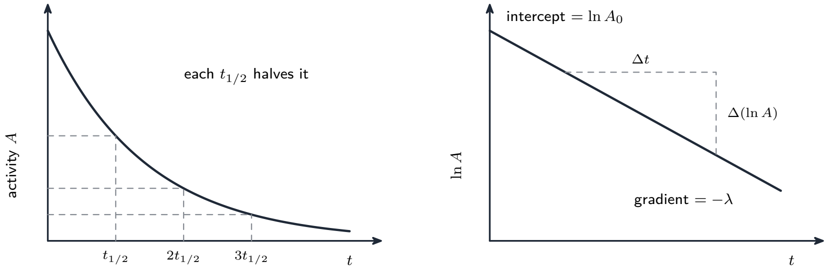
Method — The log graph – the workhorse of this topic

- so $\lambda = 3.0 \times 10^{-3}\ \mathsf{s}^{-1}$, and

$$t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{3.0 \times 10^{-3}\ \mathsf{s}^{-1}} = \mathbf{230}\ \mathsf{s}$$

Method — The log graph – the workhorse of this topic

Find the decay constant and the half-life



Take natural logs of $A = A_0 e^{-\lambda t}$ and you get $\ln A = \ln A_0 - \lambda t$. A **straight line** is then the evidence that the decay really is exponential, its **gradient is** $-\lambda$, and its **intercept** is $\ln A_0$. The same works for N and for a received count rate.

Method — Two shortcuts worth having

- **Whole numbers of half-lives.** After n half-lives the fraction left is $\left(\frac{1}{2}\right)^n$. So $\frac{1}{8}$ left means $n = 3$; falling from 800 to 100 means three halvings.
- **Any fraction at all.** Rearrange the exponential and take logs:

$$\frac{x}{x_0} = e^{-\lambda t} \quad \Longrightarrow \quad t = -\frac{1}{\lambda}\ln\left(\frac{x}{x_0}\right)$$

Keep λ and t in the **same** time unit, and remember $\ln$ of a fraction is negative, which is what makes t positive.

2 Practice

Practice 2.1 ▪ 2 marks

Explain what is meant by saying that radioactive decay is **spontaneous** and **random**.

Solution 2.1

Method: Two words, two separate marks — p.4

Worked steps

- **spontaneous**: the decay is not affected by anything outside the nucleus, so changing the temperature, pressure or chemical state does not change the rate **B1**
- **random**: it is impossible to predict which nucleus will decay next or when, and every undecayed nucleus has the same probability of decaying in the next interval **B1**

Practice 2.2 ▪ 2 marks

A student records the count rate from a long-lived source over several equal intervals and finds that the readings differ. Explain how this supports the idea that decay is random.

Solution 2.2

Method: Two words, two separate marks — p.4

Worked steps

- the source's activity is effectively constant, so any difference between readings cannot be a change in the source **B1**
- the readings **fluctuate about a mean**, which is what a random process produces; a predictable process would give the same count every time **B1**

Practice 2.3 ▪ 3 marks

A radioactive isotope has a half-life of 4.5 hours. Calculate its decay constant, in s^{-1}.

Solution 2.3

Method: The log graph – the workhorse of this topic — p.8

Worked steps

- convert the half-life to seconds first **M1**

$$t_{1/2} = 4.5 \text{ hours} \times 3600 \text{ s hour}^{-1} = 1.62 \times 10^4 \text{ s}$$

- then **M1**

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{1.62 \times 10^4 \text{ s}} = \mathbf{4.3 \times 10^{-5}} \text{ s}^{-1}$$ **A1**

Practice 2.4 ▪ 2 marks

A sample contains 6.0×10^{20} undecayed nuclei of an isotope whose decay constant is 2.2×10^{-8} s^{-1}. Calculate its activity.

Solution 2.4

Method: The exponential law, and the three things it describes — p.6

Worked steps

- use $A = \lambda N$ **M1**

$$A = 2.2 \times 10^{-8}\ \text{s}^{-1} \times 6.0 \times 10^{20} = \mathbf{1.3 \times 10^{13}}\ \text{Bq}$$ **A1**

Practice 2.5 ▪ 2 marks

Sketch a graph of $\ln A$ against t for a radioactive source, and state what its gradient represents.

Solution 2.5

Method: The log graph – the workhorse of this topic — p.8

Worked steps

- a **straight line** with a **negative** gradient, starting at $\ln A_0$ on the vertical axis B1
- the gradient is $-\lambda$, so the decay constant is minus the gradient B1

Practice 2.6 ▪ 1 mark

The activity of a source falls to $\frac{1}{8}$ of its initial value. State how many half-lives have passed.

Solution 2.6

Method: Two shortcuts worth having — p.11

Worked steps

- $\frac{1}{8} = \left(\frac{1}{2}\right)^3$, so **3 half-lives** B1

3 Exam-style

Exam-style 3.1 ▪ 1 mark

The activity of a source falls from 800 Bq to 100 Bq in 27 minutes. Its half-life is

A 3.0 minutes

B 9.0 minutes

C 13.5 minutes

D 27 minutes

Solution 3.1

Method: The log graph – the workhorse of this topic — p.8

A 3.0 minutes

B 9.0 minutes ✓

C 13.5 minutes

D 27 minutes

Worked steps

B

- $800 \to 400 \to 200 \to 100$ is **three** halvings **B1**
- 27 min $\div\, 3 = 9.0$ min. **A** divides by the wrong number of halvings; **C** assumes two; **D** assumes one

Exam-style 3.2 ▪ 6 marks

A student measures the count rate from a source whose half-life is many years, recording the number of counts in five successive 10 s intervals:

$$412 \qquad 389 \qquad 405 \qquad 421 \qquad 398$$

Exam-style 3.2 (a) ▪ 2 marks

The activity of this source is effectively constant over the measurement. Explain why the five readings are nevertheless different.

Solution 3.2 (a)

Worked steps

- decay is a **random** process, so the number of nuclei decaying in any 10 s interval is not fixed B1
- the counts therefore **fluctuate about a mean** value; the variation is in the counting, not in the source B1

Exam-style 3.2 (b) ▪ 2 marks

Calculate the mean count rate, in s^{-1}.

Solution 3.2 (b)

Worked steps

- add the five readings and divide by five, then by the interval **M1**

$$\overline{C} = \frac{412 + 389 + 405 + 421 + 398}{5} = 405 \text{ counts per } 10 \text{ s} = \mathbf{40.5} \text{ s}^{-1}$$ **A1**

Exam-style 3.2 (c) ▪ 2 marks

The detector receives and records only 1.2% of the radiation emitted by the source. Calculate the activity of the source.

Solution 3.2 (c)

Worked steps

- the detector records a fixed fraction of the emitted radiation **M1**

$$A = \frac{40.5\ \mathrm{s}^{-1}}{0.012} = \mathbf{3.4} \times \mathbf{10^3}\ \mathrm{Bq}$$ **A1**

Exam-style 3.3 ▪ 10 marks

A student measured the count rate C from a radioactive source at intervals, subtracted the background count, and plotted $\ln(C/\mathrm{s}^{-1})$ against time t. The points lay on a straight line passing through $(0,\ 5.60)$ and $(900\ \mathrm{s},\ 3.44)$.

Exam-style 3.3 (a) ▪ 2 marks

Explain how the straight line shows that the decay is exponential.

Solution 3.3 (a)

Method: The log graph – the workhorse of this topic — p.8

Worked steps

- taking logs of $C = C_0 e^{-\lambda t}$ gives $\ln C = \ln C_0 - \lambda t$ **B1**
- this has the form $y = c + mx$, so $\ln C$ against t is a straight line **only if** the decay is exponential; the points lying on a line is therefore the evidence **B1**

Exam-style 3.3 (b) ▪ 2 marks

Use the graph to calculate the decay constant of the source.

Solution 3.3 (b)

Worked steps

- the gradient of the line is $-\lambda$ **M1**

$$\text{gradient} = \frac{3.44 - 5.60}{900\ \text{s} - 0} = -2.4 \times 10^{-3}\ \text{s}^{-1}$$

$$\lambda = \mathbf{2.4 \times 10^{-3}}\ \text{s}^{-1}$$ **A1**

Exam-style 3.3 (c) ▪ 2 marks

Calculate the half-life of the source.

Solution 3.3 (c)

Worked steps

- use $\lambda = \dfrac{0.693}{t_{1/2}}$ **M1**

$$t_{1/2} = \frac{0.693}{2.4 \times 10^{-3}\ \mathsf{s}^{-1}} = \mathbf{290}\ \mathsf{s}$$ **A1**

Exam-style 3.3 (d) ▪ 2 marks

Calculate the corrected count rate at $t = 0$.

Solution 3.3 (d)

Worked steps

- the intercept is $\ln C_0$, so take the exponential of it **M1**

$$C_0 = e^{5.60} = \mathbf{270\ s^{-1}}$$ **A1**

Exam-style 3.3 (e) ▪ 2 marks

Suggest and explain the effect on the shape of the graph if the student had **not** subtracted the background count.

Solution 3.3 (e)

Worked steps

- the background count is a **constant** added to every reading, and it does **not** decay B1
- so at long times the measured count rate tends towards the background instead of towards zero, $\ln C$ levels off, and the graph **curves away** from the straight line rather than continuing down B1