Past-paper walk-through

Mass defect and nuclear binding energy ▪ 23.1

eXercise

Questions in this set

- 9702/42 N25 Q9 ▪ 13 marks
- 9702/44 J25 Q11 ▪ 9 marks
- 9702/42 M25 Q9 ▪ 10 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 9

9702/42 N25 ▪ Q9 ▪ 13 marks

(a) State what is meant by the mass defect of a nucleus. **[2]**

Question 9

nuclide	nuclide mass / u
$^{2}_{1}H$	2.013 553
$^{4}_{2}He$	4.001 505

Question 9

9702/42 N25 ▪ Q9 ▪ 13 marks

(b) The nuclear fusion reaction for the formation of helium-4 from deuterium is represented by

$$^{2}_{1}\text{H} + ^{2}_{1}\text{H} \rightarrow ^{4}_{2}\text{He}$$

Table 9.1 shows the masses of the nuclides involved in this reaction.

Table 9.1

nuclide	nuclide mass/u
$^{2}_{1}\text{H}$	2.013553
$^{4}_{2}\text{He}$	4.001505

Question 9

Calculate the energy released in the formation of 1.00 mol of helium-4. **[4]**

Question 9

9702/42 N25 ▪ Q9 ▪ 13 marks

(c)(i) Determine a value for the luminosity of Sirius. Give a unit with your answer. **[2]**

(c)(ii) Use your answer in (c)(i) to determine the surface temperature of Sirius. **[2]**

Question 9

9702/42 N25 ▪ Q9 ▪ 13 marks

(d) Explain how cosmologists use standard candles to estimate the distance of a galaxy from the Earth. **[3]**

Solution 9

(a)

Mark scheme

- difference between mass of nucleus and mass of (constituent) nucleons **M1**
- when nucleons are separated to infinity **A1**

Solution 9

(b) Worked steps

Find the mass lost in one reaction, turn it into energy, then scale to the amount asked for.

- $\Delta m = (2 \times 2.013553) - 4.001505 = 0.025601$ u
- $E = \Delta mc^2 = 0.025601 \times 1.66 \times 10^{-27} \times (3.00 \times 10^8)^2$
- $E = 3.82 \times 10^{-12}$ J per helium-4 nucleus.
- For a stated mass of helium, divide by the mass of one nucleus to get the number of reactions, then multiply by that energy.

Work in atomic mass units until the last step: converting each mass separately loses precision in a difference of two nearly equal numbers. **Mark scheme**

Solution 9

- Δm = (2 × 2.013553) – (4.001505) (u)
- (= 0.025601 u) **C1**
- E = c2Δm **C1**
- energy from one He-4 nucleus= 0.025601 × 1.66 × 10–27 × (3.00 × 108)2
- (= 3.82 × 10–12 J) **C1**
- energy to form 1.00 mol= 3.82 × 10–12 × 6.02 × 1023
- = 2.30 × 1012 J **A1**

Solution 9

(c)(i) Worked steps

Luminosity is the rate at which the star radiates energy, and its mass loss is that energy.

- $L = \dfrac{\Delta m}{\Delta t}c^2 = 1.09 \times 10^{11} \times (3.00 \times 10^8)^2$
- $L = 9.81 \times 10^{27}$ W

Solution 9

The unit is asked for: watts.

Mark scheme

- $L = 1.09 \times 1011 \times (3.00 \times 108)2$ **C1**
- $= 9.81 \times 1027$ W **A1**

Solution 9

(c)(ii) Worked steps

- Stefan's law: $L = 4\pi r^2 \sigma T^4$, so $T = \left(\frac{L}{4\pi r^2 \sigma}\right)^{1/4}$.
- $T = \left(\frac{9.81 \times 10^{27}}{4\pi \times (1.19 \times 10^9)^2 \times 5.67 \times 10^{-8}}\right)^{1/4}$
- $T = 9930$ K

Solution 9

A fourth root is forgiving of arithmetic slips but not of a missing $4\pi r^2$ — check the answer is a few thousand kelvin, as a star's surface must be.

Mark scheme

- L $= 4\pi\sigma$ r2T4
- 9.81 $\times$ 1027 $= 4\pi \times$ 5.67 $\times$ 10–8 $\times$ (1.19 $\times$ 109)2 $\times$ T4 **C1**
- T $=$ 9930 K **A1**

Solution 9

(d)

Mark scheme

- standard candles have known luminosity **B1**
- radiant flux intensity (from star) measured (on the Earth) **B1**
- distance found from $F = L\ /\ (4\pi d2)$ **B1**

Question 11

9702/44 J25 ▪ Q11 ▪ 9 marks

11 The deuterium nucleus ($^{2}_{1}H$) has a mass defect of 0.002 388 u. The helium-4 nucleus ($^{4}_{2}He$) has a mass defect of 0.030 377 u. Helium-4 is formed from deuterium in a nuclear reaction that can be represented by the equation

$$3\,^{2}_{1}H \longrightarrow {}^{4}_{2}He + {}^{1}_{1}p + {}^{1}_{0}n.$$

(a) **(i)** State the name of this type of nuclear reaction.

Question 11

(ii) Show that the energy released when one nucleus of helium-4 is formed from deuterium is 3.47×10^{-12} J.

Question 11

[3]

(b) A star has a radius of 6.96×10^{8} m. Helium-4 is produced in this star, from deuterium, at a mass rate of 7.34×10^{11} kg s^{-1}. All the energy released from this process is radiated away from the star. All the energy that is radiated from the star is released by this process.

(i) Calculate the luminosity of the star.

Question 11

Question 11

(ii) Use your answer in **(b)(i)** to determine the surface temperature of the star.

Question 11

[Total: 9]

Solution 11

(a)(i)

Mark scheme

- (nuclear) fusion

Solution 11

(a)(ii) Worked steps

Energy comes from the **mass defect** — the mass that disappears when four hydrogen nuclei become one helium nucleus.

- $\Delta m = 0.030377 - (3 \times 0.002388) = 0.023213$ u
- Convert to kilograms with 1 u $= 1.66 \times 10^{-27}$ kg, then use $E = \Delta m\, c^2$:
- $E = 0.023213 \times 1.66 \times 10^{-27} \times (3.00 \times 10^8)^2$
- $E = 3.47 \times 10^{-12}$ J

Solution 11

The three subtracted masses are the positrons or neutrinos the chain also emits, so read the question's own list rather than assuming a two-term defect. And square the speed of light — forgetting to gives an answer 3×10^8 times too small, which is the classic slip.

Mark scheme

- m $=$ [0.030377 – (3 $\times$ 0.002388)] u
- ($=$ 0.023213 u)
- E $=$ m c2
- $=$ 0.023213 $\times$ 1.66 $\times$ 10–27 $\times$ (3.00 $\times$ 108)2 $=$ 3.47 $\times$ 10–12 J

Solution 11

(b)(i) Worked steps

Luminosity is energy per second, so find how many helium nuclei the Sun makes each second and multiply by the energy each one releases.

- One mole of helium-4 has a mass of 4 g, so the rate of nuclei formed is $\dfrac{6.02 \times 10^{23} \times 7.34 \times 10^{11}}{4 \times 10^{-3}} = 1.10 \times 10^{38}$ s^{-1}.
- The same number comes from dividing by the mass of one atom: $\dfrac{7.34 \times 10^{11}}{4 \times 1.66 \times 10^{-27}}$ — use whichever route you find easier to keep straight.
- $L = 1.10 \times 10^{38} \times 3.47 \times 10^{-12} = 3.83 \times 10^{26}$ W

Solution 11

Watch the grams: a molar mass of 4 g is 4×10^{-3} kg, and mixing the two puts the luminosity out by a thousand. **Mark scheme**

- mass of 1 mol of helium-4 $=$ 4 g
- or
- mass of 1 helium atom $=$ 4 u
- N rate $=$ (6.02 $\times$ 1023 $\times$ 7.34 $\times$ 1011) / (4 $\times$ 10–3)
- or
- N rate $=$ (7.34 $\times$ 1011) / (4 $\times$ 1.66 $\times$ 10–27)
- luminosity $=$ 1.10 $\times$ 1038 $\times$ 3.47 $\times$ 10–12

Solution 11

- $= 3.83 \times 1026\,\mathrm{W}$

Solution 11

(b)(ii) Worked steps

The Sun radiates as a black body, so use Stefan's law with the luminosity just found.

- $L = 4\pi r^2 \sigma T^4$
- $3.83 \times 10^{26} = 4\pi \times 5.67 \times 10^{-8} \times (6.96 \times 10^8)^2 \times T^4$
- The bracket evaluates to 3.45×10^{11}, so $T^4 = \dfrac{3.83 \times 10^{26}}{3.45 \times 10^{11}} = 1.11 \times 10^{15}$
- $T = (1.11 \times 10^{15})^{1/4} = 5770$ K

Solution 11

Take the **fourth** root, not the square root — and sense-check the answer: the Sun's surface is about 5800 K, so a result in the thousands is right and one in the millions is the core.

Mark scheme

- $L = 4\pi\sigma r2T4$
- $3.83 \times 1026 = 4\pi \times 5.67 \times 10–8 \times (6.96 \times 108)2 \times T4$
- $T = 5770\,K$

Question 9

9702/42 M25 ▪ Q9 ▪ 10 marks

9 Polonium-193 ($^{193}_{84}Po$) is an unstable nuclide. A nucleus of polonium-193 decays to a nucleus of lead-189 ($^{189}_{82}Pb$) by emitting an alpha-particle.

(a) Radioactive decay is both random and spontaneous.

State what is meant by:

(i) random

Question 9

(ii) spontaneous.

Question 9

(b) Define half-life.

Question 9

(c) Data for the binding energy per nucleon of the particles involved in the decay of a nucleus of polonium-193 are given in Table 9.1.

Table 9.1

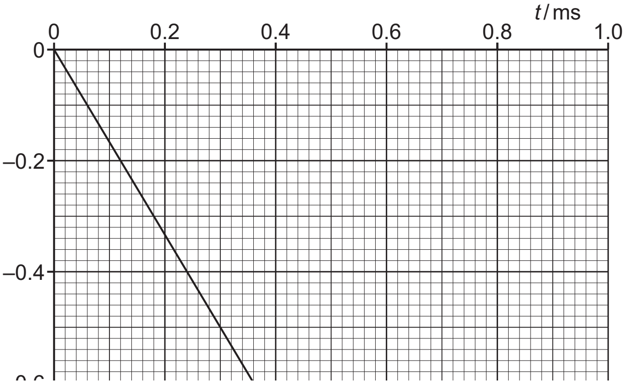
particle	binding energy per nucleon / eV
$^{193}_{84}Po$	7.774
$^{189}_{82}Pb$	7.826
$^{4}_{2}\alpha$	7.074

Determine the energy, in eV, released when a nucleus of polonium-193 decays into a nucleus of lead-189.

Question 9

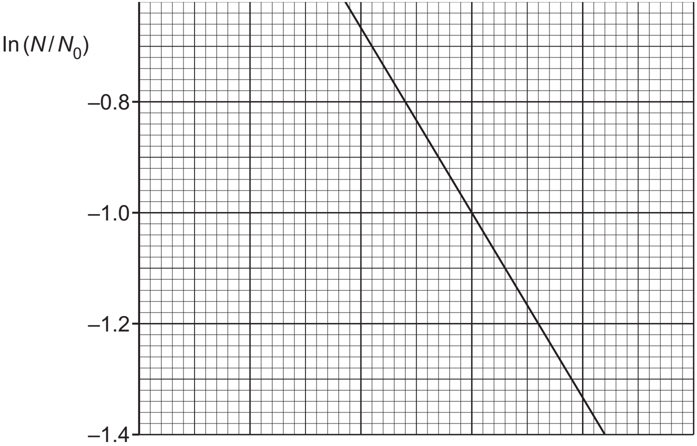
Question 9

(d) A pure sample of polonium-193 contains N_0 nuclei. After a time t the sample contains N nuclei of polonium-193. The variation of $\ln(N/N_0)$ with t is shown in Fig. 9.1.



Question 9

Question 9



Question 9

Fig. 9.1

Question 9

Fig. 9.1

(i) State the name of the quantity that is represented by the magnitude of the gradient of the line in Fig. 9.1.

Question 9

(ii) Use Fig. 9.1 to determine the half-life, in ms, of polonium-193.

Question 9

(e) Positron emission tomography (PET scanning) uses a radioactive tracer.

(i) State what happens to the positrons emitted by the tracer.

Question 9

(ii) Explain why a tracer with a half-life of approximately 2 hours is a suitable tracer to use.

Question 9

[Total: 10]

Solution 9

(a)(i)

Mark scheme

- either: cannot predict when a (particular) nucleus will decay
- or:
- cannot predict which nucleus will decay next

(a)(ii)

Mark scheme

- not affected by external / environmental factors

Solution 9

(b)

Mark scheme

- time for activity to halve

Solution 9

(c) Worked steps

Binding energy per nucleon is given, so multiply each by its nucleon number to get the total binding energy of each nuclide, then compare products with reactant.

- Products: $(189 \times 7.826) + (4 \times 7.074)$
- Reactant: (193×7.774)
- Energy released $= (189 \times 7.826) + (4 \times 7.074) - (193 \times 7.774)$
- $= 1479.1 + 28.3 - 1500.4 = 7.03$ MeV

Solution 9

Two things decide the sign and the units. The **products are more tightly bound**, so their total binding energy is larger, and that surplus is what is released — subtract the parent from the daughters, not the other way round. And the data are per nucleon in MeV, so the answer is in MeV; only convert to joules if the question asks for it.

Mark scheme

- energy $= (189 \times 7.826) + (4 \times 7.074) - (193 \times 7.774)$
- $= 7.03$ eV

Solution 9

(d)(i)

Mark scheme

- decay constant

Solution 9

(d)(ii) Worked steps

The decay constant is the **magnitude of the gradient** of the straight line, because taking logs of $A = A_0 e^{-\lambda t}$ gives $\ln A = \ln A_0 - \lambda t$.

- Read the gradient off the graph: $\lambda = \dfrac{1.4}{0.84}$ per millisecond.
- $\lambda = 1.7\ \mathrm{ms}^{-1}$

Solution 9

Use the **whole** line when reading the gradient, not two plotted points close together, and take the magnitude — the gradient itself is negative, the decay constant is not.

Mark scheme

- decay constant / magnitude of gradient $= 1.4 \,/\, 0.84$
- half-life $= \ln 2 \,/\, (1.4 \,/\, 0.84)$
- $= 0.42$ ms

Solution 9

(e)(i)

Mark scheme

- positrons collide with electrons and annihilate

Solution 9

(e)(ii) Worked steps

Half-life follows from the decay constant with one relation.

- $t_{\frac{1}{2}} = \dfrac{\ln 2}{\lambda} = \dfrac{0.693}{1.4/0.84}$
- $t_{\frac{1}{2}} = 0.42$ ms

Solution 9

The unit comes from λ: the gradient was read per millisecond, so the half-life is in milliseconds. Carrying your own λ forward earns the mark even if part (d) was wrong.

Mark scheme

- long enough to have time to conduct investigation, not so long as to cause patient unnecessary exposure to radiation
- 9702/42
- February/March 2025