Exercise walk-through

20.5.1

Induced e.m.f. from a graph: gradients, the rotating coil and Lenz in action

A-Level Physics

You must be able to

- define **magnetic flux 磁通量** and recall and use $\Phi = BA$, and use **magnetic flux linkage 磁链** $N\Phi$
- understand and explain experiments showing that a **changing** magnetic flux induces an e.m.f., that the induced e.m.f. opposes the change producing it, and what affects its magnitude
- recall and use **Faraday's law** and **Lenz's law**, including reading and sketching graphs of flux linkage and of induced e.m.f. against time

1 The method

Method — Faraday's law is a statement about a gradient

$$E = -\frac{\mathrm{d}(N\Phi)}{\mathrm{d}t}$$

Method — Faraday's law is a statement about a gradient

Read the lower graph straight off the **gradient** of the upper one:

the flux-linkage graph	**the induced e.m.f.**
a **straight** sloping section	**constant**, and not zero
a **flat** section	**zero** – the flux is not changing, so no e.m.f. at all
a **steeper** section	**larger** in magnitude
slope changes **sign**	the e.m.f. changes **sign**

Method — Faraday's law is a statement about a gradient

- **Faraday's and Lenz's laws** of **electromagnetic induction** are the two halves of this one equation: Faraday's law fixes the **size** of the e.m.f. (proportional to the rate of change of flux linkage), and Lenz's law fixes its **direction** (it opposes the change producing it).
- The minus sign **is** Lenz's law. It is not decoration: it is what makes the e.m.f. reverse when the flux starts to fall.
- A flat section is where marks are most often lost. A large, constant flux induces **nothing**.

Worked example. A coil of $N = 400$ turns and area 2.5×10^{-3} m^2 sits with its plane perpendicular to a field that falls steadily from 0.60 T to zero in 0.15 s. Find the magnitude of the induced e.m.f.

- the flux linkage falls from NBA to zero

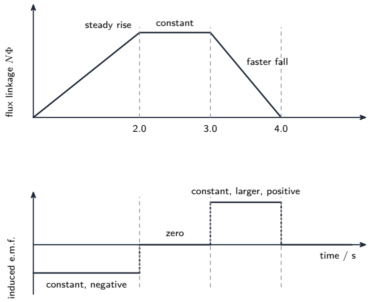
$$N\Phi = 400 \times 0.60\ \mathsf{T} \times 2.5 \times 10^{-3}\ \mathsf{m}^2 = 0.60\ \mathsf{Wb}$$

- divide the change by the time

$$|E| = \frac{\Delta(N\Phi)}{\Delta t} = \frac{0.60\ \mathsf{Wb}}{0.15\ \mathsf{s}} = \mathbf{4.0}\ \mathsf{V}$$

Method — Faraday's law is a statement about a gradient

Find the magnitude of the induced e.m.f



Faraday's law is a statement about a **gradient**: $E = -\dfrac{\mathrm{d}(N\Phi)}{\mathrm{d}t}$. A straight section of the upper graph gives a *constant* e.m.f.; a flat section gives *zero*; a steeper section gives a *bigger* one. The minus sign is Lenz's law, and it flips the sign of the e.m.f. when the flux starts to fall.

Method — A magnet dropped through a coil

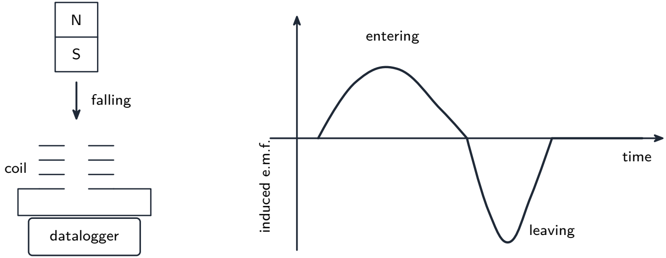
Three features, three separate marks:

1. **Two pulses of opposite sign.** As the magnet enters, the flux linkage **rises**; as it leaves, the flux linkage **falls**. Opposite gradients give opposite e.m.f.s.
2. **The second pulse is taller and narrower.** The magnet has **accelerated under gravity**, so it is moving faster when it leaves: the same change of flux happens in **less time**, so $\dfrac{\mathrm{d}(N\Phi)}{\mathrm{d}t}$ is larger and lasts for less time.
3. **The two areas are equal.** The area under an e.m.f.-time graph is $\int E\,\mathrm{d}t = \Delta(N\Phi)$, and the flux change entering equals the flux change leaving.

- Between the pulses the e.m.f. passes through **zero**, at the moment the magnet is symmetrically inside the coil and the flux linkage is momentarily stationary.

Method — A magnet dropped through a coil

Three features, three separate marks



Three features, three marks. The two pulses have **opposite signs**, because the flux first rises and then falls. The second is **taller and narrower**, because the magnet is moving faster, so the same flux change happens in less time. The two **areas are equal**, because each is the same total change of flux linkage.

Method — A coil rotating in a uniform field

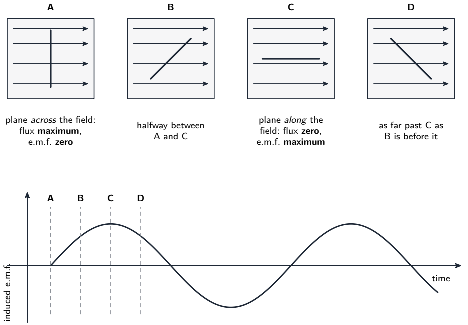
With the coil turning at a steady rate, $N\Phi = NBA\cos(2\pi ft)$, so

$$E = -\frac{\mathrm{d}(N\Phi)}{\mathrm{d}t} = 2\pi f NBA\sin(2\pi ft), \qquad \text{so} \qquad E_0 = 2\pi f NBA$$

- The e.m.f. is **zero** when the flux linkage is **greatest** (plane of the coil **across** the field), and **greatest** when the flux linkage is **zero** (plane of the coil **along** the field). The two graphs are a **quarter of a cycle** apart.
- Double the rotation rate and the peak e.m.f. **doubles** ($E_0 \propto f$) while the period **halves**. Both changes come from the same f.
- This is the a.c. generator, and it is the commonest place the "e.m.f. is biggest when the flux is biggest" error appears.

Method — A coil rotating in a uniform field

With the coil turning at a steady rate, ... , so [E = - d (N) d t = 2 f , NBA (2 f t), so E 0 = 2 f , NBA]



At **A** the flux through the coil is greatest, and the e.m.f. is **zero**: the flux is momentarily not changing. At **C** the flux is zero and the e.m.f. is **greatest**: the flux is changing fastest. The e.m.f. follows the *rate of change* of flux, so the two graphs are a quarter of a cycle apart.

Method — Lenz's law when nothing is being calculated

Every qualitative question is the same four steps. Write them in this order:

1. The motion **changes the flux** linking the conductor.
2. So an e.m.f. is induced, and because the conductor is a complete circuit, a **current** flows.
3. By **Lenz's law** the current is in the direction that **opposes the change** producing it.
4. So there is a **force opposing the motion**, and the moving object slows down.

Method — Lenz's law when nothing is being calculated

what you are shown	what to say
a magnet falls slowly down a **copper tube**	the changing flux induces **eddy currents 涡电流** in the tube wall; by Lenz's law they oppose the magnet's motion, so there is an upward force on the magnet and it falls at nearly constant, low speed
a **plastic** tube instead	plastic is an **insulator**: an e.m.f. is still induced but **no current** can flow, so there is no opposing force and the magnet falls freely
an aluminium **ring** jumps off an electromagnet switched on	the rising flux induces a current in the ring, opposing the increase, so the ring's field opposes the magnet's and it is **repelled**
a **slotted** disc is not damped	the slots **break the path** of the eddy currents, so almost no current flows

- Finish with **energy**: the work done against the opposing force is transferred to **electrical energy** and then to **thermal energy** in the conductor's resistance. If the induced current helped the motion instead, energy would be created from nothing.

Method — Lenz's law when nothing is being calculated

Write them in this order: center

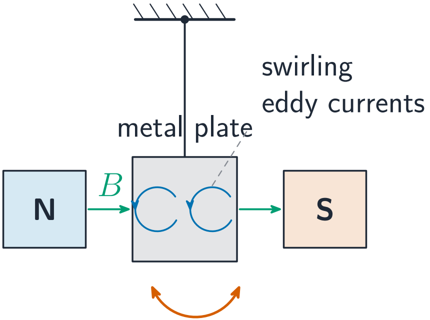


plate swings to and fro
induced eddy currents oppose the motion $\Rightarrow$ swing is damped

2 Practice

Practice 2.1 ▪ 2 marks

A coil of 150 turns and cross-sectional area 4.0×10^{-3} m^2 has its plane perpendicular to a uniform field of flux density 0.25 T. Calculate the flux linkage.

Solution 2.1

Method: Faraday's law is a statement about a gradient — p.4

Worked steps

- the plane is perpendicular to the field, so the full area links the flux **M1**

$$N\Phi = NBA = 150 \times 0.25\ \mathrm{T} \times 4.0 \times 10^{-3}\ \mathrm{m}^2 = \mathbf{0.15}\ \mathrm{Wb}$$ **A1**

Practice 2.2 ▪ 2 marks

The flux density in **2.1** then falls steadily to zero in 0.20 s.
Calculate the magnitude of the induced e.m.f.

Solution 2.2

Method: Faraday's law is a statement about a gradient — p.4

Worked steps

- divide the change in flux linkage by the time **M1**

$$|E| = \frac{\Delta(N\Phi)}{\Delta t} = \frac{0.15\ \text{Wb}}{0.20\ \text{s}} = \mathbf{0.75}\ \text{V}$$ **A1**

Practice 2.3 ▪ 3 marks

The flux linkage through a coil rises steadily for 2.0 s, stays constant for 1.0 s, and then falls to zero in 1.0 s. Sketch a graph of the induced e.m.f. against time for these 4.0 s.

Solution 2.3

Method: Faraday's law is a statement about a gradient — p.4

Worked steps

- from 0 to 2.0 s: a **constant** e.m.f., of one sign **B1**
- from 2.0 to 3.0 s: **zero** **B1**
- from 3.0 to 4.0 s: **constant**, of the **opposite** sign and **twice** the earlier magnitude, because the same flux change happens in half the time **B1**

Practice 2.4 ▪ 2 marks

A magnet is dropped through a coil connected to a datalogger. Explain why the trace shows two pulses of **opposite sign**.

Solution 2.4

Method: A magnet dropped through a coil — p.8

Worked steps

- as the magnet enters, the flux linkage through the coil **increases**; as it leaves, the flux linkage **decreases** B1
- the gradient $\dfrac{\mathrm{d}(N\Phi)}{\mathrm{d}t}$ therefore has **opposite signs**, so the induced e.m.f. does too B1

Practice 2.5 ▪ 2 marks

Explain why the second pulse is **taller and narrower** than the first.

Solution 2.5

Method: A magnet dropped through a coil — p.8

Worked steps

- the magnet **accelerates** under gravity, so it is moving faster as it leaves than as it entered B1
- the same change of flux linkage therefore happens in a **shorter time**, giving a **larger** e.m.f. that lasts for **less** time – the two areas remain equal B1

Practice 2.6 ▪ 2 marks

A coil rotates at a steady rate in a uniform magnetic field. State the position of the coil at which the induced e.m.f. is **zero**, and explain why.

Solution 2.6

Method: A coil rotating in a uniform field — p.10

Worked steps

- when the **plane of the coil is perpendicular to the field** – that is, when the flux linkage is a **maximum** B1
- at that instant the flux linkage is momentarily **not changing**, so $\dfrac{\mathrm{d}(N\Phi)}{\mathrm{d}t} = 0$ and the induced e.m.f. is zero B1

3 Exam-style

Exam-style 3.1 ▪ 1 mark

A coil rotates at a steady rate in a uniform magnetic field. At the instant when the **plane of the coil is perpendicular to the field**, the flux linkage and the induced e.m.f. are

A flux linkage maximum, e.m.f. maximum

B flux linkage maximum, e.m.f. zero

C flux linkage zero, e.m.f. maximum

D flux linkage zero, e.m.f. zero

Solution 3.1

A flux linkage maximum, e.m.f. maximum

B flux linkage maximum, e.m.f. zero ✓

C flux linkage zero, e.m.f. maximum

D flux linkage zero, e.m.f. zero

Worked steps

B

- with the plane of the coil perpendicular to the field, the full area links the flux, so the flux linkage is at its **maximum** B1
- a maximum is a turning point, so the flux linkage is momentarily constant and the e.m.f. is **zero**. **A** is the standard error: the e.m.f. follows the *rate of change*, not the flux

Exam-style 3.2 ▪ 6 marks

A strong bar magnet is dropped down a long vertical **copper** tube. It takes several seconds to fall through. The same magnet dropped down an identical **plastic** tube falls through in a fraction of a second.

Exam-style 3.2 (a) ▪ 4 marks

Explain, in terms of flux, induced current and force, why the magnet falls slowly down the copper tube.

Solution 3.2 (a)

Method: Lenz's law when nothing is being calculated — p.12

Worked steps

- as the magnet falls, the magnetic flux linking each part of the copper tube **changes** B1
- an e.m.f. is induced, and because copper is a conductor a current – an **eddy current** – flows in the tube wall B1
- by **Lenz's law** the current is in the direction that **opposes the change** producing it B1
- so the tube exerts an **upward force** on the falling magnet, which nearly balances its weight, and the magnet falls slowly at almost constant speed B1

Exam-style 3.2 (b) ▪ 2 marks

Explain why the magnet falls freely down the plastic tube.

Solution 3.2 (b)

Worked steps

- an e.m.f. is still induced in the plastic, because the changing flux does not care what the material is **B1**
- but plastic is an **insulator**, so no current flows, there is no opposing force, and the magnet falls freely under gravity **B1**

Exam-style 3.3 ▪ 11 marks

A flat coil of 250 turns and area 1.8×10^{-3} m^2 rotates at 50 revolutions per second about an axis perpendicular to a uniform magnetic field of flux density 85 mT.

Exam-style 3.3 (a) ▪ 2 marks

Calculate the maximum flux linkage through the coil.

Solution 3.3 (a)

Method: Faraday's law is a statement about a gradient — p.4

Worked steps

- the maximum flux linkage is NBA, with the plane of the coil perpendicular to the field **M1**

$$N\Phi_{\text{max}} = 250 \times 85 \times 10^{-3}\ \text{T} \times 1.8 \times 10^{-3}\ \text{m}^2 = \mathbf{3.8 \times 10^{-2}}\ \text{Wb} \qquad \textbf{A1}$$

Exam-style 3.3 (b) ▪ 2 marks

State the orientation of the coil when the flux linkage is a maximum, and state the induced e.m.f. at that instant.

Solution 3.3 (b)

Worked steps

- the **plane of the coil is perpendicular to the field** (the normal to the coil is along the field) B1
- at that instant the induced e.m.f. is **zero** B1

Exam-style 3.3 (c) ▪ 3 marks

On the same time axis, sketch the flux linkage and the induced e.m.f. against time, for one complete rotation.

Solution 3.3 (c)

Worked steps

- flux linkage: a **cosine** curve, starting at its maximum 3.8×10^{-2} Wb, through zero, to -3.8×10^{-2} Wb and back **B1**
- e.m.f.: a **sine** curve of the same period **B1**
- the e.m.f. curve is a **quarter of a cycle** ahead: its zeros are at the peaks of the flux-linkage curve, and its peaks are at the zeros **B1**

Exam-style 3.3 (d) ▪ 2 marks

The peak e.m.f. is given by $E_0 = 2\pi f NBA$. Calculate it.

Solution 3.3 (d)

Worked steps

- substitute into $E_0 = 2\pi f NBA$, using the flux linkage from part (a) **M1**

$$E_0 = 2\pi \times 50\ \text{Hz} \times 3.8 \times 10^{-2}\ \text{Wb} = \mathbf{12}\ \text{V}$$ **A1**

Exam-style 3.3 (e) ▪ 2 marks

The coil is now rotated at 100 revolutions per second. State the effect on the peak e.m.f. and on the period, and explain both.

Solution 3.3 (e)

Worked steps

- the peak e.m.f. **doubles**, to about 24 V, because $E_0 = 2\pi f NBA$ is proportional to f **B1**
- the period **halves**, to 1.0×10^{-2} s, because $T = 1/f$; both changes follow from the same doubling of f **B1**