

Exercise walk-through

20.3.1

The Hall effect, the Hall probe and velocity selection

A-Level Physics

You must be able to

- determine the direction of the force on a charge moving in a magnetic field, and recall and use $F = BQv\sin\theta$
- understand the origin of the Hall voltage, and **derive** and use $V_H = \frac{BI}{ntq}$, where t is the thickness
- understand the use of a Hall probe to measure magnetic flux density
- describe the motion of a charged particle in a uniform magnetic field perpendicular to its motion
- explain how electric and magnetic fields are used in **velocity selection** 速度选择

1

The method

Method — What you already have, in two lines

- $F = BQv \sin \theta$, with θ measured between the **velocity and the field**. The force is **zero** when the charge moves **along** the field ($\theta = 0$), and **maximum** when it moves at right angles.
- Perpendicular to a uniform field, the force is always at right angles to the velocity, so it is a **centripetal** force and the path is a **circle**:

$$BQv = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{BQ}$$

- The speed never changes, because a force at right angles to the motion does **no work**.

Method — Where the Hall voltage comes from, and the derivation

The derivation is four steps, and each one is a marking point:

- 1 A carrier drifting with speed v through the field feels a magnetic force Bqv , which pushes it to **one face** of the slab.
- 2 Charge builds up on that face, leaving the opposite face oppositely charged, so a **transverse electric field** E_H appears **across** the slab.
- 3 Charge stops building up when the **electric force balances the magnetic force**:

$$qE_H = Bqv \implies E_H = Bv$$

and since the field is uniform across the width d , the **Hall voltage** is

$$V_H = E_H d = Bvd$$

Method — Where the Hall voltage comes from, and the derivation

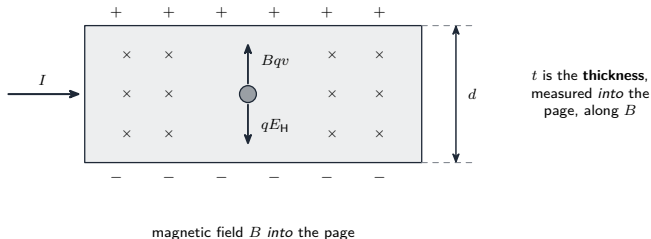
- 4 Eliminate the drift speed using $I = nAvq$ with $A = td$:

$$v = \frac{I}{ntdq} \implies V_H = B \left(\frac{I}{ntdq} \right) d = \frac{BI}{ntq}$$

- n is the **number density** of charge carriers and t is the **thickness measured along the field**, not the width.
- Notice the d **cancels**: the Hall voltage does not depend on the width of the slab.

Method — Where the Hall voltage comes from, and the derivation

The derivation is four steps, and each one is a marking point



The magnetic force pushes the carriers to one face until the electric field they build pushes back just as hard: $qE_H = Bqv$, so $E_H = Bv$ and $V_H = E_H d = Bvd$.
 Substituting the drift speed from $I = nAvq$ with $A = td$ gives $V_H = \frac{BI}{ntq}$.

Method — Why a Hall probe is made of semiconductor

$$V_H = \frac{BI}{ntq}$$

- With I , t and q fixed, $V_H \propto B$: measure the voltage and you have measured the **flux density**. That is the whole instrument.
- A **semiconductor** 半导体 has a number density n perhaps 10^7 times smaller than a metal's, and $V_H \propto \frac{1}{n}$, so the voltage is large enough to measure. A metal probe would give microvolts.
- Making the slab **thin** helps for the same reason.
- In use: the probe must be held with its **face perpendicular to the field**, because it responds to the component of B through it, and the current must be kept **constant**.

Method — Velocity selection

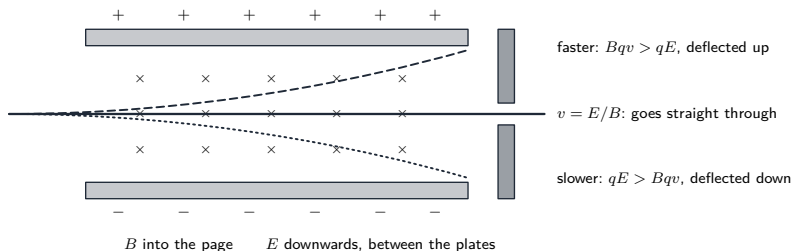
Put an electric field and a magnetic field at right angles to each other and to the beam, so that the two forces on a charge are in **opposite** directions:

$$qE = Bqv \implies v = \frac{E}{B}$$

- Only a charge with **exactly** this speed passes through undeflected and reaches the slit.
- **Faster:** the magnetic force Bqv grows with v while the electric force does not, so $Bqv > qE$ and the charge is deflected towards the magnetic force.
- **Slower:** $qE > Bqv$, and it is deflected the other way.
- The charge q **cancels**, so the selected speed does not depend on the charge or the mass – the same selector works on any ion.

Method — Velocity selection

Put an electric field and a magnetic field at right angles to each other and to the beam, so that the two...



The electric force qE and the magnetic force Bqv act in *opposite* directions, and they balance only when $qE = Bqv$, that is when $v = \frac{E}{B}$. Every other speed is deflected and stopped by the slit, whatever the particle's charge or mass.

2

Practice

Practice 2.1 ■ 2 marks

Write the equation for the force on a charge moving at an angle to a magnetic field, and state the condition for the force to be zero.

Solution 2.1

Method: What you already have, in two lines — p.4

Worked steps

- $F = BQv\sin\theta$, where θ is the angle between the velocity and the magnetic field **B1**
- the force is zero when $\theta = 0$, that is when the charge moves **along** the field **B1**

Practice 2.2 ■ 3 marks

Explain why a charged particle entering a uniform magnetic field at right angles to it follows a circular path at constant speed.

Solution 2.2

Worked steps

- the magnetic force is always at **right angles to the velocity**, so it changes the direction of motion but not its magnitude (B1)
- a force of constant magnitude always perpendicular to the velocity is a **centripetal** force, so the path is a circle (B1)
- since the force is perpendicular to the motion it does **no work**, so the kinetic energy and therefore the speed are constant (B1)

Practice 2.3 ■ 3 marks

Explain how a Hall voltage arises across a slab carrying a current in a magnetic field.

Solution 2.3

Method: Where the Hall voltage comes from, and the derivation — p.5

Worked steps

- the drifting charge carriers experience a magnetic force Bqv that pushes them towards **one face** of the slab **B1**
- charge builds up on that face, leaving the opposite face oppositely charged, so a **transverse electric field** appears across the slab **B1**
- the build-up stops when the **electric force on a carrier balances the magnetic force**, and the p.d. across the slab at that point is the Hall voltage **B1**

Practice 2.4 ■ 2 marks

Explain why a Hall probe is made from a semiconductor rather than a metal.

Solution 2.4

Method: Why a Hall probe is made of semiconductor — p.8

Worked steps

- $V_H = \frac{BI}{ntq}$, so the Hall voltage is **inversely proportional to the number density n** (B1)
- a semiconductor has a far smaller n than a metal, so the voltage is large enough to be measured (B1)

Practice 2.5 ■ 2 marks

State the condition for a charged particle to pass undeflected through a velocity selector, and give the expression for the selected speed.

Solution 2.5

Method: Velocity selection — p.9

Worked steps

- the **electric force** and the **magnetic force** must be equal and opposite, so $qE = Bqv$ **B1**

$$v = \frac{E}{B}$$

B1

3

Exam-style

Exam-style 3.1 ■ 1 mark

A Hall probe is to give the largest possible Hall voltage for a given flux density. The slab should have

- A** a large number density of charge carriers and a large thickness.
- B** a large number density of charge carriers and a small thickness.
- C** a small number density of charge carriers and a large thickness.
- D** a small number density of charge carriers and a small thickness.

Solution 3.1

Method: Why a Hall probe is made of semiconductor — p.8

- A a large number density of charge carriers and a large thickness.
- B a large number density of charge carriers and a small thickness.
- C a small number density of charge carriers and a large thickness.
- D a small number density of charge carriers and a small thickness. 

Worked steps

D

- $V_H = \frac{BI}{ntq}$, so a **small** n and a **small** t both make V_H larger **B1**

Exam-style 3.2 ■ 6 marks

A rectangular slab of width d and thickness t carries a current I . A uniform magnetic field of flux density B acts perpendicular to the face of the slab, and the charge carriers each have charge q and number density n .

Exam-style 3.2 (a) ■ 4 marks

Derive the expression $V_H = \frac{BI}{ntq}$ for the Hall voltage across the slab.

Solution 3.2 (a)

Method: Where the Hall voltage comes from, and the derivation — p.5

Worked steps

- a carrier drifting at speed v feels a magnetic force Bqv pushing it to one face, so charge collects there and a transverse electric field E_H builds up **B1**
- equilibrium is reached when the electric force balances the magnetic force **M1**

$$qE_H = Bqv \implies E_H = Bv, \quad V_H = E_H d = Bvd \quad \text{A1}$$

- eliminate v with $I = nAvq$ and $A = td$ **M1**

$$v = \frac{I}{ntdq} \implies V_H = B \frac{I}{ntdq} d = \frac{BI}{ntq}$$

Exam-style 3.2 (b) ■ 2 marks

A Hall probe uses a slab of thickness 0.10 mm in which $n = 2.0 \times 10^{22} \text{ m}^{-3}$ and $q = 1.6 \times 10^{-19} \text{ C}$. The current through it is 15 mA. Calculate the Hall voltage when the probe is placed in a field of flux density 0.25 T.

Solution 3.2 (b)

Worked steps

- substitute, with the thickness in metres **M1**

$$V_H = \frac{BI}{ntq} = \frac{0.25 \text{ T} \times 0.015 \text{ A}}{2.0 \times 10^{22} \text{ m}^{-3} \times 1.0 \times 10^{-4} \text{ m} \times 1.6 \times 10^{-19} \text{ C}} = 1.2 \times 10^{-2} \text{ V}$$
A1

Exam-style 3.3 ■ 11 marks

Positive ions, each of charge $+1.6 \times 10^{-19}$ C, pass through a velocity selector in which the electric field strength is 3.0×10^4 V m⁻¹ and the magnetic flux density is 0.12 T. They then enter a second region of uniform magnetic field of flux density 0.35 T, perpendicular to their motion.

Exam-style 3.3 (a) ■ 2 marks

Calculate the speed of the ions that pass through the selector undeflected.

Solution 3.3 (a)

Worked steps

- the two forces balance, so $qE = Bqv$ and the charge cancels (M1)

$$v = \frac{E}{B} = \frac{3.0 \times 10^4 \text{ V m}^{-1}}{0.12 \text{ T}} = 2.5 \times 10^5 \text{ m s}^{-1} \quad (\text{A1})$$

Exam-style 3.3 (b) ■ 2 marks

Explain why ions travelling faster than this do not pass through.

Solution 3.3 (b)

Worked steps

- the magnetic force Bqv **increases with speed**, while the electric force qE does not (B1)
- so for a faster ion $Bqv > qE$, the forces no longer balance, and the ion is deflected out of the beam and stopped by the slit (B1)

Exam-style 3.3 (c) ■ 3 marks

In the second region an ion follows a circular path of radius 0.086 m. Calculate its mass.

Solution 3.3 (c)

Worked steps

- the magnetic force provides the centripetal force M1

$$Bqv = \frac{mv^2}{r} \quad \Rightarrow \quad m = \frac{rBq}{v} \quad \text{M1}$$

$$m = \frac{rBq}{v} = \frac{0.086 \text{ m} \times 0.35 \text{ T} \times 1.6 \times 10^{-19} \text{ C}}{2.5 \times 10^5 \text{ m s}^{-1}} = 1.9 \times 10^{-26} \text{ kg} \quad \text{A1}$$

Exam-style 3.3 (d) ■ 2 marks

State and explain the effect on the radius of the path of using an ion of the same charge and speed but **twice** the mass.

Solution 3.3 (d)

Worked steps

- the radius **doubles** B1
- $r = \frac{mv}{Bq}$, and with v , B and q unchanged the radius is proportional to the mass
B1

Exam-style 3.3 (e) ■ 2 marks

Explain why the magnetic field in the second region does no work on the ion.

Solution 3.3 (e)

Worked steps

- the magnetic force is always **perpendicular to the velocity**, so it has no component along the direction of motion **B1**
- work done is $Fs \cos \theta$ with $\theta = 90^\circ$, so it is zero; the kinetic energy and the speed are unchanged and only the direction alters **B1**