Exercise walk-through

Equations of motion ▪ 2.1

eXercise

You must be able to

- find velocity from a displacement–time gradient, and acceleration + displacement from a velocity–time graph
- choose and use the four SUVAT equations for straight-line motion
- solve free-fall problems ($g = 9.81$ m s^{-2}, no air resistance)
- treat the horizontal and vertical motion of a projectile independently
- **describe** the experiment that measures the acceleration of free fall
- **sketch** a motion graph with both axes drawn and labelled

Method — Choosing a SUVAT equation

The five symbols are u (start velocity), v (final velocity), a (acceleration), s (displacement) and t (time). Each equation uses **four** of them and leaves one out:

equation	**leaves out**
$v = u + at$	s
$s = \frac{1}{2}(u + v)t$	a
$s = ut + \frac{1}{2}at^2$	v
$v^2 = u^2 + 2as$	t

Method — Choosing a SUVAT equation

Method: list what you know and what you want, then pick the row that leaves out the symbol you don't have.

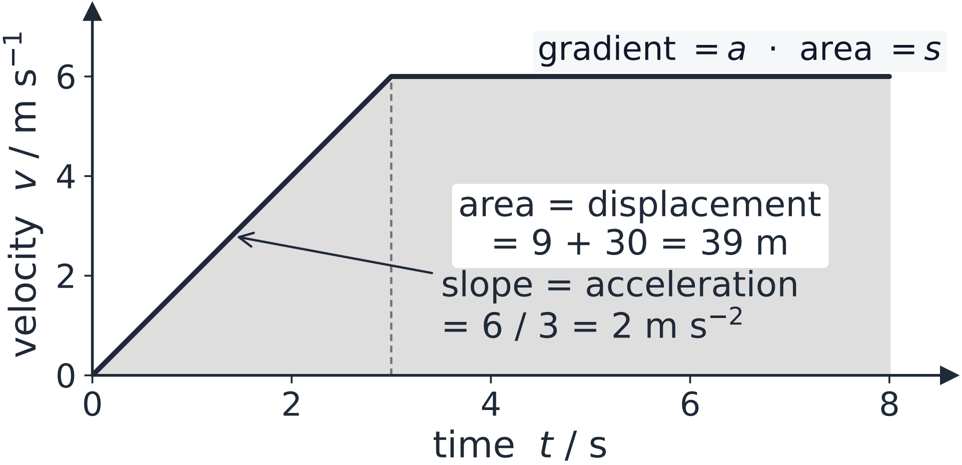
Example. A cyclist speeds up uniformly from 4.0 to 13 m s^{-1} in 6.0 s: so $u = 4.0$, $v = 13$, $t = 6.0$; find a, then s.

- For a (no s): $v = u + at \Rightarrow a = \dfrac{v-u}{t} = \dfrac{13-4.0}{6.0} = 1.5$ m s^{-2}.
- For s (no a): $s = \frac{1}{2}(u+v)t = \frac{1}{2}(4.0+13)(6.0) = 51$ m.

Method — Reading a velocity–time graph

On a velocity–time graph the **gradient 斜率** (steepness) is the acceleration, and the **area** under the line is the displacement.
(For an *acceleration*–time graph, the area under the line is instead the *change in velocity*.)

Method — Reading a velocity–time graph



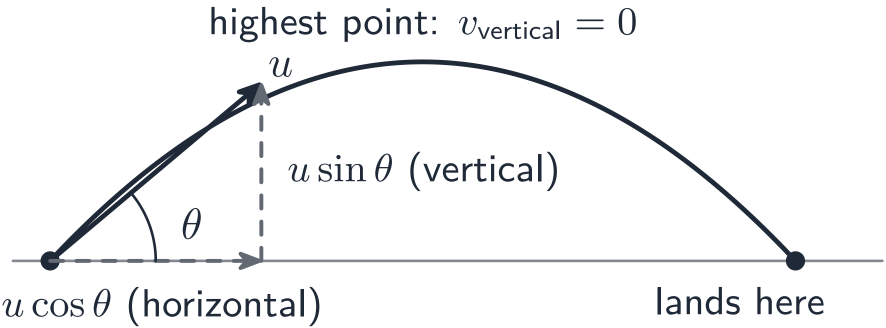
Method — A projectile

Horizontal and vertical motion are **independent** — they share only the time t, so solve each as its own SUVAT problem. For a launch at angle θ, first **resolve** the speed: horizontal $u\cos\theta$ (stays constant) and vertical $u\sin\theta$ (changes).

Worked case — a ball thrown **horizontally** at 12 m s^{-1} from a cliff 25 m high ($g = 9.81$ m s^{-2}):

- Vertical (start 0): $25 = \frac{1}{2}g\,t^2 \Rightarrow t = \sqrt{2 \times 25/9.81} = 2.3$ s.
- Horizontal: $x = 12 \times 2.3 = 27$ m.

Method — A projectile



Method — Measuring the acceleration of free fall

The syllabus asks you to **describe** this experiment, so learn it as five steps:

1. A steel ball is held by an **electromagnet** above a **trapdoor** switch, a measured height h above it.
2. Switching the magnet off starts an electronic **timer** and releases the ball at the same instant.
3. The ball opens the trapdoor on landing, which **stops** the timer, giving the fall time t.
4. The ball falls from rest, so $h = \frac{1}{2}gt^2$. Repeat for **several heights**, plot h against t^2, and the **gradient** is $\frac{1}{2}g$.
5. Measure h from the **bottom** of the ball to the trapdoor, and repeat each timing — the graph, not a single pair of readings, is what removes the random error.

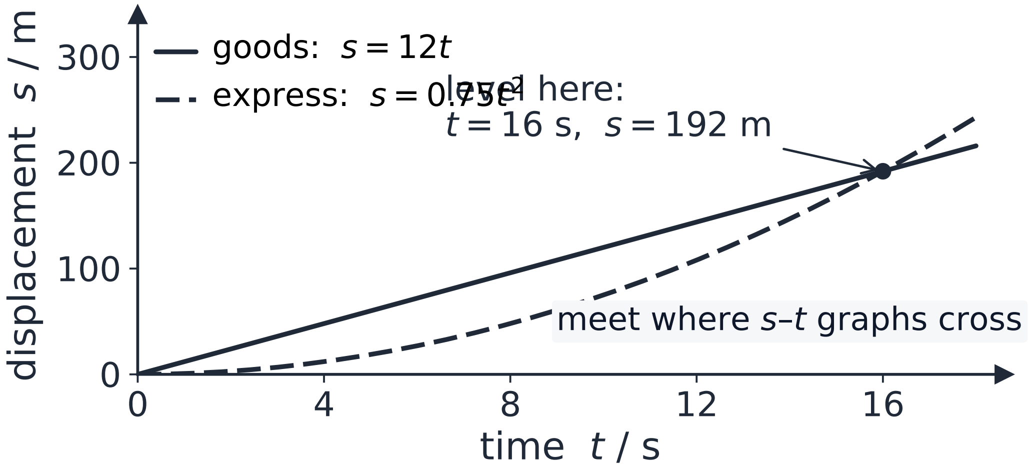
A steel ball is used because **air resistance** on it is negligible; the graph is a straight line **through the origin** only if there is no systematic timing delay.

Method — Two objects meeting

Write a displacement equation for each object using the same start time and positive direction, then set the displacements equal.
A goods train moves at a constant 12 m s^{-1}. As it passes a signal, an express train starts from rest there with acceleration 1.5 m s^{-2}.

- Goods: $s = 12t$. Express: $s = \frac{1}{2}(1.5)t^2 = 0.75t^2$.
- Level when $12t = 0.75t^2 \Rightarrow t = \dfrac{12}{0.75} = 16$ s ($s = 12 \times 16 = 192$ m).

Method — Two objects meeting



Practice 2.1 ▪ 2 marks

For each quantity, write **scalar 标量** or **vector 矢量**, and state its SI unit: (a) velocity (b) acceleration.

quantity	**scalar or vector?**	**SI unit**
velocity		
acceleration		

Solution 2.1

Worked steps

velocity — vector, m s^{-1}; acceleration — vector, m s^{-2} **B1** *each*.

Practice 2.2 ▪ 3 marks

A car travelling at 24 m s^{-1} brakes uniformly at 3.0 m s^{-2} until it stops. Find the braking distance. *(Hint: which SUVAT equation links u, v, a and s?)*

Solution 2.2

Method: Choosing a SUVAT equation

Worked steps

Use $v^2 = u^2 + 2as$ with $v = 0$: $0 = 24^2 + 2(-3.0)s$ **M1** $\Rightarrow s = 576/6.0 = 96$ m **A1** **A1**.

Practice 2.3 ▪ 3 marks

The displacement–time graph of a cyclist is a straight line passing through $(2.0\ \text{s},\ 12\ \text{m})$ and $(8.0\ \text{s},\ 42\ \text{m})$. **Determine** her velocity, and state how you know her acceleration is zero.

Solution 2.3

Method: Choosing a SUVAT equation

Worked steps

velocity $= $ gradient $= \dfrac{42 - 12}{8.0 - 2.0} = \dfrac{30}{6.0} = 5.0$ m s^{-1} M1 A1; the graph is a **straight** line, so the gradient — and therefore the velocity — never changes, and acceleration is the rate of change of velocity, so it is zero B1.

Practice 2.4 ▪ 4 marks

Describe an experiment to determine the acceleration of free fall using a falling object. State the measurements you would take, how you would use them, and one way to reduce the uncertainty.

Solution 2.4

Method: Measuring the acceleration of free fall

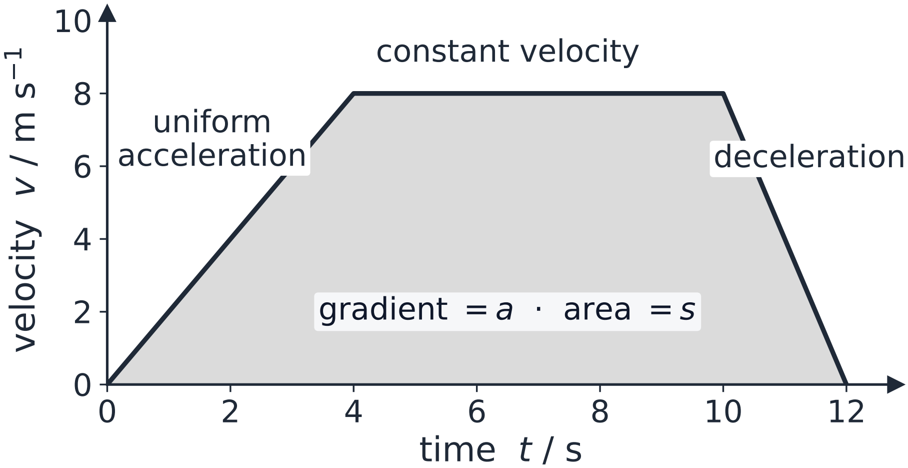
Worked steps

Release a steel ball from an electromagnet a measured height h above a trapdoor switch **B1**; switching the magnet off starts an electronic timer and releases the ball, and the ball opens the trapdoor to stop it, giving t **B1**; use $h = \frac{1}{2}gt^2$, or better plot h against t^2 and take $g = 2\times$ gradient **B1**; reduce uncertainty by repeating each timing and by using several heights so a graph averages the readings **B1**.

Practice 2.5 ▪ 1 mark

The velocity–time graph shows a tram's journey.
(a) Find the acceleration during the first 4 s.
(b) Find the total displacement of the tram. *(Hint: split the area into a triangle, a rectangle and a triangle.)*

Practice 2.5 ▪ 1 mark



Solution 2.5

Worked steps

(a) gradient $= 8/4 = 2.0$ m s^{-2} **A1**.

(b) triangle $\frac{1}{2}(4)(8) = 16$; rectangle $(6)(8) = 48$; triangle $\frac{1}{2}(2)(8) = 8$ **M1** *for the areas*; total $= 16 + 48 + 8 = 72$ m **A1** **A1**.

Exam-style 3.1 ▪ 1 mark

A stone is thrown vertically upwards at 15 m s^{-1}. Take $g = 9.81$ m s^{-2} and ignore air resistance. Its height above the launch point after 2.0 s is closest to:

A 10 m

B 20 m

C 30 m

D 50 m

Solution 3.1

Method: A projectile

A 10 m ✓

B 20 m

C 30 m

D 50 m

Worked steps

A. $s = (15)(2.0) - \frac{1}{2}(9.81)(2.0)^2 = 30 - 19.6 = 10.4$ m, closest to 10 m.

Exam-style 3.2 ▪ 1 mark

The area under an **acceleration–time graph** gives which quantity?

A displacement

B change in velocity

C distance

D force

Solution 3.2

Method: Reading a velocity–time graph

A displacement

B change in velocity ✓

C distance

D force

Worked steps

B. The area under an acceleration–time graph is the change in velocity.

Exam-style 3.3 ▪ 2 marks

A train starts from rest and accelerates uniformly at 0.50 m s^{-2} for 40 s.

(a) Calculate the speed it reaches.

(b) Calculate the distance travelled during these 40 s.

(c) State one assumption you have made in your calculation.

Solution 3.3

Method: Two objects meeting

Worked steps

(a) $v = u + at = 0 + (0.50)(40) = 20$ m s^{-1} M1 A1.
(b) $s = \frac{1}{2}at^2 = \frac{1}{2}(0.50)(40)^2 = 400$ m M1 A1. [or $s = \frac{1}{2}(0 + 20)(40)$]
(c) e.g. the acceleration stays constant / the track is straight B1.

Exam-style 3.4 ▪ 1 mark

A ball is kicked from level ground at 20 m s^{-1}, at 30° above the horizontal. Take $g = 9.81$ m s^{-2} and ignore air resistance.

(a) Show that the vertical **component 分量** of the initial velocity is about 10 m s^{-1}.

(b) Calculate the time the ball is in the air before it lands.

(c) Calculate the horizontal distance it travels (its range).

Solution 3.4

Method: A projectile

Worked steps

(a) $u_V = 20\sin 30° = (20)(0.50) = 10$ m s^{-1} **B1**.
(b) time to the top $= u_V/g = 10/9.81 = 1.02$ s; total time $= 2 \times 1.02 = 2.0$ s **M1** **A1**.
(c) $u_H = 20\cos 30° = 17.3$ m s^{-1}; range $= u_H \times t = 17.3 \times 2.04 = 35$ m **M1** **A1**.

Exam-style 3.6 ▪ 1 mark

A drone of mass 1.8 kg takes off from the ground and rises vertically. The graph shows its velocity for the first 8.0 s of the flight.

(a) **Define** acceleration.

(b) **Determine** the acceleration of the drone.

(c) **Show that** the drone rises 96 m in these 8.0 s.

(d) Calculate the gain in **gravitational potential energy** of the drone.

(e) Calculate the gain in **kinetic energy** of the drone.

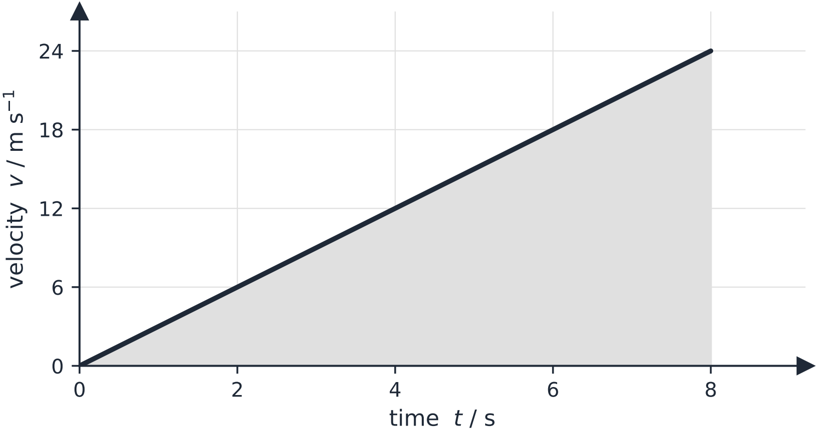
(f) **Determine** the average useful power output of the drone's motors during these 8.0 s.

(g) In the space below, **sketch** the variation with time of the **displacement** of the drone over the same 8.0 s. **Draw** and **label** both axes.

Exam-style 3.6 ▪ 1 mark

(h) A second drone of mass 2.7 kg takes off and follows exactly the same velocity–time graph. **State and explain** whether the average useful power output of its motors is less than, the same as, or greater than your answer to **(f)**.

Exam-style 3.6 ▪ 1 mark



Solution 3.6

Method: Reading a velocity–time graph

Worked steps

(a) Acceleration is the **rate of change of velocity** B1.

(b) gradient $= \dfrac{24-0}{8.0-0}$ M1 $= 3.0$ m s^{-2} A1.

(c) height = area under the line $= \frac{1}{2}(8.0)(24)$ M1 $= 96$ m A1. [or $s = \frac{1}{2}at^2 = \frac{1}{2}(3.0)(8.0)^2$]

(d) $\Delta E_P = mgh = 1.8 \times 9.81 \times 96$ M1 $= 1.7 \times 10^3$ J A1.

(e) $\Delta E_K = \frac{1}{2}mv^2 = \frac{1}{2}(1.8)(24)^2$ M1 $= 5.2 \times 10^2$ J A1.

(f) $P = \dfrac{\text{total energy gained}}{t} = \dfrac{1.7 \times 10^3 + 5.2 \times 10^2}{8.0}$ M1 $= 2.8 \times 10^2$ W A1.

Solution 3.6

(g) Axes drawn and labelled *displacement* / *m* (vertical) and *time* / *s* (horizontal) **B1**; a curve starting at the origin **B1**; getting **steeper** with time (a parabola, since $s \propto t^2$), reaching 96 m at 8.0 s **B1**.

(h) **Greater** **B1**. The graph is the same, so it gains the same height and the same speed, but both $\Delta E_\mathsf{P} = mgh$ and $\Delta E_\mathsf{K} = \frac{1}{2}mv^2$ are **proportional to the mass**, so a heavier drone gains more energy in the same 8.0 s **B1**.

Exam-style 3.5 ▪ 3 marks

A car passes a stationary motorcyclist at a constant 18 m s^{-1}. At that instant the motorcyclist sets off from rest with a uniform acceleration of 3.0 m s^{-2}.

(a) Calculate the time the motorcyclist takes to catch up with the car.

(b) Calculate how far each has travelled at that moment.

(c) Calculate the motorcyclist's speed when she catches the car.

Solution 3.5

Method: Two objects meeting

Worked steps

(a) car: $s = 18t$; motorcyclist: $s = \frac{1}{2}(3.0)t^2 = 1.5t^2$ **M1**. They are level when $18t = 1.5t^2$ **M1** $\Rightarrow t = 18/1.5 = 12$ s **A1**.

(b) $s = (18)(12) = 216$ m **A1**.

(c) $v = at = (3.0)(12) = 36$ m s^{-1} **A1**.