

Exercise walk-through

2.1.5

Projectile motion

A-Level Physics

You must be able to

- explain why the horizontal and vertical motions of a **projectile** 抛体 are independent, and linked only by the time
- solve a horizontal launch: the time of flight from the height, then the distance from the horizontal velocity
- **resolve** 分解 an angled launch velocity into horizontal and vertical **components** 分量
- find the time of flight, the greatest height and the **range** 射程
- give the velocity at any moment as a size and a direction
- sketch how the horizontal and vertical components of the velocity change with time

1

The method

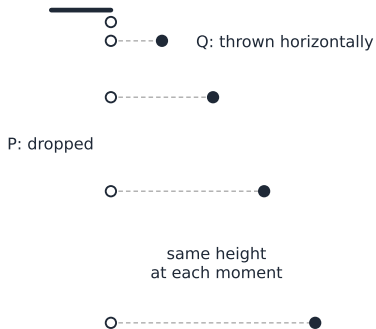
Method — Two independent motions

Ball P is dropped from rest. At the same moment, ball Q is thrown **horizontally** 水平 from the same height. Air resistance is negligible. The figure shows both balls at equal time intervals.

- vertically, both balls start with zero vertical velocity and accelerate at g , so they are at the same height at every moment and land together
- horizontally, nothing changes Q's horizontal velocity, so it moves the same distance in each interval
- the two motions share only the time t : find t from the vertical motion, then use it in the horizontal motion

Method — Two independent motions

The figure shows both balls at equal time intervals



Method — A horizontal launch

A ball rolls off a horizontal shelf 1.8 m above the floor at 2.0 m s^{-1} .

- vertical motion: $u_V = 0$, $s = 1.8 \text{ m}$ and $a = 9.81 \text{ m s}^{-2}$, taking down as positive

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 1.8 \text{ m}}{9.81 \text{ m s}^{-2}}} = 0.606 \text{ s}$$

- horizontal motion: constant velocity

$$x = (2.0 \text{ m s}^{-1})(0.606 \text{ s}) = 1.2 \text{ m}$$

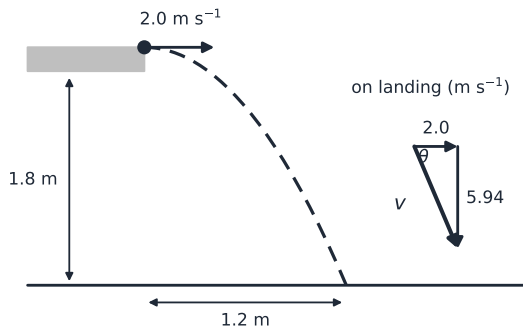
- on landing, $v_H = 2.0 \text{ m s}^{-1}$ and $v_V = at = (9.81 \text{ m s}^{-2})(0.606 \text{ s}) = 5.94 \text{ m s}^{-1}$
- add the components like any two vectors at right angles

$$v = \sqrt{(2.0 \text{ m s}^{-1})^2 + (5.94 \text{ m s}^{-1})^2} = 6.3 \text{ m s}^{-1}, \quad \tan \theta = \frac{5.94}{2.0} \Rightarrow \theta = 71^\circ$$

- the velocity is 6.3 m s^{-1} at 71° below the horizontal

Method — A horizontal launch

A ball rolls off a horizontal shelf ... above the floor at



Method — An angled launch

A ball is kicked from level ground at 25 m s^{-1} at 40° above the horizontal.

- resolve the launch velocity

$$u_H = (25 \text{ m s}^{-1}) \cos 40^\circ = 19.2 \text{ m s}^{-1}, \quad u_V = (25 \text{ m s}^{-1}) \sin 40^\circ = 16.1 \text{ m s}^{-1}$$

- take up as positive; at the top $v_V = 0$

$$t_{\text{top}} = \frac{0 - 16.1 \text{ m s}^{-1}}{-9.81 \text{ m s}^{-2}} = 1.64 \text{ s}, \quad H = \frac{0 - (16.1 \text{ m s}^{-1})^2}{2(-9.81 \text{ m s}^{-2})} = 13 \text{ m}$$

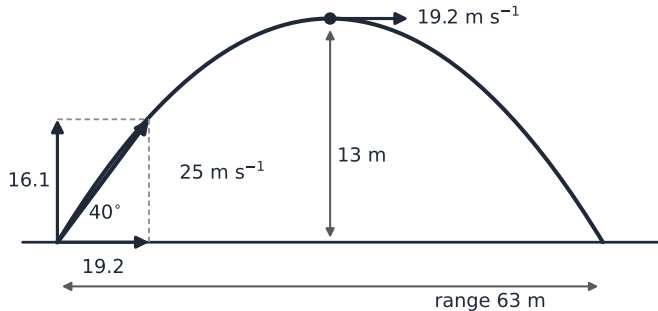
- it lands at the same level, so the whole flight takes twice as long, 3.28 s

$$R = (19.2 \text{ m s}^{-1})(3.28 \text{ s}) = 63 \text{ m}$$

- at the top the velocity is **not** zero: it is 19.2 m s^{-1} horizontally

Method — An angled launch

A ball is kicked from level ground at ... at ... above the horizontal



Method — Velocity at any moment

For the same ball at $t = 2.5$ s, up positive:

- the horizontal component is unchanged: $v_H = 19.2 \text{ m s}^{-1}$
- the vertical component

$$v_V = u_V + at = 16.1 \text{ m s}^{-1} + (-9.81 \text{ m s}^{-2})(2.5 \text{ s}) = -8.4 \text{ m s}^{-1}$$

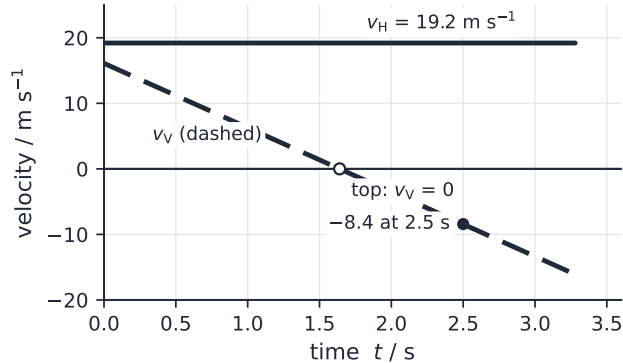
- size and direction

$$v = \sqrt{(19.2 \text{ m s}^{-1})^2 + (8.4 \text{ m s}^{-1})^2} = 21 \text{ m s}^{-1}, \quad \tan \theta = \frac{8.4}{19.2} \Rightarrow \theta = 24^\circ \text{ below the horizontal}$$

- on the graph, v_H is a horizontal line and v_V is a straight line of gradient -9.81 m s^{-2} that crosses zero at the top

Method — Velocity at any moment

For the same ball at ... , up positive



2

Practice

Practice 2.1 ■ 2 marks

Two identical balls leave the edge of a horizontal table at the same moment. Ball X falls from rest and ball Y is pushed off horizontally. Air resistance is negligible. State which ball reaches the floor first, and explain your answer.

Solution 2.1

Method: Two independent motions — p.4

Worked steps

- they reach the floor **at the same time** B1
- both start with zero vertical velocity, fall the same height and have the same acceleration g ; the horizontal velocity of Y does not change its vertical motion B1

Practice 2.2 ■ 6 marks

A delivery drone flies horizontally at 12 m s^{-1} , 45 m above flat ground, and releases a parcel. Air resistance is negligible.

Practice 2.2 (a) ■ 1 mark

State the horizontal and vertical components of the parcel's velocity just after it is released.

Solution 2.2 (a)

Method: Two independent motions — p.4

Worked steps

horizontal 12 m s^{-1} (the drone's velocity) and vertical 0 **B1**

Practice 2.2 (b) ■ 2 marks

Calculate the time taken for the parcel to reach the ground.

Solution 2.2 (b)

Worked steps

vertical motion from rest, down positive **M1**

$$t = \sqrt{\frac{2 \times 45 \text{ m}}{9.81 \text{ m s}^{-2}}} = \mathbf{3.0 \text{ s}}$$

A1

Practice 2.2 (c) ■ 1 mark

Calculate the horizontal distance the parcel travels.

Solution 2.2 (c)

Worked steps

$$(12 \text{ m s}^{-1})(3.03 \text{ s}) = \mathbf{36 \text{ m}} \quad \text{B1}$$

Practice 2.2 (d) ■ 2 marks

Calculate the speed of the parcel as it reaches the ground.

Solution 2.2 (d)

Worked steps

$$v_V = (9.81 \text{ m s}^{-2})(3.03 \text{ s}) = 29.7 \text{ m s}^{-1} \quad \text{M1}$$

$$v = \sqrt{(12 \text{ m s}^{-1})^2 + (29.7 \text{ m s}^{-1})^2} = \mathbf{32 \text{ m s}^{-1}}$$

A1

Practice 2.3 ■ 7 marks

A football is kicked from level ground at 18 m s^{-1} at 35° above the horizontal. Air resistance is negligible.

Practice 2.3 (a) ■ 2 marks

Calculate the horizontal and vertical components of the initial velocity.

Solution 2.3 (a)

Method: An angled launch — p.8

Worked steps

resolve **M1**

$$u_H = (18 \text{ m s}^{-1}) \cos 35^\circ = \mathbf{14.7 \text{ m s}^{-1}}, \quad u_V = (18 \text{ m s}^{-1}) \sin 35^\circ = \mathbf{10.3 \text{ m s}^{-1}}$$

A1

Practice 2.3 (b) ■ 1 mark

Calculate the time taken for the ball to reach its greatest height.

Solution 2.3 (b)

Worked steps

at the top $v_v = 0$

$$t = \frac{0 - 10.3 \text{ m s}^{-1}}{-9.81 \text{ m s}^{-2}} = 1.05 \text{ s}$$

B1

Practice 2.3 (c) ■ 2 marks

Calculate the greatest height.

Solution 2.3 (c)

Worked steps

use $v^2 = u^2 + 2as$ vertically **M1**

$$H = \frac{0 - (10.3 \text{ m s}^{-1})^2}{2(-9.81 \text{ m s}^{-2})} = 5.4 \text{ m}$$

A1

Practice 2.3 (d) ■ 2 marks

Calculate the range.

Solution 2.3 (d)

Worked steps

$$\text{time of flight} = 2 \times 1.05 \text{ s} = 2.10 \text{ s} \quad \text{M1}$$

$$R = (14.7 \text{ m s}^{-1})(2.10 \text{ s}) = \mathbf{31 \text{ m}}$$

A1

Practice 2.4 ■ 3 marks

For the football in **2.3**, sketch on one set of axes how the horizontal and vertical components of its velocity change with time, from the kick until it lands. Take up as positive, label each line, and mark the values at the start and at the end.

Solution 2.4

Method: Velocity at any moment — p.10

Worked steps

- horizontal component: a horizontal line at 14.7 m s^{-1} from 0 to 2.1 s (B1)
- vertical component: a straight line from $+10.3 \text{ m s}^{-1}$ at $t = 0$ (B1)
- through zero at 1.05 s to -10.3 m s^{-1} at 2.1 s, with both lines labelled (B1)

3

Exam-style

Exam-style 3.1 ■ 1 mark

A ball is thrown at an angle above horizontal ground. Air resistance is negligible. Which row describes the ball at the highest point of its path?

	velocity	acceleration
A	zero	zero
B	zero	9.81 m s^{-2} downwards
C	horizontal	zero
D	horizontal	9.81 m s^{-2} downwards

Solution 3.1

Method: An angled launch — p.8

Worked steps

D

- at the top the vertical component is zero, but the horizontal component is not, so the velocity is horizontal **B1**
- the acceleration is g downwards during the whole flight, including at the top

Exam-style 3.2 ■ 1 mark

A ball is thrown horizontally from the top of a building 20 m high. It lands 30 m from the base of the building. Air resistance is negligible. What was its initial speed?

A 7.4 m s^{-1}

B 15 m s^{-1}

C 20 m s^{-1}

D 21 m s^{-1}

Solution 3.2

Method: Two independent motions — p.4

A 7.4 m s^{-1}

B 15 m s^{-1}



C 20 m s^{-1}

D 21 m s^{-1}

Worked steps

B

Solution 3.2

- vertical motion from rest gives the time

$$t = \sqrt{\frac{2 \times 20 \text{ m}}{9.81 \text{ m s}^{-2}}} = 2.02 \text{ s}$$

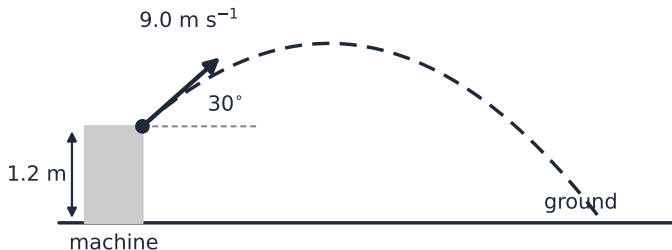
- horizontal motion gives the speed

$$u = \frac{30 \text{ m}}{2.02 \text{ s}} = \mathbf{15 \text{ m s}^{-1}}$$

B1

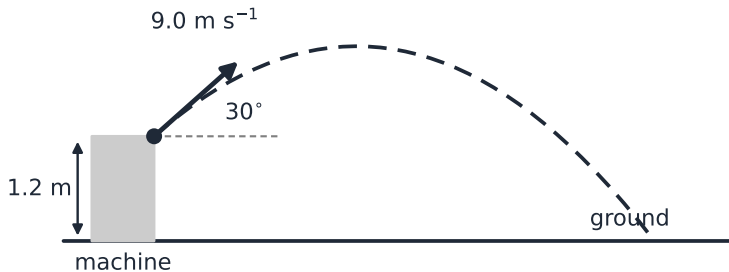
Exam-style 3.3 ■ 11 marks

A ball-launching machine is 1.2 m above level ground. It fires a ball at 9.0 m s^{-1} at 30° above the horizontal. Air resistance is negligible.



Exam-style 3.3 (a) ■ 2 marks

Calculate the horizontal and vertical components of the ball's initial velocity.



Solution 3.3 (a)

Method: An angled launch — p.8

Worked steps

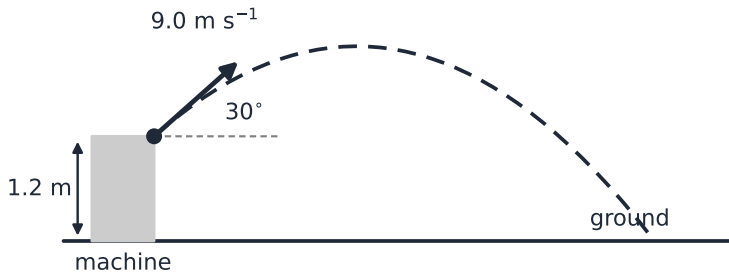
resolve **M1**

$$u_H = (9.0 \text{ m s}^{-1}) \cos 30^\circ = \mathbf{7.8 \text{ m s}^{-1}}, \quad u_V = (9.0 \text{ m s}^{-1}) \sin 30^\circ = \mathbf{4.5 \text{ m s}^{-1}}$$

A1

Exam-style 3.3 (b) ■ 2 marks

Calculate the greatest height of the ball above the ground.



Solution 3.3 (b)

Worked steps

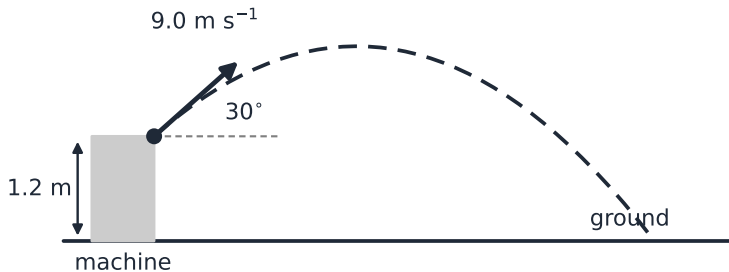
rise above the machine, with $v_V = 0$ at the top **M1**

$$\frac{0 - (4.5 \text{ m s}^{-1})^2}{2(-9.81 \text{ m s}^{-2})} = 1.03 \text{ m}$$

■ height above the ground = $1.03 \text{ m} + 1.2 \text{ m} = \mathbf{2.2 \text{ m}}$ **A1**

Exam-style 3.3 (c) ■ 3 marks

Show that the ball takes about 1.1 s to reach the ground.



Solution 3.3 (c)

Worked steps

up positive: $s = -1.2 \text{ m}$, $u = +4.5 \text{ m s}^{-1}$, $a = -9.81 \text{ m s}^{-2}$ **M1**

$$-1.2 = 4.5t - 4.905t^2 \quad \Rightarrow \quad 4.905t^2 - 4.5t - 1.2 = 0$$

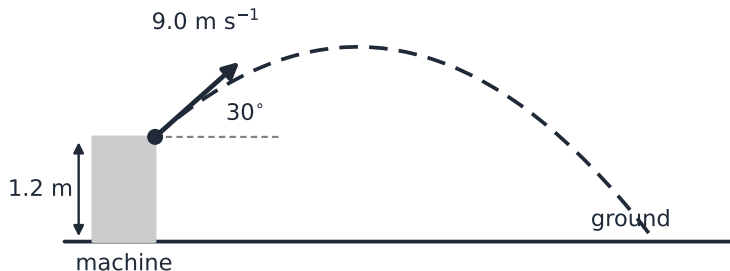
■ quadratic formula, values in m and s **M1**

$$t = \frac{4.5 + \sqrt{20.25 + 23.5}}{9.81} = 1.13 \text{ s} \approx 1.1 \text{ s}$$

A1

Exam-style 3.3 (d) ■ 1 mark

Calculate the horizontal distance from the machine to where the ball lands.



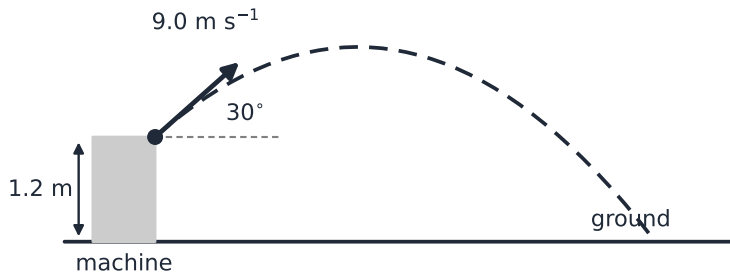
Solution 3.3 (d)

Worked steps

$$(7.8 \text{ m s}^{-1})(1.13 \text{ s}) = 8.8 \text{ m} \quad \text{B1}$$

Exam-style 3.3 (e) ■ 3 marks

Calculate the speed of the ball as it lands, and the angle its velocity makes with the ground.



Solution 3.3 (e)

Worked steps

$$v_y = 4.5 \text{ m s}^{-1} + (-9.81 \text{ m s}^{-2})(1.13 \text{ s}) = -6.6 \text{ m s}^{-1} \quad \text{M1}$$

$$v = \sqrt{(7.8 \text{ m s}^{-1})^2 + (6.6 \text{ m s}^{-1})^2} = 10 \text{ m s}^{-1}$$

A1

■ $\tan \theta = 6.6/7.8$, so the velocity is at 40° below the horizontal A1

Exam-style 3.4 ■ 6 marks

A marble rolls off the edge of a horizontal table 0.80 m high. It lands on the floor 0.52 m from the foot of the table. Air resistance is negligible.

Exam-style 3.4 (a) ■ 2 marks

Show that the marble takes 0.40 s to fall to the floor.

Solution 3.4 (a)

Method: A horizontal launch — p.6

Worked steps

vertical motion from rest **M1**

$$t = \sqrt{\frac{2 \times 0.80 \text{ m}}{9.81 \text{ m s}^{-2}}} = 0.404 \text{ s} \approx 0.40 \text{ s}$$

A1

Exam-style 3.4 (b) ■ 1 mark

Calculate the speed of the marble as it leaves the table.

Solution 3.4 (b)

Worked steps

$$u = \frac{0.52 \text{ m}}{0.404 \text{ s}} = \mathbf{1.3 \text{ m s}^{-1}}$$

B1

Exam-style 3.4 (c) ■ 3 marks

A second marble leaves the same table at twice the speed. State the time it takes to reach the floor and how far from the table it lands. Explain your answers.

Solution 3.4 (c)

Worked steps

- it still takes **0.40 s** (B1)
- the vertical motion is unchanged: zero vertical velocity at the start, the same height and the same g (B1)
- it lands **1.0 m** from the table, because the horizontal distance is the horizontal speed multiplied by the same time (B1)