Exercise walk-through

2.1.4
Free fall and measuring g

A-Level Physics

You must be able to

- use $g = 9.81$ m s^{-2} with a sign convention for an object that goes up and comes down
- find the greatest height, the time to reach it, and the time to land
- give an object released from a moving balloon its correct initial velocity
- draw and read the velocity–time graph of a thrown or bouncing ball
- **describe** an experiment to measure g, and find g from the gradient of a graph
- estimate the uncertainty in g, and explain the effect of a systematic error

1 The method

Method — Up and down with one sign convention

A ball is thrown vertically upwards at $12\ \mathrm{m\ s^{-1}}$ from a hand $1.5\ \mathrm{m}$ above the ground. **Air resistance 空气阻力** is negligible. Take **up as positive**, so $u = +12\ \mathrm{m\ s^{-1}}$ and $a = -9.81\ \mathrm{m\ s^{-2}}$ for the whole flight, on the way up and on the way down.

- greatest height above the hand: at the top $v = 0$

$$s = \frac{v^2 - u^2}{2a} = \frac{0 - (12\ \mathrm{m\ s^{-1}})^2}{2(-9.81\ \mathrm{m\ s^{-2}})} = 7.3\ \mathrm{m}$$

- time to reach the top

$$t = \frac{v - u}{a} = \frac{0 - 12\ \mathrm{m\ s^{-1}}}{-9.81\ \mathrm{m\ s^{-2}}} = 1.2\ \mathrm{s}$$

Method — Up and down with one sign convention

- the ground is 1.5 m **below** the hand, so $s = -1.5$ m (values in m and s)

$$-1.5 = 12t - 4.905t^2 \quad \Rightarrow \quad 4.905t^2 - 12t - 1.5 = 0 \quad \Rightarrow \quad t = 2.6 \text{ s}$$

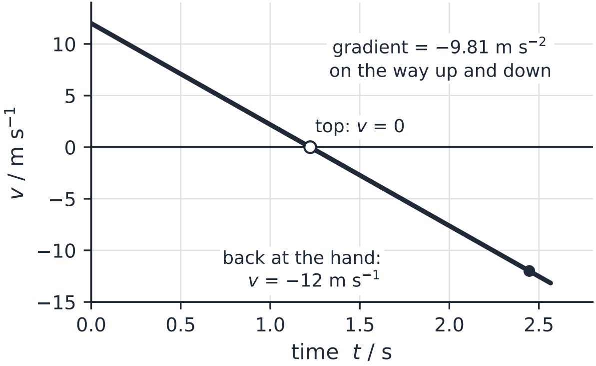
- the other root of the quadratic is negative, a time before the throw, so it is rejected
- velocity on landing

$$v^2 = (12 \text{ m s}^{-1})^2 + 2(-9.81 \text{ m s}^{-2})(-1.5 \text{ m}) \quad \Rightarrow \quad v = -13 \text{ m s}^{-1}$$

- the minus sign means the ball is moving downwards

Method — Up and down with one sign convention

A ball is thrown vertically upwards at … from a hand … above the ground.



Method — Released from something moving

A sandbag is released from a balloon that is rising at 4.0 m s^{-1}, 30 m above the ground. At the moment of release the sandbag moves with the balloon, so $u = +4.0$ m s^{-1}, **not zero**. Take up as positive: $s = -30$ m and $a = -9.81$ m s^{-2}.

- time to reach the ground (values in m and s)

$$-30 = 4.0t - 4.905t^2 \quad \Rightarrow \quad 4.905t^2 - 4.0t - 30 = 0 \quad \Rightarrow \quad t = 2.9 \text{ s}$$

- taking $u = 0$ gives 2.5 s, which is wrong: the sandbag rises a little before it falls

Method — The velocity–time graph of a bouncing ball

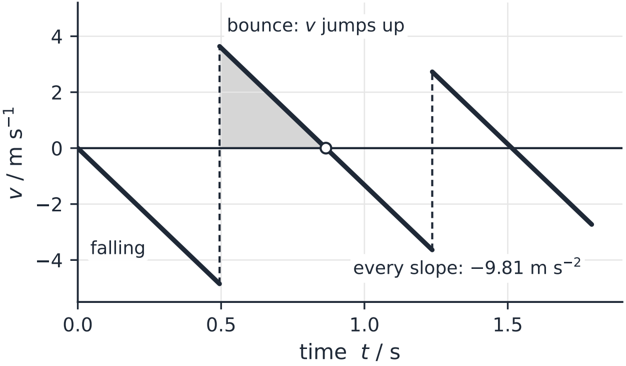
A ball is dropped from rest from a height of 1.2 m and bounces. Up is positive.

- every sloping line has the same gradient, -9.81 m s^{-2}: while the ball is in the air its acceleration is g downwards
- at a bounce the velocity jumps from negative (moving down) to positive (moving up), and it is smaller after the bounce
- the ball is at the top of a bounce where the line crosses the time axis, $v = 0$
- the height of the bounce is the area of the triangle above the axis

$$\frac{1}{2} \times 0.37 \text{ s} \times 3.6 \text{ m s}^{-1} = 0.67 \text{ m}$$

Method — The velocity–time graph of a bouncing ball

A ball is dropped from rest from a height of ... and bounces.



Method — Measuring \$g\$

The syllabus asks you to **describe** this experiment.

1. Measure the height h from the bottom of the steel ball to the trapdoor with a metre rule.
2. Switching off the electromagnet releases the ball and starts the electronic timer. The ball opens the trapdoor, which stops the timer, so the timer shows the fall time t.
3. Repeat each timing and find the mean. Do this for at least five different heights.
4. The ball starts from rest, so $h = \frac{1}{2}gt^2$. Plot h against t^2: the gradient is $\frac{1}{2}g$.

h / m	0.200	0.400	0.600	0.800	1.000
t / s	0.202	0.286	0.350	0.404	0.452
t^2 / s^2	0.0408	0.0818	0.1225	0.1632	0.2043

Method — Measuring g

- gradient of the best-fit line, from the large triangle

$$\text{gradient} = \frac{0.980\ \text{m}}{0.200\ \text{s}^2} = 4.90\ \text{m s}^{-2}$$

- $g = 2 \times \text{gradient} = 2 \times 4.90\ \text{m s}^{-2} = 9.80\ \text{m s}^{-2}$

Uncertainty. $g = 2h/t^2$, so the percentage uncertainties add, and the time counts twice because it is squared (Topic 1). For $h = 0.800 \pm 0.002$ m and $t = 0.404 \pm 0.004$ s:

Method — Measuring \$g\$

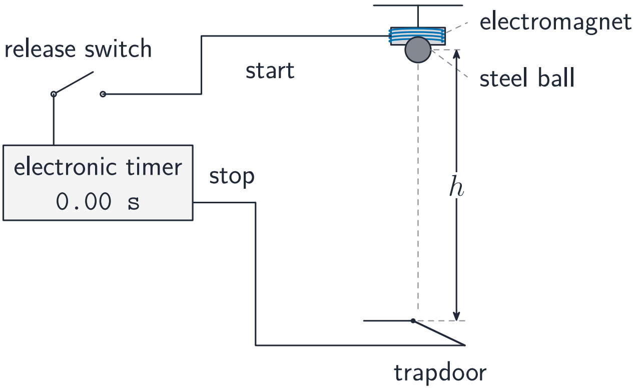
$$\frac{\Delta g}{g} = \frac{0.002\ \text{m}}{0.800\ \text{m}} + 2 \times \frac{0.004\ \text{s}}{0.404\ \text{s}} = 0.25\,\% + 2.0\,\% = 2.2\,\%$$

so $g = 9.8 \pm 0.2$ m s^{-2}.

Systematic error. If the ball is released a short time **after** the timer starts, every measured time is too long by the same amount. Each calculated value of g is then too small, and the graph of h against t^2 no longer passes through the origin.

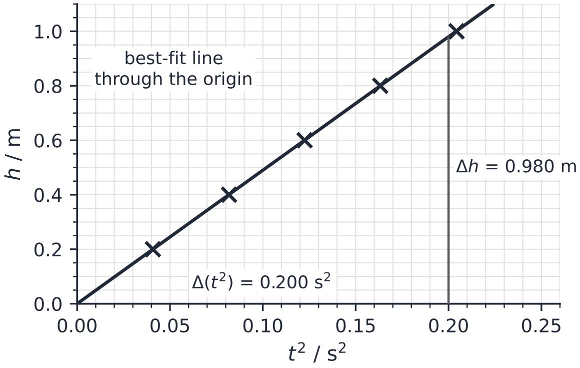
Method — Measuring g

The syllabus asks you to describe this experiment.



Method — Measuring g

The syllabus asks you to describe this experiment.



2 Practice

Practice 2.1 ▪ 4 marks

A stone is dropped from rest from a bridge 20 m above a river. Air resistance is negligible.

Practice 2.1 (a) ▪ 2 marks

Calculate the time taken for the stone to reach the water.

Solution 2.1 (a)

Method: Up and down with one sign convention — p.4

Worked steps

$u = 0$, $s = 20$ m, $a = 9.81$ m s^{-2} (down positive); use $s = \frac{1}{2}at^2$ **M1**

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 20 \text{ m}}{9.81 \text{ m s}^{-2}}} = \mathbf{2.0} \text{ s}$$

A1

Practice 2.1 (b) ▪ 2 marks

Calculate the speed of the stone as it reaches the water.

Solution 2.1 (b)

Worked steps

use $v^2 = u^2 + 2as$ **M1**

$$v = \sqrt{2 \times 9.81 \text{ m s}^{-2} \times 20 \text{ m}} = \mathbf{20} \text{ m s}^{-1}$$

A1

Practice 2.2 ▪ 5 marks

A ball is thrown vertically upwards at 18 m s^{-1} and is caught at the same height. Take up as positive.

Practice 2.2 (a) ▪ 2 marks

Calculate the greatest height the ball reaches.

Solution 2.2 (a)

Method: Up and down with one sign convention — p.4

Worked steps

at the top $v = 0$ **M1**

$$s = \frac{0 - (18\ \mathsf{m\ s}^{-1})^2}{2(-9.81\ \mathsf{m\ s}^{-2})} = \mathbf{17}\ \mathsf{m}$$

A1

Practice 2.2 (b) ▪ 2 marks

Calculate the total time the ball is in the air.

Solution 2.2 (b)

Worked steps

time to the top, then double it: the flight is symmetrical **M1**

$$t_{\text{up}} = \frac{0 - 18\ \text{m s}^{-1}}{-9.81\ \text{m s}^{-2}} = 1.83\ \text{s}, \qquad t = 2 \times 1.83\ \text{s} = \mathbf{3.7}\ \text{s}$$

A1

Practice 2.2 (c) ▪ 1 mark

State the velocity of the ball just before it is caught.

Solution 2.2 (c)

Worked steps

$-\mathbf{18}$ m s^{-1}: the same speed as the throw, now downwards **B1**

Practice 2.3 ▪ 4 marks

A camera falls out of a hot-air balloon that is rising at 3.0 m s^{-1}. At that moment the camera is 45 m above the ground.

Practice 2.3 (a) ▪ 1 mark

State the velocity of the camera at the moment it leaves the balloon.

Solution 2.3 (a)

Method: Released from something moving — p.7

Worked steps

$\mathbf{3.0}$ m s^{-1} **upwards**, the velocity of the balloon B1

Practice 2.3 (b) ▪ 3 marks

Calculate the time taken for the camera to reach the ground.
(Hint: with up as positive, $s = -45$ m.)

Solution 2.3 (b)

Worked steps

$u = +3.0$ m s^{-1}, $a = -9.81$ m s^{-2}, $s = -45$ m in $s = ut + \frac{1}{2}at^2$ **M1**

$$-45 = 3.0t - 4.905t^2 \quad \Rightarrow \quad 4.905t^2 - 3.0t - 45 = 0$$

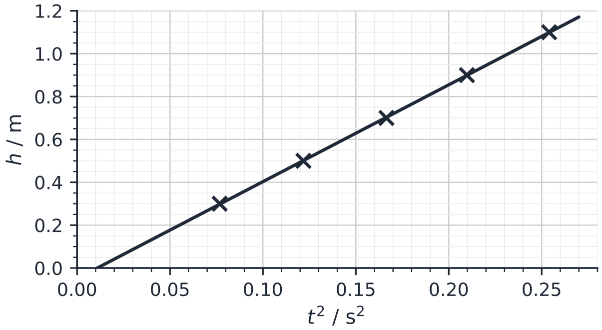
- quadratic formula (values in m and s) **M1**

$$t = \frac{3.0 + \sqrt{9.0 + 883}}{9.81} = \mathbf{3.4} \text{ s}$$

A1

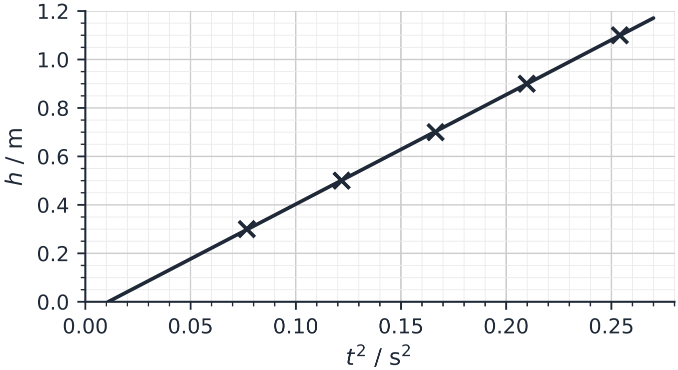
Practice 2.4 ▪ 4 marks

A student measures the time t for a ball to fall from rest through different heights h. The graph shows her results and her best-fit line.



Practice 2.4 (a) ▪ 2 marks

Determine the gradient of the line.



Solution 2.4 (a)

Method: Measuring g — p.10

Worked steps

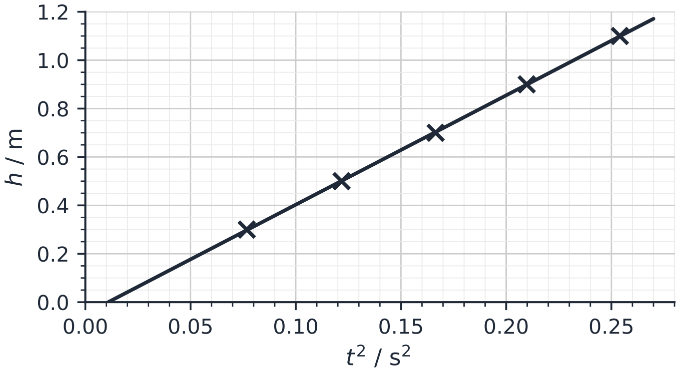
large triangle on the line, for example from $(0.050\ \mathrm{s}^2,\ 0.18\ \mathrm{m})$ to $(0.250\ \mathrm{s}^2,\ 1.08\ \mathrm{m})$ **M1**

$$\text{gradient} = \frac{1.08\ \mathrm{m} - 0.18\ \mathrm{m}}{0.250\ \mathrm{s}^2 - 0.050\ \mathrm{s}^2} = \mathbf{4.5}\ \mathrm{m\ s}^{\mathbf{-2}}$$

(accept 4.4 to 4.6) **A1**

Practice 2.4 (b) ▪ 1 mark

Use your answer to (a) to find a value for g.



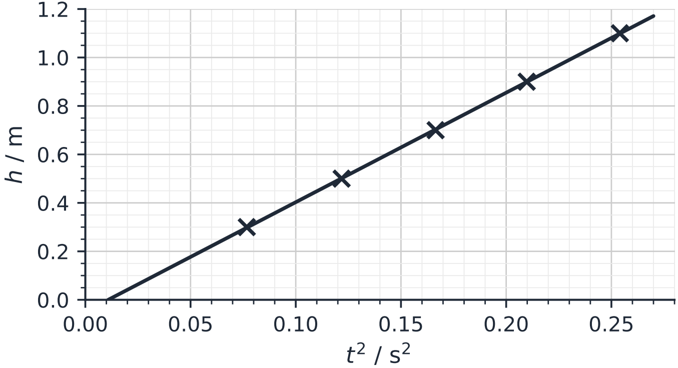
Solution 2.4 (b)

Worked steps

$g = 2 \times \text{gradient} = \mathbf{9.0}$ m $\text{s}^{\mathbf{-2}}$ (accept 8.8 to 9.2) **B1**

Practice 2.4 (c) ▪ 1 mark

The line does not pass through the origin. Suggest one reason.



Solution 2.4 (c)

Worked steps

any one: every time reading is too long by the same amount, for example the ball is released a short time after the timer starts; or h is measured from the wrong point on the ball **B1**

3 Exam-style

Exam-style 3.1 ▪ 1 mark

An object falls from rest. Air resistance is negligible. How far does it fall during the third second of its fall, from $t = 2.0$ s to $t = 3.0$ s?

A 9.8 m

B 25 m

C 29 m

D 44 m

Solution 3.1

Method: Measuring g — p.10

A 9.8 m

B 25 m ✓

C 29 m

D 44 m

Worked steps

B

- displacement after 3.0 s minus displacement after 2.0 s

$$\frac{1}{2}(9.81\ \text{m s}^{-2})(3.0\ \text{s})^2 - \frac{1}{2}(9.81\ \text{m s}^{-2})(2.0\ \text{s})^2 = 44.1\ \text{m} - 19.6\ \text{m} = \mathbf{25}\ \text{m}$$

B1

Exam-style 3.2 ▪ 1 mark

A ball is dropped from rest through different heights h, and its speed v at a light gate is measured each time. A graph of v^2 against h is a straight line through the origin with gradient 19.4 m s^{-2}. What is g?

A 4.9 m s^{-2}

B 9.7 m s^{-2}

C 19 m s^{-2}

D 39 m s^{-2}

Solution 3.2

Method: Measuring g — p.10

A 4.9 m s^{-2}

B 9.7 m s^{-2} ✓

C 19 m s^{-2}

D 39 m s^{-2}

Worked steps

B

- from $v^2 = u^2 + 2as$ with $u = 0$: $v^2 = 2gh$, so the gradient is $2g$

$$g = \frac{19.4 \text{ m s}^{-2}}{2} = \mathbf{9.7} \text{ m s}^{-2}$$

B1

Exam-style 3.3 ▪ 11 marks

A student has a steel ball, an electromagnet, a trapdoor switch, an electronic timer and a metre rule. She uses them to determine the acceleration of free fall g.

Exam-style 3.3 (a) ▪ 4 marks

Describe how she carries out the experiment. Include the measurements she takes and how she finds g from a graph.

Solution 3.3 (a)

Method: Measuring g — p.10

Worked steps

- measure h from the bottom of the ball to the trapdoor with the metre rule **B1**
- switching off the electromagnet releases the ball and starts the timer; the ball hitting the trapdoor stops it, giving t **B1**
- repeat for several different heights, repeating each timing and taking the mean **B1**
- plot h against t^2 and find the gradient; $g = 2\times$ gradient **B1**

Exam-style 3.3 (b) ▪ 2 marks

For one height, $h = 0.750 \pm 0.002$ m and $t = 0.393 \pm 0.005$ s.
Calculate g.

Solution 3.3 (b)

Worked steps

the ball starts from rest, so $h = \frac{1}{2}gt^2$ **M1**

$$g = \frac{2h}{t^2} = \frac{2 \times 0.750\ \text{m}}{(0.393\ \text{s})^2} = \mathbf{9.71}\ \text{m s}^{\mathbf{-2}}$$

A1

Exam-style 3.3 (c) ▪ 3 marks

Calculate the percentage uncertainty in her value of g, and hence give g with its absolute uncertainty.

Solution 3.3 (c)

Worked steps

percentage uncertainties **M1**

$$\frac{0.002\ \text{m}}{0.750\ \text{m}} = 0.27\,\%, \qquad \frac{0.005\ \text{s}}{0.393\ \text{s}} = 1.27\,\%$$

- the time is squared, so it counts twice: $0.27\,\% + 2 \times 1.27\,\% = \mathbf{2.8}\,\%$ **A1**
- $2.8\,\%$ of 9.71 m s^{-2} is 0.27 m s^{-2}, so $g = \mathbf{9.7} \pm \mathbf{0.3}$ m s$^{\mathbf{-2}}$ **A1**

Exam-style 3.3 (d) ▪ 2 marks

The electromagnet takes a short time to release the ball after the timer starts. State and explain the effect of this on the value of g calculated from one measurement.

Solution 3.3 (d)

Worked steps

- the measured time is longer than the real time of fall **B1**
- $g = 2h/t^2$, so the calculated value of g is too small **B1**

Exam-style 3.4 ▪ 8 marks

A ball is thrown vertically upwards at 8.0 m s^{-1} from the top of a cliff. It then falls past the cliff top into the sea 25 m below. Air resistance is negligible. Take up as positive.

Exam-style 3.4 (a) ▪ 2 marks

Calculate the greatest height of the ball above the cliff top.

Solution 3.4 (a)

Method: Up and down with one sign convention — p.4

Worked steps

at the top $v = 0$ **M1**

$$s = \frac{0 - (8.0\ \text{m s}^{-1})^2}{2(-9.81\ \text{m s}^{-2})} = \mathbf{3.3}\ \text{m}$$

A1

Exam-style 3.4 (b) ▪ 3 marks

Calculate the time from the throw until the ball reaches the sea.

Solution 3.4 (b)

Worked steps

$s = -25$ m, $u = +8.0$ m s^{-1}, $a = -9.81$ m s^{-2} **M1**

$$-25 = 8.0t - 4.905t^2 \quad \Rightarrow \quad 4.905t^2 - 8.0t - 25 = 0$$

- quadratic formula, keeping the positive root **M1**

$$t = \frac{8.0 + \sqrt{64 + 490.5}}{9.81} = \mathbf{3.2}\text{ s}$$

A1

Exam-style 3.4 (c) ▪ 3 marks

Sketch the velocity–time graph of the ball from the throw until it reaches the sea. Mark the values of the velocity at the start and at the end.

Solution 3.4 (c)

Worked steps

- a single straight line with a negative gradient **B1**
- starting at $+8.0$ m s^{-1} and crossing $v = 0$ at about 0.8 s **B1**
- ending at about -24 m s^{-1} at 3.2 s **B1**