Exercise walk-through

2.1.3

The equations of uniformly accelerated motion

A-Level Physics

You must be able to

- **derive 推导** the equations from the definition of acceleration and the area under a velocity–time graph
- state when the equations can be used
- choose the equation that links what you know to what you want
- choose a positive direction and give every vector a sign
- solve problems with two stages, or with two objects
- solve a **quadratic 二次方程** with two answers, and decide which answers make sense

1 The method

Method — Deriving the equations

An object has **uniform acceleration 匀加速** a in a straight line. Its velocity changes from u to v in a time t, and its displacement is s.

- from the definition of acceleration (the gradient)

$$a = \frac{v-u}{t} \quad \Rightarrow \quad v = u + at$$

- displacement = area under the line, a **trapezium 梯形**

$$s = \tfrac{1}{2}(u+v)t$$

- put $v = u + at$ into the trapezium; this is the rectangle plus the triangle in the graph

$$s = \tfrac{1}{2}(u+u+at)t = ut + \tfrac{1}{2}at^2$$

Method — Deriving the equations

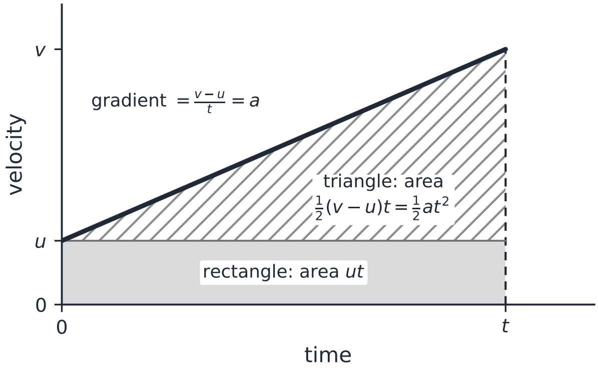
- from the first equation $t = (v - u)/a$; put this into $s = \frac{1}{2}(u + v)t$

$$s = \frac{(v+u)(v-u)}{2a} \quad \Rightarrow \quad v^2 = u^2 + 2as$$

The equations work only when the acceleration is **constant** and the motion is in a **straight line**. $s = \frac{1}{2}at^2$ also needs the object to start from rest $(u = 0)$.

Method — Deriving the equations

An object has uniform acceleration 匀加速 ... in a straight line.



Method — Choosing an equation

Write down the five symbols s, u, v, a, t. Find the one you are not given and do not want: use the equation without it.

not given and not wanted	**use**
s	$v = u + at$
a	$s = \frac{1}{2}(u + v)t$
v	$s = ut + \frac{1}{2}at^2$
t	$v^2 = u^2 + 2as$

Method — Choosing an equation

Example. A cyclist brakes uniformly from $9.0\ \mathrm{m\ s^{-1}}$ to rest in a distance of $12\ \mathrm{m}$.

- $u = 9.0\ \mathrm{m\ s^{-1}}$, $v = 0$, $s = 12\ \mathrm{m}$; to find a, there is no t

$$a = \frac{v^2 - u^2}{2s} = \frac{0 - (9.0\ \mathrm{m\ s^{-1}})^2}{2 \times 12\ \mathrm{m}} = -3.4\ \mathrm{m\ s^{-2}}$$

- to find t, there is no a

$$t = \frac{2s}{u + v} = \frac{2 \times 12\ \mathrm{m}}{9.0\ \mathrm{m\ s^{-1}}} = 2.7\ \mathrm{s}$$

- the minus sign shows the acceleration is opposite to the velocity: a **deceleration** **减速度** of $3.4\ \mathrm{m\ s^{-2}}$

Method — Signs, and two answers

Choose one direction as positive and keep it for the whole problem. A ball rolls up a slope at 4.0 m s^{-1}. Its acceleration is 2.0 m s^{-2} down the slope, on the way up and on the way back down. Take **up the slope as positive**: $u = +4.0$ m s^{-1} and $a = -2.0$ m s^{-2}.

- when does it stop? Use $v = u + at$

$$0 = 4.0 \text{ m s}^{-1} + (-2.0 \text{ m s}^{-2})t \quad \Rightarrow \quad t = 2.0 \text{ s}$$

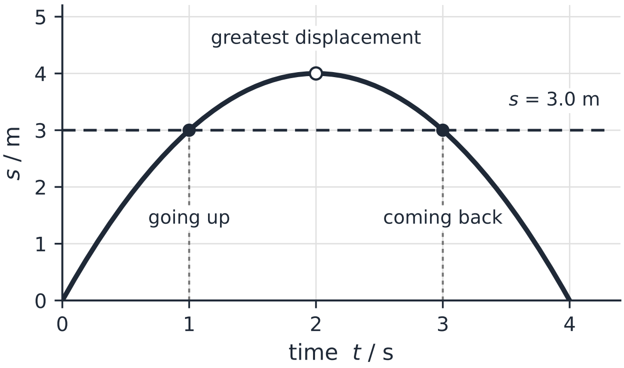
- when is it 3.0 m up the slope? Use $s = ut + \frac{1}{2}at^2$ (values in m and s)

$$3.0 = 4.0t - 1.0t^2 \quad \Rightarrow \quad t^2 - 4.0t + 3.0 = 0 \quad \Rightarrow \quad t = 1.0 \text{ s or } 3.0 \text{ s}$$

- both answers are real: at 1.0 s the ball is going up, and at 3.0 s it is coming back down

Method — Signs, and two answers

Choose one direction as positive and keep it for the whole problem.



Method — Two stages

A tram starts from rest with an acceleration of 0.80 m s^{-2} for 15 s. It then keeps a steady speed for 50 s.

- stage 1: the final velocity

$$v = u + at = 0 + (0.80 \text{ m s}^{-2})(15 \text{ s}) = 12 \text{ m s}^{-1}$$

- stage 1: the displacement

$$s_1 = \frac{1}{2}at^2 = \frac{1}{2}(0.80 \text{ m s}^{-2})(15 \text{ s})^2 = 90 \text{ m}$$

- the final velocity of stage 1 is the initial velocity of stage 2

$$s_2 = (12 \text{ m s}^{-1})(50 \text{ s}) = 600 \text{ m}$$

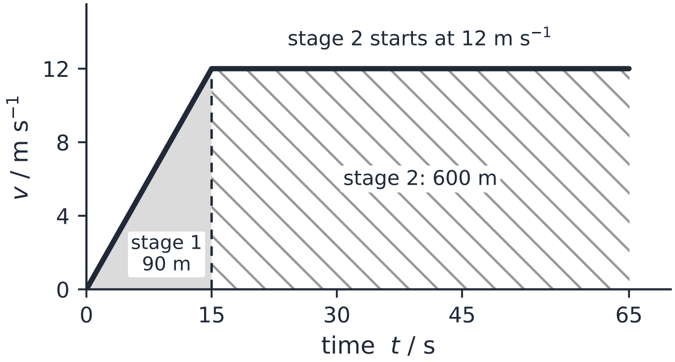
- total displacement $= 90$ m $+ 600$ m $= 690$ m

Method — Two stages

A driver's **reaction time 反应时间** is a stage too: the car keeps its speed, so the **thinking distance 反应距离** is speed $\times$ reaction time. The **braking distance 制动距离** comes from $v^2 = u^2 + 2as$, and the **stopping distance 停车距离** is the sum of the two.

Method — Two stages

A tram starts from rest with an acceleration of ... for ... .



Method — Two objects

Car A passes a point P at a constant 15 m s^{-1}. 2.0 s later, car B starts from rest at P with an acceleration of 3.0 m s^{-2}. Measure the time T from when A passes P.

- car A has moved for T and car B for $(T - 2.0)$, so (values in m and s)

$$s_A = 15T, \qquad s_B = \tfrac{1}{2}(3.0)(T-2.0)^2 = 1.5(T-2.0)^2$$

- they are level when the displacements are equal

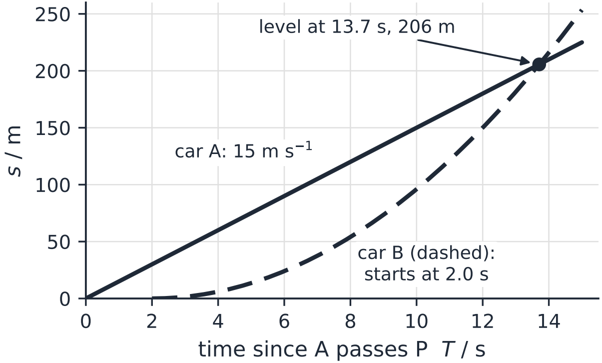
$$15T = 1.5(T-2.0)^2 \quad \Rightarrow \quad T^2 - 14T + 4.0 = 0 \quad \Rightarrow \quad T = 13.7 \text{ s or } 0.29 \text{ s}$$

- 0.29 s is before B starts, so it is not a meeting: B catches A at $T = 13.7$ s

$$s = (15 \text{ m s}^{-1})(13.7 \text{ s}) = 206 \text{ m}$$

Method — Two objects

Measure the time ... from when A passes P



2 Practice

Practice 2.1 ▪ 2 marks

Show that $s = ut + \frac{1}{2}at^2$ follows from $v = u + at$ and $s = \frac{1}{2}(u + v)t$.

Solution 2.1

Worked steps

- substitute $v = u + at$ into $s = \frac{1}{2}(u + v)t$ **M1**

$$s = \frac{1}{2}(u + u + at)t = \frac{1}{2}(2u + at)t$$

- multiply out the bracket: $s = ut + \frac{1}{2}at^2$ **A1**

Practice 2.2 ▪ 3 marks

The equations of uniformly accelerated motion can only be used under certain conditions.

Practice 2.2 (a) ▪ 2 marks

State two conditions that must both be true.

Solution 2.2 (a)

Method: Deriving the equations — p.4

Worked steps

the acceleration is constant **B1**; the motion is in a straight line **B1**

Practice 2.2 (b) ▪ 1 mark

State the extra condition needed to use $s = \frac{1}{2}at^2$.

Solution 2.2 (b)

Worked steps

the object starts from rest, so $u = 0$ **B1**

Practice 2.3 ▪ 4 marks

Choose the right equation each time.

Practice 2.3 (a) ▪ 1 mark

$u = 2.0$ m s^{-1}, $a = 1.5$ m s^{-2}, $t = 4.0$ s. Find v.

Solution 2.3 (a)

Method: Signs, and two answers — p.9

Worked steps

no s, so $v = u + at = 2.0\ \mathrm{m\ s^{-1}} + (1.5\ \mathrm{m\ s^{-2}})(4.0\ \mathrm{s}) = \mathbf{8.0}\ \mathrm{m\ s}^{\mathbf{-1}}$ B1

Practice 2.3 (b) ▪ 1 mark

$u = 0$, $v = 12\ \text{m s}^{-1}$, $s = 18\ \text{m}$. Find a.

Solution 2.3 (b)

Worked steps

no t, so use $v^2 = u^2 + 2as$

$$a = \frac{v^2 - u^2}{2s} = \frac{(12\ \text{m s}^{-1})^2}{2 \times 18\ \text{m}} = \mathbf{4.0}\ \text{m s}^{\mathbf{-2}}$$

B1

Practice 2.3 (c) ▪ 1 mark

$u = 5.0$ m s^{-1}, $v = 15$ m s^{-1}, $t = 4.0$ s. Find s.

Solution 2.3 (c)

Worked steps

no a, so use $s = \frac{1}{2}(u + v)t$

$$s = \frac{1}{2}(5.0\ \mathrm{m\ s^{-1}} + 15\ \mathrm{m\ s^{-1}})(4.0\ \mathrm{s}) = \mathbf{40}\ \mathrm{m}$$

B1

Practice 2.3 (d) ▪ 1 mark

$u = 3.0$ m s^{-1}, $a = 2.0$ m s^{-2}, $s = 10$ m. Find v.

Solution 2.3 (d)

Worked steps

no t, so use $v^2 = u^2 + 2as$

$$v^2 = (3.0\ \mathrm{m\ s^{-1}})^2 + 2(2.0\ \mathrm{m\ s^{-2}})(10\ \mathrm{m}) = 49\ \mathrm{m^2\,s^{-2}} \Rightarrow v = \mathbf{7.0}\ \mathrm{m\ s^{-1}}$$

B1

Practice 2.4 ▪ 4 marks

A car is travelling at 25 m s^{-1} when the driver sees a hazard. The driver's reaction time is 0.60 s. The car then brakes with a constant deceleration of 6.25 m s^{-2}.

Practice 2.4 (a) ▪ 1 mark

Calculate the thinking distance.

Solution 2.4 (a)

Method: Two stages — p.11

Worked steps

the speed is constant during the reaction time: $(25 \text{ m s}^{-1})(0.60 \text{ s}) = \mathbf{15} \text{ m}$ B1

Practice 2.4 (b) ▪ 2 marks

Calculate the braking distance.

Solution 2.4 (b)

Worked steps

$u = 25$ m s^{-1}, $v = 0$, $a = -6.25$ m s^{-2}; use $v^2 = u^2 + 2as$ **M1**

$$s = \frac{0 - (25 \text{ m s}^{-1})^2}{2(-6.25 \text{ m s}^{-2})} = \mathbf{50} \text{ m}$$

A1

Practice 2.4 (c) ▪ 1 mark

State the stopping distance.

Solution 2.4 (c)

Worked steps

$15\ \text{m} + 50\ \text{m} = \mathbf{65}\ \text{m}$ B1

Practice 2.5 ▪ 6 marks

A puck slides up a smooth icy ramp at 6.0 m s^{-1}. Its acceleration is 1.5 m s^{-2} down the ramp, on the way up and on the way back down. Take up the ramp as positive.

Practice 2.5 (a) ▪ 1 mark

Calculate the time taken for the puck to stop.

Solution 2.5 (a)

Method: Signs, and two answers — p.9

Worked steps

at the top $v = 0$

$$t = \frac{v - u}{a} = \frac{0 - 6.0 \text{ m s}^{-1}}{-1.5 \text{ m s}^{-2}} = \mathbf{4.0} \text{ s}$$

B1

Practice 2.5 (b) ▪ 2 marks

Calculate the greatest distance the puck travels up the ramp.

Solution 2.5 (b)

Worked steps

at the top $v = 0$; use $v^2 = u^2 + 2as$ **M1**

$$s = \frac{0 - (6.0\ \text{m s}^{-1})^2}{2(-1.5\ \text{m s}^{-2})} = \mathbf{12}\ \text{m}$$

A1

Practice 2.5 (c) ▪ 3 marks

Calculate the two times at which the puck is 10 m up the ramp.
(Hint: you will need to solve a quadratic.)

Solution 2.5 (c)

Worked steps

$s = ut + \frac{1}{2}at^2$ with $s = 10$, $u = 6.0$, $a = -1.5$ (values in m and s) M1

$$10 = 6.0t - 0.75t^2 \quad \Rightarrow \quad 0.75t^2 - 6.0t + 10 = 0$$

- use the quadratic formula M1

$$t = \frac{6.0 \pm \sqrt{36 - 30}}{1.5}$$

- $t = \mathbf{2.4}$ s (going up) and $t = \mathbf{5.6}$ s (coming back down) A1

3 Exam-style

Exam-style 3.1 ▪ 1 mark

A car moving at 5.0 m s^{-1} accelerates uniformly. In the next 4.0 s it travels 44 m. What is its acceleration?

A 1.5 m s^{-2}

B 3.0 m s^{-2}

C 5.5 m s^{-2}

D 6.0 m s^{-2}

Solution 3.1

Method: Choosing an equation — p.7

A $1.5\ \mathrm{m\ s^{-2}}$

B $3.0\ \mathrm{m\ s^{-2}}$ ✓

C $5.5\ \mathrm{m\ s^{-2}}$

D $6.0\ \mathrm{m\ s^{-2}}$

Worked steps

B

Solution 3.1

- $u = 5.0\ \mathrm{m\ s^{-1}}$, $t = 4.0\ \mathrm{s}$, $s = 44\ \mathrm{m}$; there is no v, so use $s = ut + \frac{1}{2}at^2$

$$44\ \mathrm{m} = (5.0\ \mathrm{m\ s^{-1}})(4.0\ \mathrm{s}) + \tfrac{1}{2}a(4.0\ \mathrm{s})^2 \Rightarrow a = \frac{24\ \mathrm{m}}{8.0\ \mathrm{s}^2} = \mathbf{3.0}\ \mathrm{m\ s^{-2}}$$

B1

Exam-style 3.2 ▪ 1 mark

A lift starts from rest and accelerates uniformly at 1.2 m s^{-2} for 2.5 s. It then moves at a constant velocity for 6.0 s, and finally slows uniformly to rest in 2.0 s. How far does the lift travel?

A 18 m

B 25 m

C 32 m

D 33 m

Solution 3.2

Method: Two stages — p.11

A 18 m

B 25 m ✓

C 32 m

D 33 m

Worked steps

B

- top speed: $(1.2\ \mathrm{m\ s^{-2}})(2.5\ \mathrm{s}) = 3.0\ \mathrm{m\ s^{-1}}$

Solution 3.2

- distance = area under the velocity–time graph

$$\frac{1}{2}(2.5\ \mathrm{s})(3.0\ \mathrm{m\ s^{-1}}) + (6.0\ \mathrm{s})(3.0\ \mathrm{m\ s^{-1}}) + \frac{1}{2}(2.0\ \mathrm{s})(3.0\ \mathrm{m\ s^{-1}}) = \mathbf{25}\ \mathrm{m}$$

B1

Exam-style 3.3 ▪ 10 marks

A delivery van travels along a straight road at a constant velocity of 16 m s^{-1}. It passes a police car that is at rest. 3.0 s later, the police car sets off after the van. It accelerates uniformly at 2.5 m s^{-2} until it reaches 30 m s^{-1}, and then keeps this velocity.

Exam-style 3.3 (a) ▪ 1 mark

Calculate the time the police car takes to reach 30 m s^{-1}.

Solution 3.3 (a)

Worked steps

$$t = \frac{v - u}{a} = \frac{30 \text{ m s}^{-1} - 0}{2.5 \text{ m s}^{-2}} = \mathbf{12} \text{ s}$$

B1

Exam-style 3.3 (b) ▪ 2 marks

Calculate the distance the police car travels while it is accelerating.

Solution 3.3 (b)

Worked steps

no a needed once t is known **M1**

$$s = \frac{1}{2}(u + v)t = \frac{1}{2}(0 + 30 \text{ m s}^{-1})(12 \text{ s}) = \mathbf{180} \text{ m}$$

A1

Exam-style 3.3 (c) ▪ 2 marks

Show that the police car has not caught the van when it reaches 30 m s^{-1}.

Solution 3.3 (c)

Worked steps

the van has been moving for $3.0\ \mathrm{s} + 12\ \mathrm{s} = 15\ \mathrm{s}$ **M1**

$$s_{\mathrm{van}} = (16\ \mathrm{m\ s^{-1}})(15\ \mathrm{s}) = 240\ \mathrm{m}$$

- $240\ \mathrm{m}$ is more than $180\ \mathrm{m}$, so the van is still ahead **A1**

Exam-style 3.3 (d) ▪ 3 marks

Determine the time, measured from when the van passes the police car, at which the police car catches the van.

Solution 3.3 (d)

Worked steps

the gap at 15 s is 240 m $-$ 180 m $= 60$ m **M1**

- the police car closes the gap at 30 m s^{-1} $-$ 16 m s^{-1} $= 14$ m s^{-1} **M1**

$$\frac{60\ \text{m}}{14\ \text{m s}^{-1}} = 4.3\ \text{s} \quad \Rightarrow \quad t = 15\ \text{s} + 4.3\ \text{s} = \mathbf{19}\ \text{s}$$

A1

Exam-style 3.3 (e) ▪ 2 marks

On one set of axes, **sketch** the velocity–time graphs of the van and of the police car up to the moment the police car catches the van. Label each line.

Solution 3.3 (e)

Worked steps

- van: a horizontal line at 16 m s^{-1} from $t = 0$ **B1**
- police car: zero until 3.0 s, a straight line up to 30 m s^{-1} at 15 s, then horizontal to about 19 s **B1**

Exam-style 3.4 ▪ 6 marks

The equation $v^2 = u^2 + 2as$ links velocity and displacement.

Exam-style 3.4 (a) ▪ 3 marks

Starting from $v = u + at$ and $s = \frac{1}{2}(u + v)t$, **derive** $v^2 = u^2 + 2as$.

Solution 3.4 (a)

Worked steps

- from $v = u + at$, $t = (v - u)/a$ **M1**
- substitute into $s = \frac{1}{2}(u + v)t$ **M1**

$$s = \frac{(u + v)(v - u)}{2a} = \frac{v^2 - u^2}{2a}$$

- so $2as = v^2 - u^2$, which gives $v^2 = u^2 + 2as$ **A1**

Exam-style 3.4 (b) ▪ 2 marks

A skateboarder accelerates uniformly from rest to 9.0 m s^{-1} over a distance of 18 m. Calculate her acceleration.

Solution 3.4 (b)

Worked steps

$u = 0$ **M1**

$$a = \frac{v^2}{2s} = \frac{(9.0\ \text{m s}^{-1})^2}{2 \times 18\ \text{m}} = \mathbf{2.3}\ \text{m s}^{\mathbf{-2}}$$

A1

Exam-style 3.4 (c) ▪ 1 mark

Explain why this equation could not be used if her acceleration decreased as she sped up.

Solution 3.4 (c)

Worked steps

the equation is true only when the acceleration is constant **B1**