Exercise walk-through

2.1.2

Velocity–time and acceleration–time graphs

A-Level Physics

You must be able to

- find an acceleration from the gradient of a velocity–time graph, including at one instant on a curve
- find a displacement from the area under a velocity–time graph, including area below the time axis
- tell distance from displacement on a velocity–time graph
- estimate the area under a curve by counting squares
- find a change in velocity from the area under an acceleration–time graph
- **sketch** one motion graph from another

1 The method

Method — Gradient and area of a velocity–time graph

On a velocity–time graph the **gradient 斜率** is the **acceleration 加速度**, and the **area 面积** between the line and the time axis is the **displacement 位移**. Area **below** the time axis is negative displacement: the object is moving backwards. The **distance 距离** adds up the sizes of all the areas.

A trolley is pushed up a track at 6.0 m s^{-1}. It slows down, stops for an instant at $t = 3.0$ s, and rolls back.

- acceleration = gradient

$$a = \frac{\Delta v}{\Delta t} = \frac{-4.0 \text{ m s}^{-1} - 6.0 \text{ m s}^{-1}}{5.0 \text{ s}} = -2.0 \text{ m s}^{-2}$$

- the line is straight, so the acceleration is still -2.0 m s^{-2} at $t = 3.0$ s, when the velocity is zero

Method — Gradient and area of a velocity–time graph

- area above the axis

$$\frac{1}{2} \times 3.0\ \text{s} \times 6.0\ \text{m s}^{-1} = +9.0\ \text{m}$$

- area below the axis

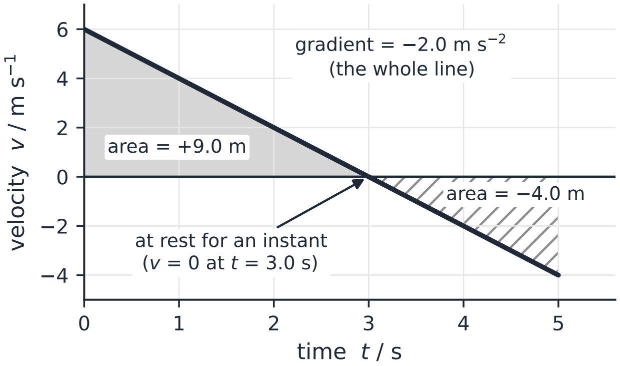
$$\frac{1}{2} \times 2.0\ \text{s} \times (-4.0\ \text{m s}^{-1}) = -4.0\ \text{m}$$

- displacement after 5.0 s: $9.0\ \text{m} - 4.0\ \text{m} = 5.0\ \text{m}$ up the track; distance: $9.0\ \text{m} + 4.0\ \text{m} = 13\ \text{m}$

A negative gradient means the acceleration points in the negative direction. On an acceleration–time graph, the area under the line is the **change in velocity 速度变化**.

Method — Gradient and area of a velocity–time graph

On a velocity–time graph the gradient 斜率 is the acceleration 加速度, and the area 面积 between the line and the...



Method — One journey, three graphs

A train starts from rest and speeds up uniformly to 12 m s^{-1} in 20 s. It keeps this speed for 30 s, then brakes uniformly to rest in 15 s.

- **acceleration–time**: each acceleration is a gradient of the velocity–time graph

$$a_1 = \frac{12 \text{ m s}^{-1}}{20 \text{ s}} = 0.60 \text{ m s}^{-2}, \qquad a_3 = \frac{-12 \text{ m s}^{-1}}{15 \text{ s}} = -0.80 \text{ m s}^{-2}$$

- while the speed is steady, $a = 0$
- **displacement–time**: its gradient is the velocity, so it gets steeper while the train speeds up, is a straight line while the speed is steady, and flattens to horizontal as the train stops
- total displacement $=$ area under the velocity–time graph

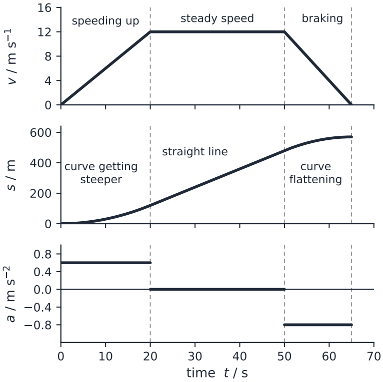
$$s = \tfrac{1}{2}(20 \text{ s})(12 \text{ m s}^{-1}) + (30 \text{ s})(12 \text{ m s}^{-1}) + \tfrac{1}{2}(15 \text{ s})(12 \text{ m s}^{-1}) = 570 \text{ m}$$

Method — One journey, three graphs

When you **sketch 画草图** a graph, draw and label both axes, and give each part the right shape: straight or curved, getting steeper or getting flatter.

Method — One journey, three graphs

When you sketch 画草图 a graph, draw and label both axes, and give each part the right shape: straight or...



Method — A curve: a tangent and counting squares

When a velocity–time graph is a curve, the acceleration at one instant is the gradient of the **tangent 切线** there, and the area is estimated by counting grid squares.

- the tangent at $t = 2.5$ s passes through $(1.0\ \text{s},\ 4.1\ \text{m s}^{-1})$ and $(4.0\ \text{s},\ 8.5\ \text{m s}^{-1})$

$$a = \frac{8.5\ \text{m s}^{-1} - 4.1\ \text{m s}^{-1}}{4.0\ \text{s} - 1.0\ \text{s}} = 1.5\ \text{m s}^{-2}$$

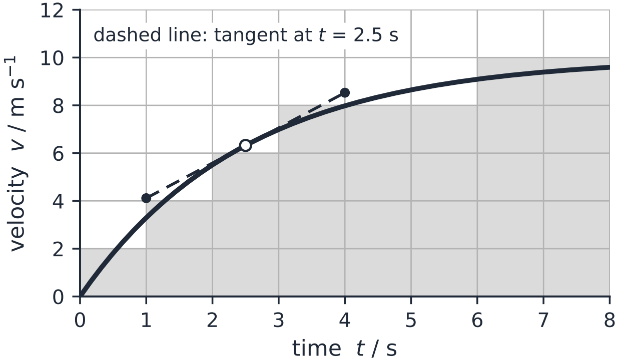
- one square is 1 s wide and $2\ \text{m s}^{-1}$ tall, so it is worth

$$1\ \text{s} \times 2\ \text{m s}^{-1} = 2\ \text{m}$$

- count every square that is **at least half** under the curve: 28 squares
- displacement $\approx 28 \times 2\ \text{m} = 56\ \text{m}$

Method — A curve: a tangent and counting squares

When a velocity–time graph is a curve, the acceleration at one instant is the gradient of the tangent 切线 …



2 Practice

Practice 2.1 ▪ 2 marks

State what is represented by (a) the gradient of a velocity–time graph; (b) the area under a velocity–time graph.

Solution 2.1

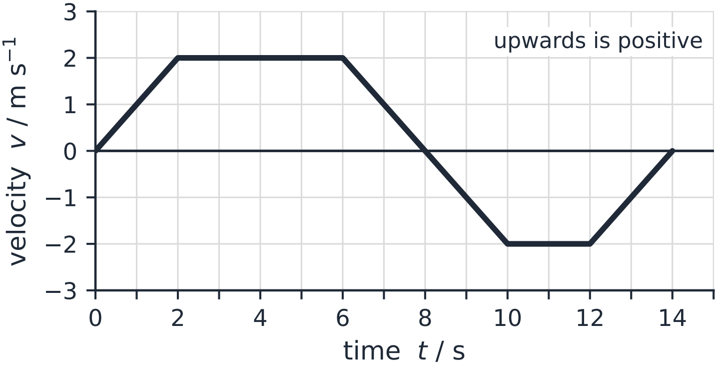
Worked steps

(a) the gradient is the **acceleration** B1

(b) the area is the **displacement** B1

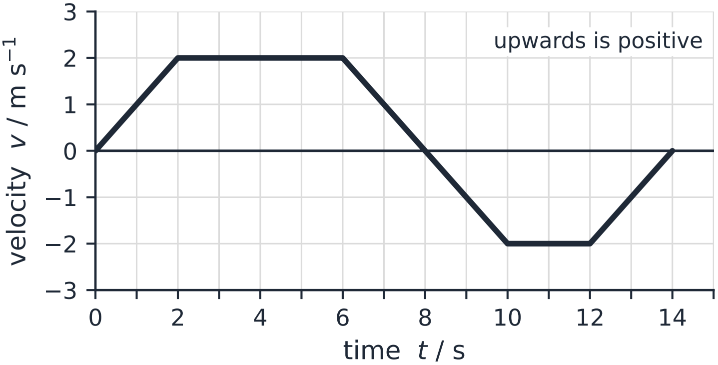
Practice 2.2 ▪ 5 marks

The velocity–time graph shows a lift that goes up and then comes back down. Upwards is positive.



Practice 2.2 (a) ▪ 1 mark

Calculate the acceleration of the lift between $t = 0$ and $t = 2.0$ s.



Solution 2.2 (a)

Worked steps

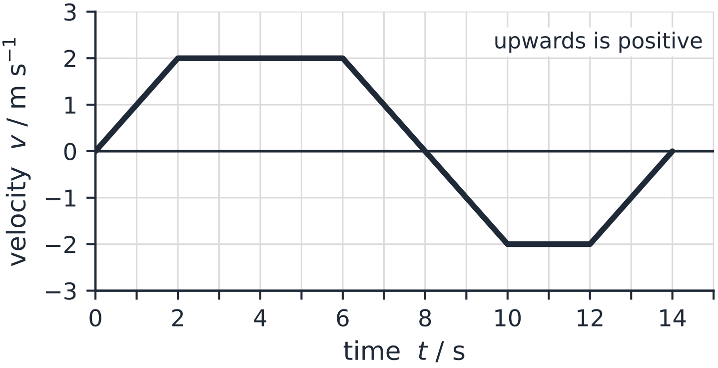
gradient of the first line

$$a = \frac{2.0\ \text{m s}^{-1} - 0}{2.0\ \text{s}} = \mathbf{1.0}\ \text{m s}^{\mathbf{-2}}$$

B1

Practice 2.2 (b) ▪ 1 mark

State the times at which the lift is at rest.



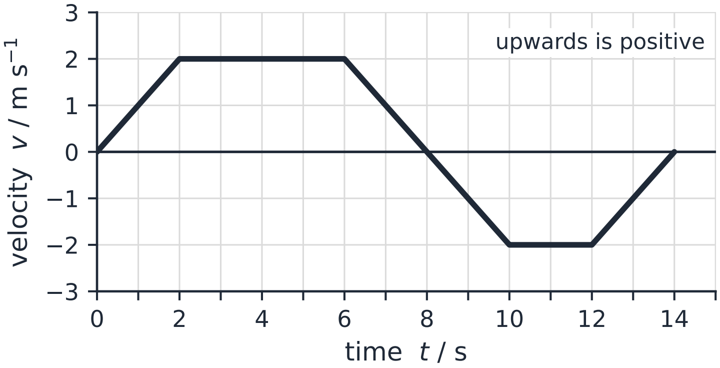
Solution 2.2 (b)

Worked steps

$v = 0$ at $t = \mathbf{0}$, $\mathbf{8.0}$ s and $\mathbf{14}$ s B1

Practice 2.2 (c) ▪ 2 marks

Calculate the total distance travelled by the lift in the 14 s. *(Hint: for distance, every area counts as positive.)*



Solution 2.2 (c)

Worked steps

going up, the area above the axis is a trapezium (or two triangles and a rectangle) M1

$$\frac{1}{2}(8.0\ \mathsf{s} + 4.0\ \mathsf{s})(2.0\ \mathsf{m\ s}^{-1}) = 12\ \mathsf{m}$$

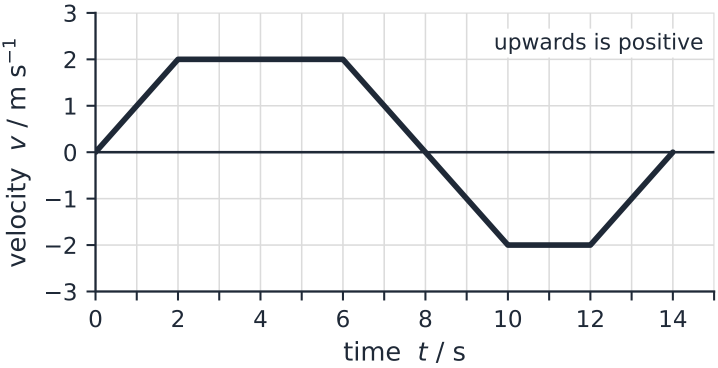
- coming down, the area below the axis

$$\frac{1}{2}(6.0\ \mathsf{s} + 2.0\ \mathsf{s})(2.0\ \mathsf{m\ s}^{-1}) = 8.0\ \mathsf{m}$$

- total distance $= 12\ \mathsf{m} + 8.0\ \mathsf{m} = \mathbf{20}\ \mathsf{m}$ A1

Practice 2.2 (d) ▪ 1 mark

Determine the displacement of the lift from its starting point at $t = 14$ s.



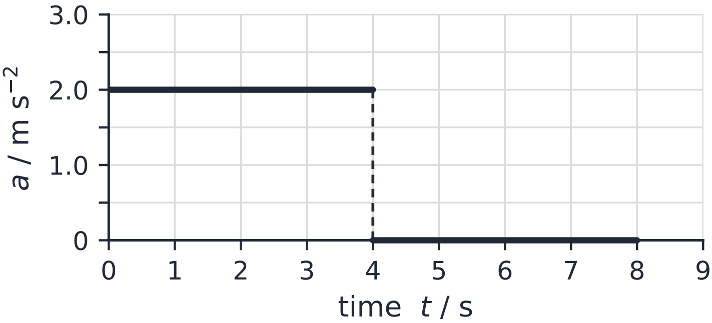
Solution 2.2 (d)

Worked steps

displacement $= 12$ m $- 8.0$ m $= \mathbf{4.0}$ m above the starting point B1

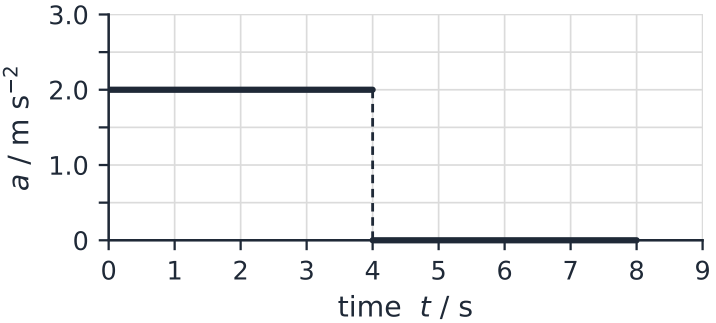
Practice 2.3 ▪ 4 marks

A car is moving at 5.0 m s^{-1} at time $t = 0$. The graph shows its acceleration.



Practice 2.3 (a) ▪ 1 mark

Determine the change in velocity of the car between $t = 0$ and $t = 4.0$ s.



Solution 2.3 (a)

Method: One journey, three graphs — p.7

Worked steps

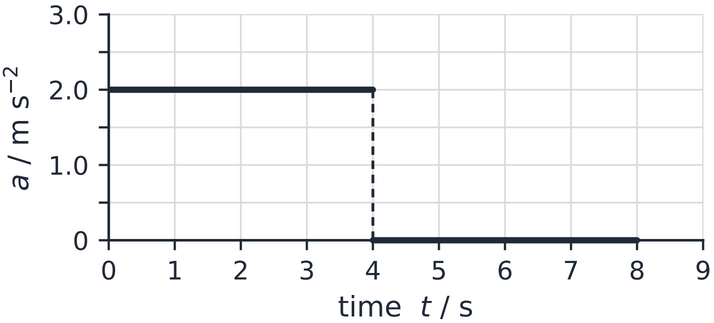
change in velocity = area under the acceleration–time graph

$$\Delta v = 2.0 \text{ m s}^{-2} \times 4.0 \text{ s} = \mathbf{8.0} \text{ m s}^{\mathbf{-1}}$$

B1

Practice 2.3 (b) ▪ 1 mark

State the velocity of the car at $t = 6.0$ s.



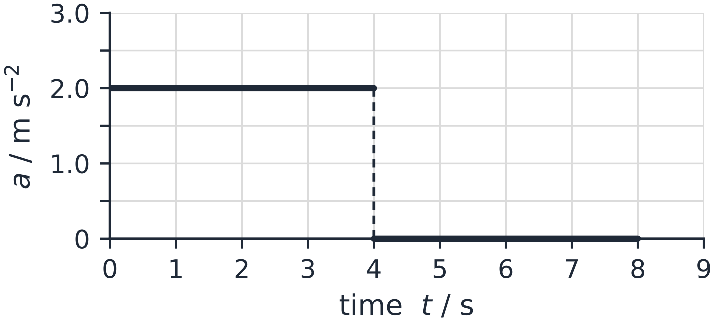
Solution 2.3 (b)

Worked steps

5.0 m s^{-1} $+ 8.0$ m s^{-1} $= 13$ m s^{-1} at 4.0 s; the acceleration is then zero, so at 6.0 s the velocity is still $\mathbf{13}$ m s^{-1} B1

Practice 2.3 (c) ▪ 2 marks

Sketch the velocity–time graph of the car from $t = 0$ to $t = 8.0$ s. Label both axes and mark the values of velocity.



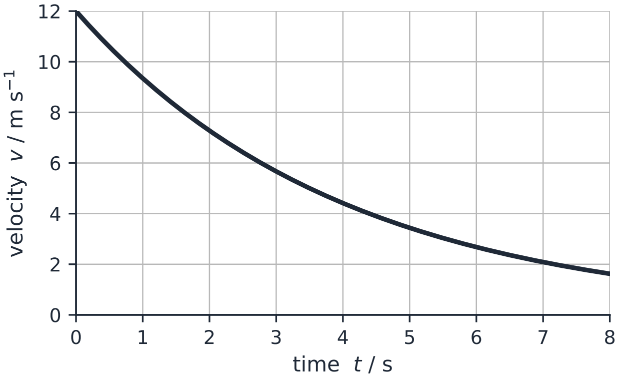
Solution 2.3 (c)

Worked steps

- a straight line from $(0,\ 5.0$ m s$^{-1})$ to $(4.0$ s$,\ 13$ m s$^{-1})$ **B1**
- then a horizontal line at 13 m s^{-1} to 8.0 s, with the axes labelled *v* / m s^{-1} and *t* / s **B1**

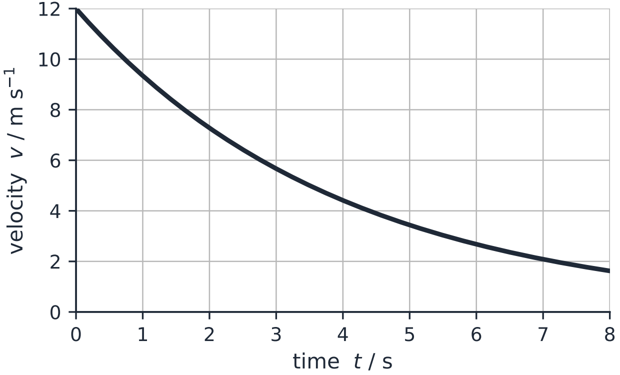
Practice 2.4 ▪ 4 marks

A cyclist stops pedalling and slows down. The graph shows the velocity of the cyclist.



Practice 2.4 (a) ▪ 2 marks

By drawing a tangent, determine the acceleration of the cyclist at $t = 4.0$ s.



Solution 2.4 (a)

Method: A curve: a tangent and counting squares — p.10

Worked steps

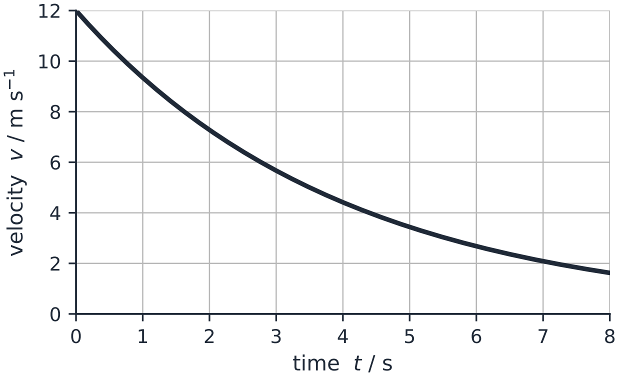
tangent drawn at $t = 4.0$ s, for example through $(0,\ 8.8\text{ m s}^{-1})$ and $(8.0\text{ s},\ 0)$ **M1**

$$a = \frac{0 - 8.8\text{ m s}^{-1}}{8.0\text{ s} - 0} = \mathbf{-1.1\text{ m s}^{-2}}$$

(accept -1.0 to -1.2) **A1**

Practice 2.4 (b) ▪ 2 marks

Estimate the distance travelled by the cyclist from $t = 0$ to $t = 8.0$ s by counting squares.



Solution 2.4 (b)

Worked steps

about 21 squares are at least half under the curve, and each is worth 1 s $\times$ 2 m s^{-1} $= 2$ m **M1**

$$\text{distance} \approx 21 \times 2 \text{ m} = \mathbf{42} \text{ m}$$

(accept 40 m to 44 m) **A1**

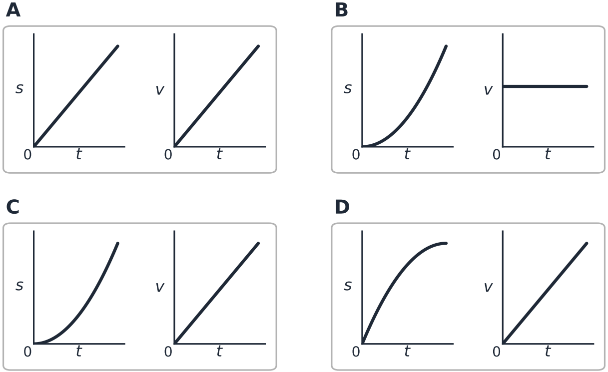
3 Exam-style

Exam-style 3.1 ▪ 1 mark

An object starts from rest and moves in a straight line. Which pair of graphs could show how its displacement s and its velocity v change with time t?

Exam-style 3.1 ▪ 1 mark

Which pair of graphs could show how its displacement … and its velocity … change with time … ?



Solution 3.1

Method: One journey, three graphs — p.7

Worked steps

C

- the curve in **C** gets steeper, so the velocity increases from zero, which matches a velocity line rising from the origin B1
- **A**: a straight displacement line means a constant velocity, not an increasing one
- **B**: a curve that gets steeper cannot have a constant velocity
- **D**: a curve that gets flatter means the velocity is decreasing

Exam-style 3.2 ▪ 1 mark

The velocity–time graph of an object is a straight line from 8.0 m s^{-1} at $t = 0$ to -4.0 m s^{-1} at $t = 6.0$ s. What is the displacement of the object after 6.0 s?

A 12 m

B 20 m

C 24 m

D 36 m

Solution 3.2

Method: Gradient and area of a velocity–time graph — p.4

A 12 m ✓

B 20 m

C 24 m

D 36 m

Worked steps

A

- the gradient is -2.0 m s^{-2}, so $v = 0$ at $t = 4.0$ s

Solution 3.2

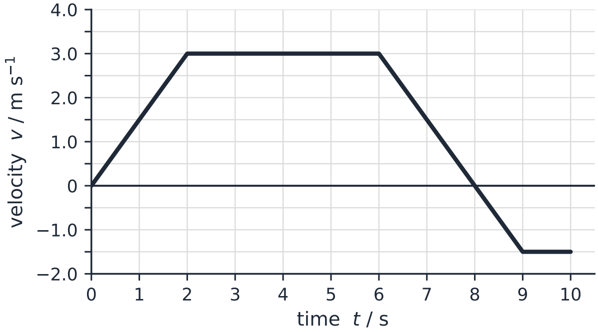
- area above the axis and area below it

$$\tfrac{1}{2}(4.0\ \mathrm{s})(8.0\ \mathrm{m\ s^{-1}}) = 16\ \mathrm{m}, \qquad \tfrac{1}{2}(2.0\ \mathrm{s})(-4.0\ \mathrm{m\ s^{-1}}) = -4.0\ \mathrm{m}$$

- displacement $= 16\ \mathrm{m} - 4.0\ \mathrm{m} = \mathbf{12}\ \mathrm{m}$ B1

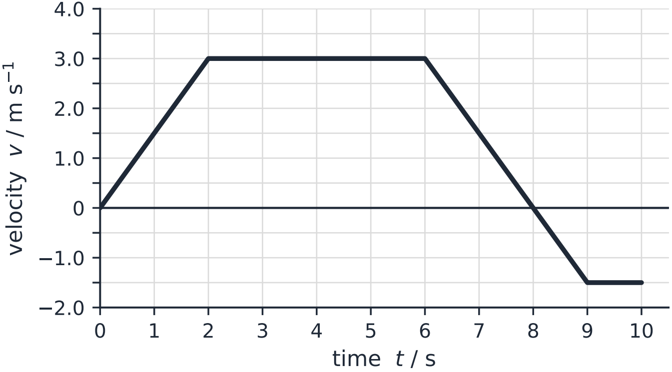
Exam-style 3.3 ▪ 13 marks

A remote-controlled toy car moves along a straight track. A **motion sensor 运动传感器** records its velocity v. The graph shows how v varies with time t.



Exam-style 3.3 (a) ▪ 1 mark

Define acceleration.



Solution 3.3 (a)

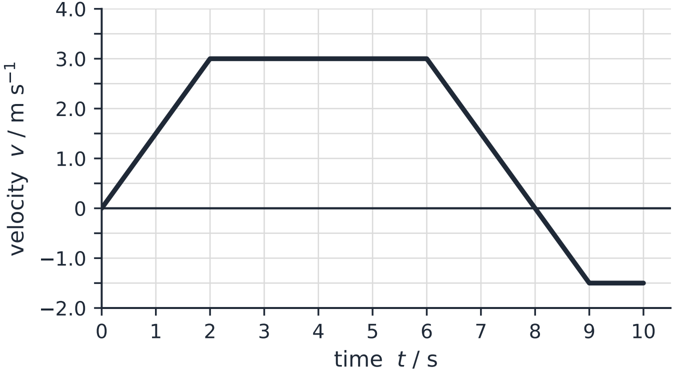
Method: One journey, three graphs — p.7

Worked steps

acceleration is the **rate of change of velocity** B1

Exam-style 3.3 (b) ▪ 2 marks

Compare the acceleration of the car at $t = 1.0$ s with its acceleration at $t = 7.0$ s, in terms of magnitude and direction.



Solution 3.3 (b)

Worked steps

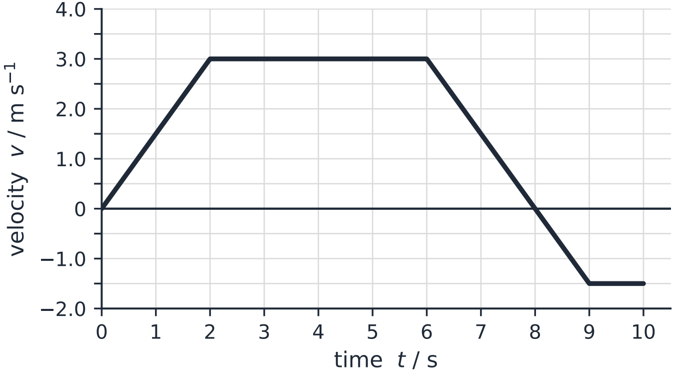
the gradients are

$$\frac{3.0\ \text{m s}^{-1}}{2.0\ \text{s}} = 1.5\ \text{m s}^{-2} \quad \text{and} \quad \frac{-1.5\ \text{m s}^{-1} - 3.0\ \text{m s}^{-1}}{9.0\ \text{s} - 6.0\ \text{s}} = -1.5\ \text{m s}^{-2}$$

- magnitude: **the same** B1
- direction: **opposite** B1

Exam-style 3.3 (c) ▪ 2 marks

The car is at rest for an instant at $t = 8.0$ s. Determine the acceleration of the car at this instant.



Solution 3.3 (c)

Worked steps

the acceleration is the gradient of the straight line from 6.0 s to 9.0 s, which includes 8.0 s **M1**

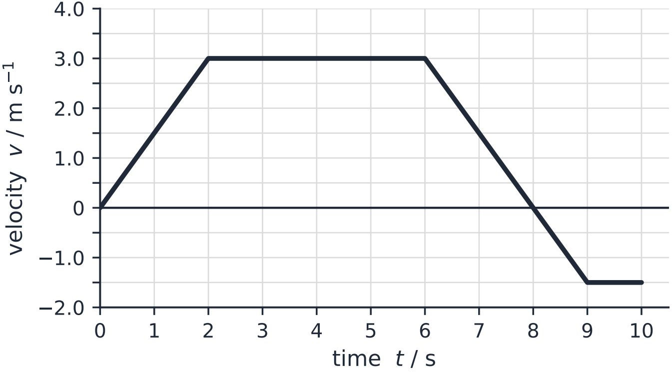
$$a = \frac{-1.5 \text{ m s}^{-1} - 3.0 \text{ m s}^{-1}}{9.0 \text{ s} - 6.0 \text{ s}} = \mathbf{-1.5} \text{ m s}^{\mathbf{-2}}$$

A1

- it is not zero: the velocity is zero for an instant, but it is still changing

Exam-style 3.3 (d) ▪ 3 marks

Determine the greatest displacement of the car from its starting point, and the time at which it happens.



Solution 3.3 (d)

Worked steps

the car moves forwards until $v = 0$, so the greatest displacement is at $t = \mathbf{8.0}$ s **B1**

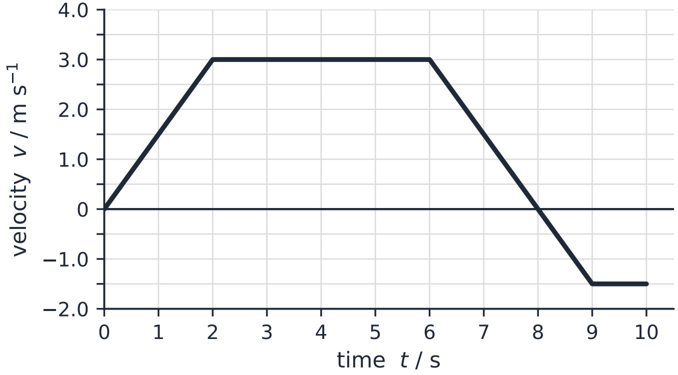
- displacement = area under the line from 0 to 8.0 s **M1**

$$\tfrac{1}{2}(2.0\ \mathrm{s})(3.0\ \mathrm{m\ s^{-1}}) + (4.0\ \mathrm{s})(3.0\ \mathrm{m\ s^{-1}}) + \tfrac{1}{2}(2.0\ \mathrm{s})(3.0\ \mathrm{m\ s^{-1}}) = \mathbf{18}\ \mathrm{m}$$

A1

Exam-style 3.3 (e) ▪ 2 marks

Determine the displacement of the car from its starting point at $t = 10.0$ s.



Solution 3.3 (e)

Worked steps

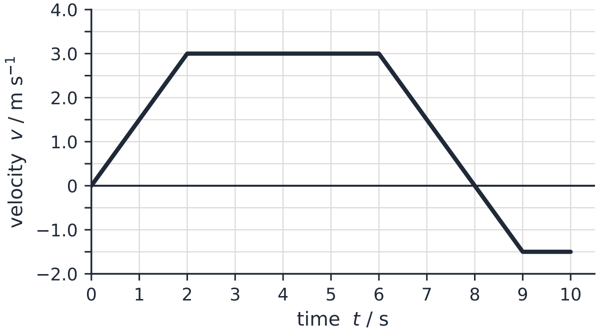
area below the axis from 8.0 s to 10.0 s M1

$$\frac{1}{2}(1.0\ \text{s})(-1.5\ \text{m s}^{-1}) + (1.0\ \text{s})(-1.5\ \text{m s}^{-1}) = -2.25\ \text{m}$$

- displacement $= 18$ m $- 2.25$ m $= \mathbf{16}$ m A1

Exam-style 3.3 (f) ▪ 3 marks

Sketch the variation with time of the displacement of the car from $t = 0$ to $t = 10.0$ s. Label both axes and mark the greatest displacement.



Solution 3.3 (f)

Worked steps

- axes labelled s / m and t / s; a curve getting steeper to 3.0 m at 2.0 s, then a straight line to 15 m at 6.0 s **B1**
- a curve getting flatter to a maximum of 18 m at 8.0 s, flat at the top **B1**
- the displacement then decreasing, to about 16 m at 10.0 s **B1**