Exercise walk-through

2.1.1

Describing motion and displacement–time graphs

A-Level Physics

You must be able to

- **define** distance 距离, displacement 位移, speed 速率, velocity 速度 and acceleration 加速度
- tell average speed from average velocity, and change km h^{-1} into m s^{-1}
- add two displacements that are at right angles
- find a velocity from the gradient of a displacement–time graph, using a tangent for a curve
- **describe** a motion from the shape of its displacement–time graph

1 The method

Method — Definitions the examiner accepts

quantity	**definition**	**scalar 标量 or vector 矢量**
distance	the total length of the path travelled	scalar
displacement	the distance in a straight line from the start, in a stated direction	vector
speed	the rate of change of distance	scalar
velocity	the rate of change of displacement	vector
acceleration	the rate of change of velocity	vector

Method — Distance or displacement?

An athlete runs half a lap of a circular track of radius 40 m, from S to T, in 25 s.

- distance: the path is half the **circumference 周长**

$$\pi r = \pi \times 40 \text{ m} = 126 \text{ m}$$

- displacement: the straight line from S to T is the **diameter 直径**, 80 m towards T
- **average speed 平均速率** $=$ total distance $\div$ total time

$$\frac{126 \text{ m}}{25 \text{ s}} = 5.0 \text{ m s}^{-1}$$

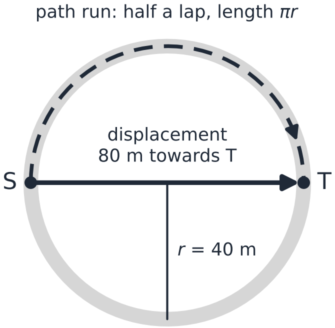
- **average velocity 平均速度** $=$ displacement $\div$ total time

$$\frac{80 \text{ m}}{25 \text{ s}} = 3.2 \text{ m s}^{-1} \text{ towards T}$$

After a **whole** lap the displacement is zero, so the average velocity is zero. The average speed is not.

Method — Distance or displacement?

An athlete runs half a lap of a circular track of radius … , from S to T, in … .



Method — Adding two displacements

A walker goes 600 m due west and then 800 m due south. The two displacements are at right angles, so they are the two short sides of a right-angled triangle (Topic 1).

- size of the displacement, by Pythagoras

$$\sqrt{(600\ \mathrm{m})^2 + (800\ \mathrm{m})^2} = 1000\ \mathrm{m}$$

- direction, as an angle measured from west towards south

$$\tan\theta = \frac{800\ \mathrm{m}}{600\ \mathrm{m}} \Rightarrow \theta = 53^\circ$$

- the displacement is 1000 m at 53° south of west; the distance walked is 1400 m

Units. To change km h^{-1} into m s^{-1}, multiply by 1000 (km to m) and divide by 3600 (h to s):

$$54\ \mathrm{km\ h^{-1}} = \frac{54 \times 1000\ \mathrm{m}}{3600\ \mathrm{s}} = 15\ \mathrm{m\ s^{-1}}$$

Method — Velocity from a displacement–time graph

The **gradient 斜率** of a displacement–time graph is the velocity. Use the shape to **describe 描述** the motion:

- flat line: the object is at rest
- straight sloping line: constant velocity; a line sloping down means moving back towards the start (negative velocity)
- curve getting steeper: speeding up; curve getting flatter: slowing down

For a curve, draw a **tangent 切线** at the time you need, make a **large** triangle on it, and find its gradient.

Example: the velocity of the car at P, where $t = 4.0$ s.

- the tangent passes through $(2.0\ \text{s},\ 0)$ and $(6.0\ \text{s},\ 24\ \text{m})$

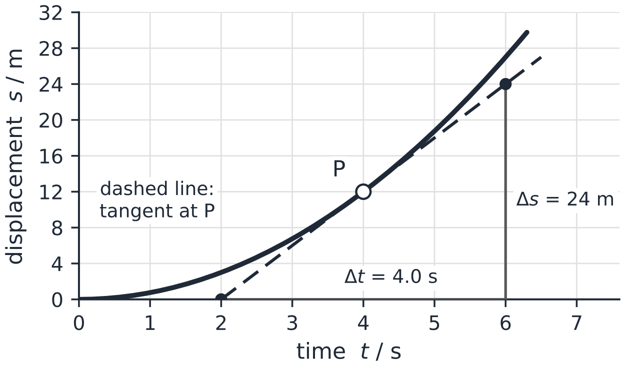
$$v = \frac{\Delta s}{\Delta t} = \frac{24\ \text{m} - 0}{6.0\ \text{s} - 2.0\ \text{s}} = 6.0\ \text{m s}^{-1}$$

Method — Velocity from a displacement–time graph

- a small triangle would make your reading errors much larger
- this is the **instantaneous 瞬时** velocity at P; the average velocity over a time is the total displacement divided by that time

Method — Velocity from a displacement–time graph

Use the shape to describe **描述** the motion: For a curve, draw a tangent **切线** at the time you need, make a large...



2 Practice

Practice 2.1 ▪ 2 marks

The examiner often starts a question with a definition.

Practice 2.1 (a) ▪ 1 mark

Define velocity.

Solution 2.1 (a)

Method: Definitions the examiner accepts — p.4

Worked steps

velocity is the **rate of change of displacement** B1

Practice 2.1 (b) ▪ 1 mark

State one way in which velocity is different from speed.

Solution 2.1 (b)

Worked steps

velocity is a vector, so it has a direction; speed is a scalar, so it has none **B1**

Practice 2.2 ▪ 6 marks

A cyclist rides 3.0 km due east and then 5.0 km due north. The whole ride takes 25 minutes.

Practice 2.2 (a) ▪ 1 mark

State the total distance travelled.

Solution 2.2 (a)

Method: Adding two displacements — p.7

Worked steps

$3.0\ \text{km} + 5.0\ \text{km} = \mathbf{8.0}\ \text{km}$ **B1**

Practice 2.2 (b) ▪ 1 mark

Determine the size of the cyclist's displacement from the start.
(Hint: the two displacements are at right angles.)

Solution 2.2 (b)

Worked steps

the displacements are at right angles, so use Pythagoras

$$\sqrt{(3.0\ \text{km})^2 + (5.0\ \text{km})^2} = \mathbf{5.8}\ \text{km}$$

B1

Practice 2.2 (c) ▪ 1 mark

Determine the direction of the displacement, as an angle measured from east towards north.

Solution 2.2 (c)

Worked steps

opposite side north, adjacent side east

$$\tan\theta = \frac{5.0\ \text{km}}{3.0\ \text{km}} \Rightarrow \theta = \mathbf{59}^\circ$$

north of east **B1**

Practice 2.2 (d) ▪ 2 marks

Calculate the average speed of the cyclist, in m s^{-1}.

Solution 2.2 (d)

Worked steps

time $= 25 \times 60$ s $= 1500$ s and distance $= 8000$ m **M1**

$$\text{average speed} = \frac{8000 \text{ m}}{1500 \text{ s}} = \mathbf{5.3} \text{ m s}^{-1}$$

A1

Practice 2.2 (e) ▪ 1 mark

Calculate the size of the average velocity of the cyclist, in m s^{-1}.

Solution 2.2 (e)

Worked steps

use the displacement, not the distance

$$\text{average velocity} = \frac{5830\ \text{m}}{1500\ \text{s}} = \mathbf{3.9}\ \text{m s}^{-1}$$

B1

Practice 2.3 ▪ 2 marks

Change the units. *(Hint:* 1 *km* $= 1000$ *m and* 1 *h* $= 3600$ *s.)* (a) 72 km h^{-1} into m s^{-1}; (b) 25 m s^{-1} into km h^{-1}.

Solution 2.3

Method: Adding two displacements — p.7

Worked steps

(a) multiply by 1000 and divide by 3600

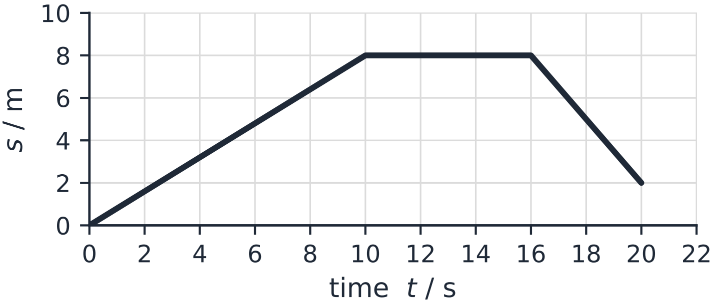
$$72 \text{ km h}^{-1} = \frac{72 \times 1000 \text{ m}}{3600 \text{ s}} = \mathbf{20} \text{ m s}^{-1}$$

B1

(b) in one hour the car travels 25 m $\times\, 3600 = 90\,000$ m $= 90$ km, so the speed is $\mathbf{90}$ km h^{-1} **B1**

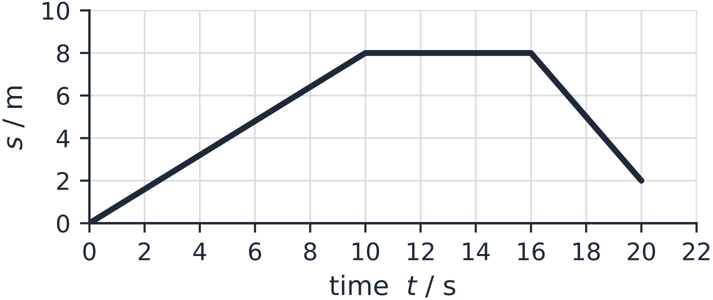
Practice 2.4 ▪ 3 marks

The displacement–time graph shows a model train on a straight track.



Practice 2.4 (a) ▪ 1 mark

Determine the velocity of the train between $t = 0$ and $t = 10$ s.



Solution 2.4 (a)

Method: Velocity from a displacement–time graph — p.8

Worked steps

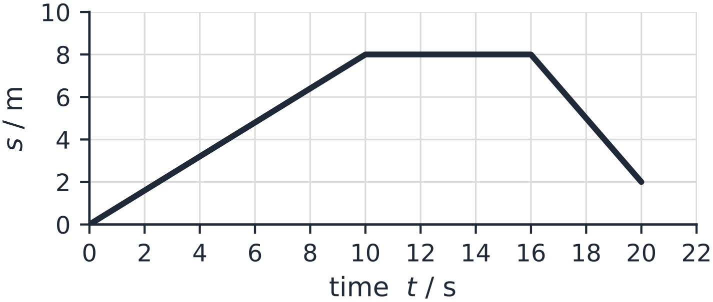
velocity = gradient

$$v = \frac{8.0\ \text{m} - 0}{10\ \text{s} - 0} = \mathbf{0.80}\ \text{m s}^{\mathbf{-1}}$$

B1

Practice 2.4 (b) ▪ 1 mark

Describe the motion of the train between $t = 10$ s and $t = 16$ s.



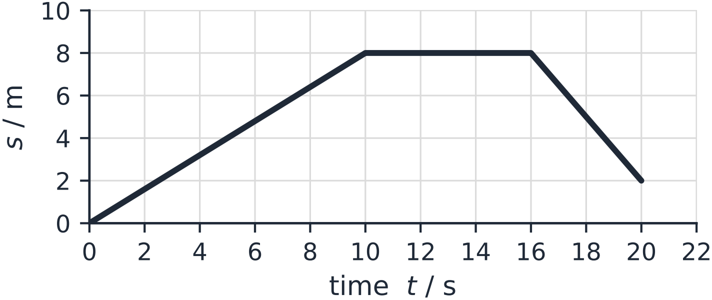
Solution 2.4 (b)

Worked steps

the line is flat, so the displacement does not change: the train is **at rest** B1

Practice 2.4 (c) ▪ 1 mark

Determine the velocity of the train between $t = 16$ s and $t = 20$ s.



Solution 2.4 (c)

Worked steps

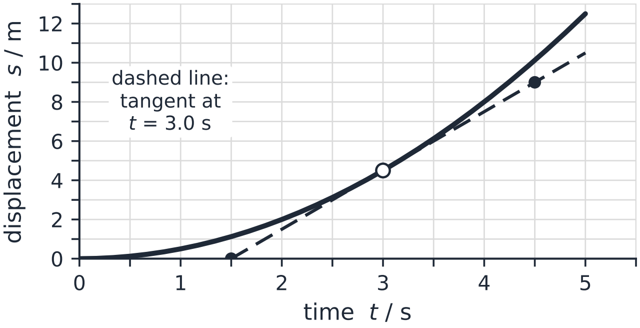
gradient of the line from 16 s to 20 s

$$v = \frac{2.0 \text{ m} - 8.0 \text{ m}}{20 \text{ s} - 16 \text{ s}} = -\mathbf{1.5} \text{ m s}^{-1}$$

- the minus sign means the train moves back towards its start **B1**

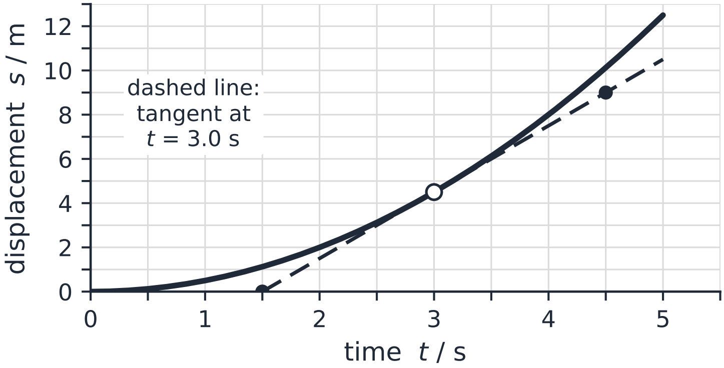
Practice 2.5 ▪ 3 marks

A **trolley 小车** is released from rest at the top of a **ramp 斜坡**. The graph shows its displacement down the ramp. A tangent has been drawn at $t = 3.0$ s.



Practice 2.5 (a) ▪ 2 marks

Use the tangent to determine the velocity of the trolley at $t = 3.0$ s.



Solution 2.5 (a)

Method: Velocity from a displacement–time graph — p.8

Worked steps

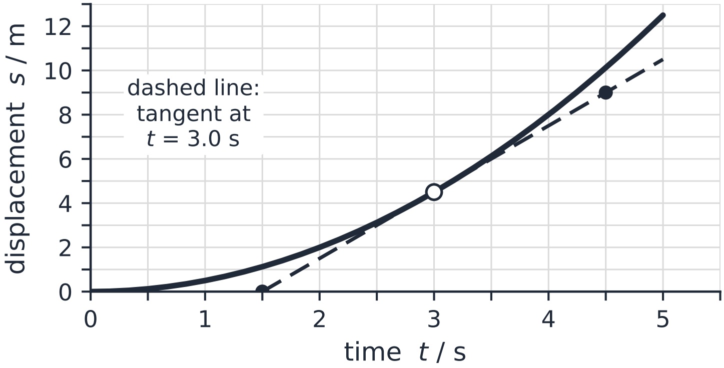
the tangent passes through $(1.5\text{ s},\ 0)$ and $(4.5\text{ s},\ 9.0\text{ m})$ **M1**

$$v = \frac{9.0\text{ m} - 0}{4.5\text{ s} - 1.5\text{ s}} = \mathbf{3.0}\text{ m s}^{-1}$$

A1

Practice 2.5 (b) ▪ 1 mark

State how the graph shows that the trolley is speeding up.



Solution 2.5 (b)

Worked steps

the curve gets **steeper**: its gradient, the velocity, increases with time **B1**

3 Exam-style

Exam-style 3.1 ▪ 1 mark

A runner completes 2.5 laps of a circular track of circumference 400 m in 300 s. What are her average speed and the size of her average velocity?

A speed 3.3 m s^{-1}, velocity 0

B speed 3.3 m s^{-1}, velocity 0.42 m s^{-1}

C speed 0.42 m s^{-1}, velocity 3.3 m s^{-1}

D speed 3.3 m s^{-1}, velocity 3.3 m s^{-1}

Solution 3.1

Method: Distance or displacement? — p.5

A speed 3.3 m s^{-1}, velocity 0

B speed 3.3 m s^{-1}, velocity 0.42 m s^{-1} ✓

C speed 0.42 m s^{-1}, velocity 3.3 m s^{-1}

D speed 3.3 m s^{-1}, velocity 3.3 m s^{-1}

Worked steps

B

Solution 3.1

- distance $= 2.5 \times 400$ m $= 1000$ m

$$\text{average speed} = \frac{1000 \text{ m}}{300 \text{ s}} = 3.3 \text{ m s}^{-1}$$

- after 2.5 laps the runner is half a lap from the start, so the displacement is one diameter

$$d = \frac{400 \text{ m}}{\pi} = 127 \text{ m}, \qquad \text{average velocity} = \frac{127 \text{ m}}{300 \text{ s}} = \mathbf{0.42} \text{ m s}^{-1}$$

B1

Exam-style 3.2 ▪ 1 mark

A bus travels 12 km at an average speed of 40 km h^{-1}, then another 12 km at 60 km h^{-1}. What is its average speed for the whole journey?

A 13 m s^{-1}

B 14 m s^{-1}

C 48 m s^{-1}

D 50 m s^{-1}

Solution 3.2

Method: Distance or displacement? — p.5

A $13\ \mathrm{m\ s^{-1}}$ ✓

B $14\ \mathrm{m\ s^{-1}}$

C $48\ \mathrm{m\ s^{-1}}$

D $50\ \mathrm{m\ s^{-1}}$

Worked steps

A

Solution 3.2

- time for each part

$$t_1 = \frac{12\ \text{km}}{40\ \text{km h}^{-1}} = 0.30\ \text{h}, \qquad t_2 = \frac{12\ \text{km}}{60\ \text{km h}^{-1}} = 0.20\ \text{h}$$

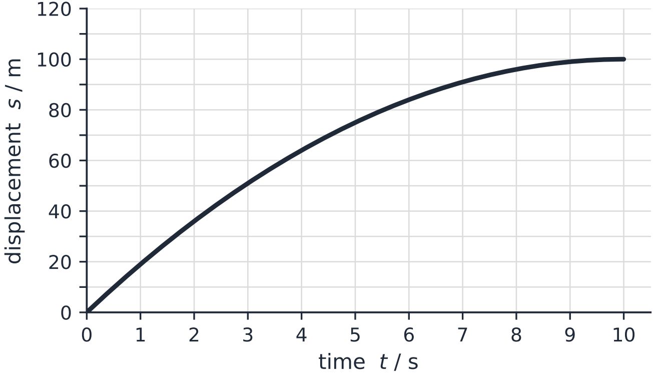
- average speed = total distance ÷ total time

$$\frac{24\ \text{km}}{0.50\ \text{h}} = 48\ \text{km h}^{-1} = \frac{48 \times 1000\ \text{m}}{3600\ \text{s}} = \mathbf{13}\ \text{m s}^{-\mathbf{1}}$$

B1

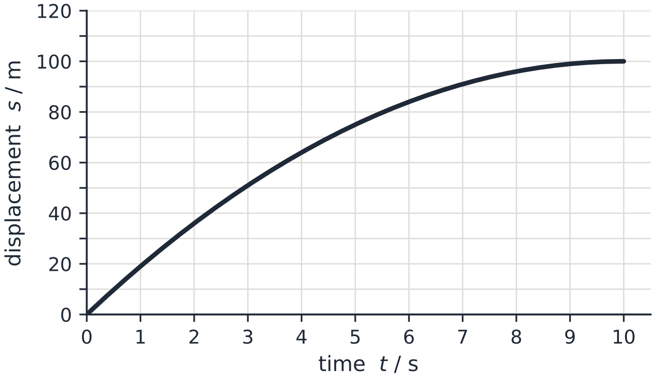
Exam-style 3.3 ▪ 8 marks

The graph shows the displacement s of a car along a straight road as the driver brakes to a stop.



Exam-style 3.3 (a) ▪ 2 marks

Describe the motion of the car from $t = 0$ to $t = 10$ s.



Solution 3.3 (a)

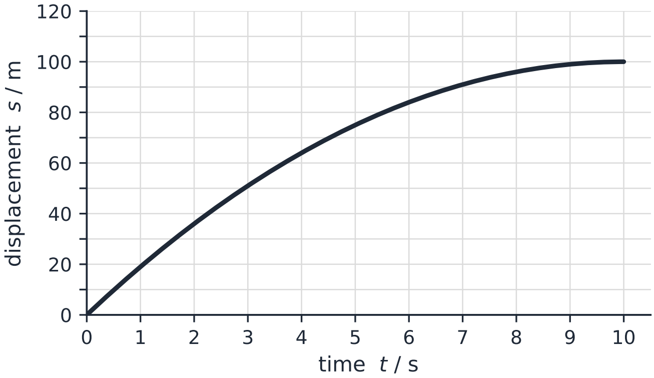
Method: Velocity from a displacement–time graph — p.8

Worked steps

- the displacement increases, so the car moves forwards **B1**
- the gradient decreases to zero, so the car slows down and stops at $t = 10$ s **B1**

Exam-style 3.3 (b) ▪ 3 marks

By drawing a tangent, determine the velocity of the car at $t = 4.0$ s.



Solution 3.3 (b)

Worked steps

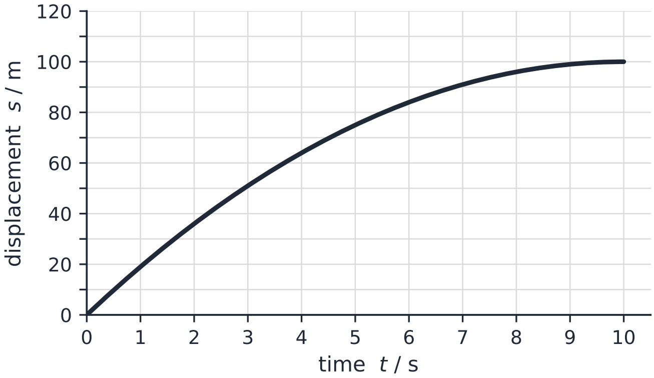
- tangent drawn at $t = 4.0$ s **M1**
- gradient from a large triangle, for example through $(0,\ 16\text{ m})$ and $(8.0\text{ s},\ 112\text{ m})$ **M1**

$$v = \frac{112\text{ m} - 16\text{ m}}{8.0\text{ s} - 0} = \mathbf{12}\text{ m s}^{-1}$$

(accept 11 to 13) **A1**

Exam-style 3.3 (c) ▪ 2 marks

Determine the average velocity of the car from $t = 0$ to $t = 10$ s.



Solution 3.3 (c)

Worked steps

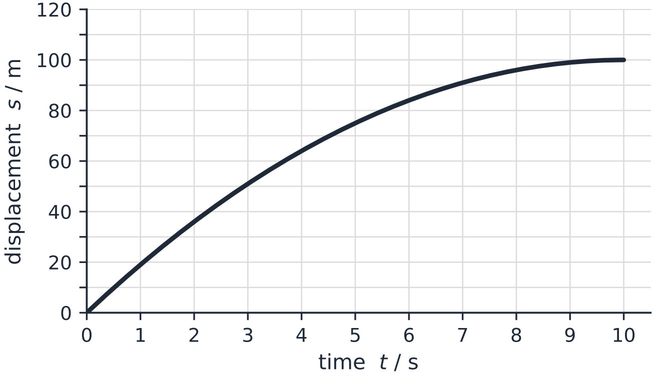
average velocity = displacement ÷ time M1

$$\frac{100\ \text{m}}{10\ \text{s}} = \mathbf{10}\ \text{m s}^{-1}$$

A1

Exam-style 3.3 (d) ▪ 1 mark

Explain why your answers to (b) and (c) are different.



Solution 3.3 (d)

Worked steps

(b) is the velocity at one instant; (c) is the total displacement divided by the total time **B1**