Past-paper walk-through

Discharging a capacitor ▪ 19.3

eXercise

Questions in this set

- 9702/42 J25 Q7 ▪ 10 marks
- 9702/41 N24 Q6 ▪ 11 marks
- 9702/43 J24 Q6 ▪ 10 marks
- 9702/43 J23 Q5 ▪ 9 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 7

9702/42 J25 ▪ Q7 ▪ 10 marks

Fig. 7.1 shows a circuit containing a capacitor of capacitance C and a resistor of resistance R.

Initially, the switch is open and the potential difference (p.d.) across the capacitor is 12 V.

The switch is closed at time $t = 0$ and the capacitor discharges through the resistor.

Fig. 7.2 shows the variation of the charge Q on the capacitor with the p.d. V_C across the capacitor as the capacitor discharges. Fig. 7.3 shows the variation of the current I in the resistor with the p.d. V_R across the resistor as the capacitor discharges.

(a) State the relationship between V_C and V_R. **[1]**

(b)(i) the capacitance C, in μF **[2]**

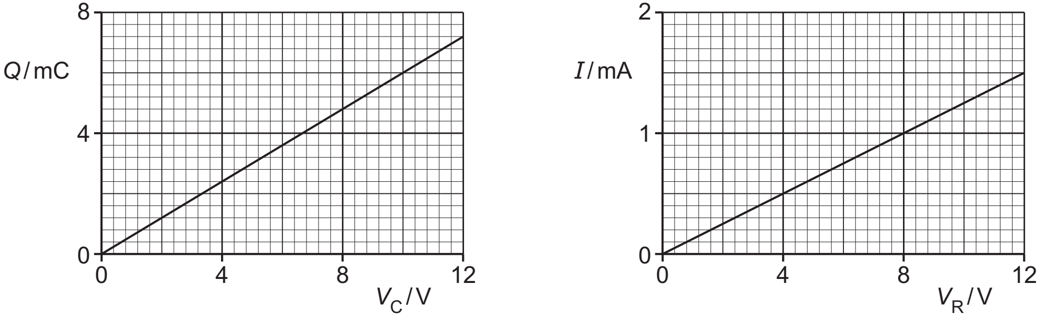
(b)(ii) the resistance R, in kΩ **[2]**

(b)(iii) the time constant τ of the circuit. **[2]**

Question 7

(c) Use Fig. 7.2, Fig. 7.3 and your answer in (a) to explain why the variation of Q with t is exponential in nature. **[3]**

Question 7



Solution 7

(a)

Mark scheme

- VC = VR **B1**

Solution 7

(b)(i) Worked steps

Capacitance is charge per unit p.d., and both are given for the moment before discharge begins.

- $C = \dfrac{Q}{V} = \dfrac{7.2 \times 10^{-3}}{12} = 6.0 \times 10^{-4}$ F
- The question asks for **microfarads**, so multiply by 10^6: $C = 600\ \mu$F.

Solution 7

Both marks depend on finishing the conversion. 6.0×10^{-4} F is right but is not what was asked for, and the answer line carries the unit μF to say so.

Mark scheme

- C = Q / V **C1**
- = (7.2 × 10–3) / 12 (= 6.0 × 10–4 F)
- = 600 μF **A1**

Solution 7

(b)(ii) Worked steps

The resistor carries the whole p.d. at the instant the discharge starts, and the initial current is given.

- $R = \dfrac{V}{I} = \dfrac{12}{1.5 \times 10^{-3}} = 8000\ \Omega$
- In kilohms: $R = 8.0\ \text{k}\Omega$

Solution 7

Use the **initial** current with the **initial** p.d. — both fall together as the capacitor discharges, so pairing a starting p.d. with a later current gives the wrong resistance.

Mark scheme

- R = V / I C1
- $= 12 / (1.5 \times 10–3)$ $(= 8000\ \Omega)$
- $= 8.0\ \mathrm{k}\Omega$ A1

Solution 7

(b)(iii) Worked steps

The time constant is the product of the two quantities just found.

- $\tau = RC = 8000 \times 6.0 \times 10^{-4}$
- $\tau = 4.8$ s

Substitute in **SI units** — ohms and farads, not kilohms and microfarads — or the answer is out by a factor of a thousand. The units are their own check: $\Omega \times$ F gives seconds.

Solution 7

What 4.8 s means physically: after that long the p.d. has fallen to $1/e$, about 37%, of 12 V, and after about five time constants the capacitor is effectively discharged.

Mark scheme

- $\tau =$ RC **C1**
- $=$ 8000 $\times$ 6.0 $\times$ 10–4
- $=$ 4.8 s **A1**

Solution 7

(c)

Mark scheme

- Any two points from:
- ▪ charge and current are both (directly) proportional to voltage
- ▪ charge is (directly) proportional to current
- ▪ current is the rate of change of charge **B2**
- Q is proportional to the rate of change of Q (so exponential variation) **B1**

Question 6

9702/41 N24 ▪ Q6 ▪ 11 marks

(a)(i) State what is meant by rectification of an alternating voltage. **[1]**

(a)(ii) State the difference between half-wave rectification and full-wave rectification. **[2]**

(b)(i) Complete Fig. 6.1 to show a circuit that produces half-wave rectification of an alternating input voltage V_{IN} to produce output voltage V_{OUT} across the resistor R. **[2]**

(b)(ii) State the purpose of the capacitor C in the circuit of Fig. 6.1. **[1]**

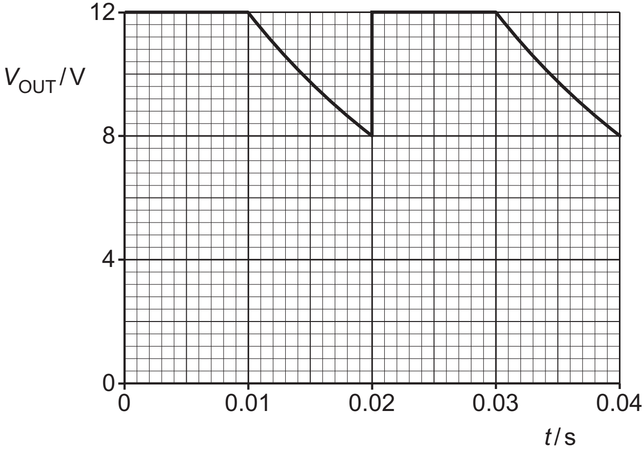
(c)(i) Fig. 6.3 shows the variation of V_{OUT} with t.

The maximum energy stored in the capacitor is 0.041 J.

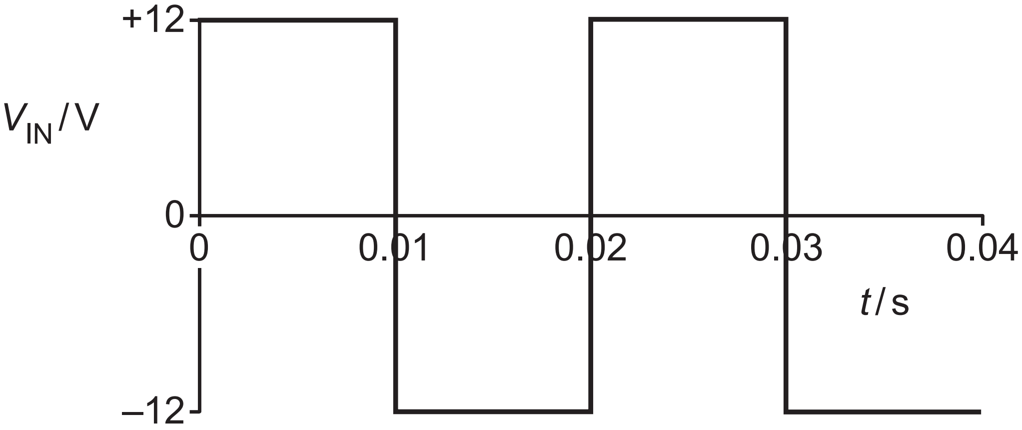
Show that the capacitance of C is $570\ \mu$F. **[2]**

(c)(ii) Determine the resistance of R. **[3]**

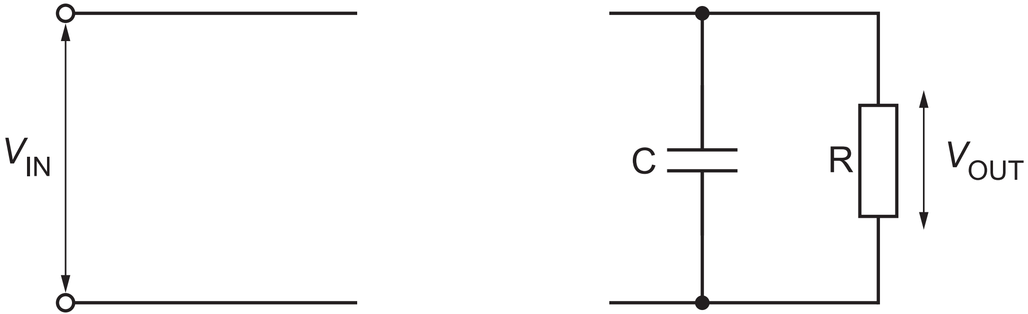
Question 6



Question 6



Question 6



Solution 6

(a)(i)

Mark scheme

- conversion (from a.c.) to d.c.

(a)(ii)

Mark scheme

- half-wave: voltage in one direction is removed
- full-wave: voltage in one direction is reversed

Solution 6

(b)(i)

Mark scheme

- one gap connected by a single diode and other gap connected directly
- diode drawn (in a circuit) with correct circuit symbol

(b)(ii)

Mark scheme

- smoothing

Solution 6

(c)(i) Worked steps

Know. The capacitor holds most energy when the p.d. across it is greatest. Read that peak off the output graph: $V = 12$ V.

Equation. $E = \frac{1}{2}CV^2$, so $C = \dfrac{2E}{V^2}$.

Substitute. $C = \dfrac{2 \times 0.041}{12^2}$.

Answer. $C = 5.7 \times 10^{-4}$ F $= 570\ \mu$F, as required.

Solution 6

Check. Take V as the **peak** of the smoothed output, not its mean and not the r.m.s. value of the supply. The maximum energy goes with the maximum p.d.

Mark scheme

- $E = \frac{1}{2}CV^2$
- $C = 2 \times 0.041/12^2 = 5.7 \times 10^{-4}\ \mathrm{F} = 570\ \mu\mathrm{F}$

Solution 6

(c)(ii) Worked steps

Know. Between one peak and the next, the capacitor discharges through R, so the ripple is an exponential decay with time constant $\tau = RC$.

Equation. $V = V_0 \exp\left(-\frac{t}{RC}\right)$.

Read the graph. The p.d. falls from 12.0 V to 8.0 V in 0.010 s.

Solution 6

Substitute. $8.0 = 12.0\,\exp\left(-\dfrac{0.010}{R \times 5.7 \times 10^{-4}}\right)$, so $\ln\left(\dfrac{8.0}{12.0}\right) = -\dfrac{0.010}{R \times 5.7 \times 10^{-4}}$.

Answer. $R = 43\ \Omega$.

Solution 6

Check. $\ln\left(\frac{8}{12}\right)$ is **negative**, and so is the right-hand side; the two minus signs cancel to give a positive resistance. An answer that comes out negative means one of them was dropped.

Mark scheme

- $8.0 = 12.0\exp(-0.010/RC)$
- $\ln(8.0/12.0) = -0.010/(R \times 5.7 \times 10^{-4})$
- $R = 43\ \Omega$

Question 6

9702/43 J24 ▪ Q6 ▪ 10 marks

Fig. 6.1 shows a capacitor of capacitance C connected in series with a resistor of resistance R.

Initially the switch is open and there is a p.d. of 12 V across the capacitor.

At time $t = 0$, the switch is closed so that there is a current I in the resistor.

Fig. 6.2 shows the variation of I with t.

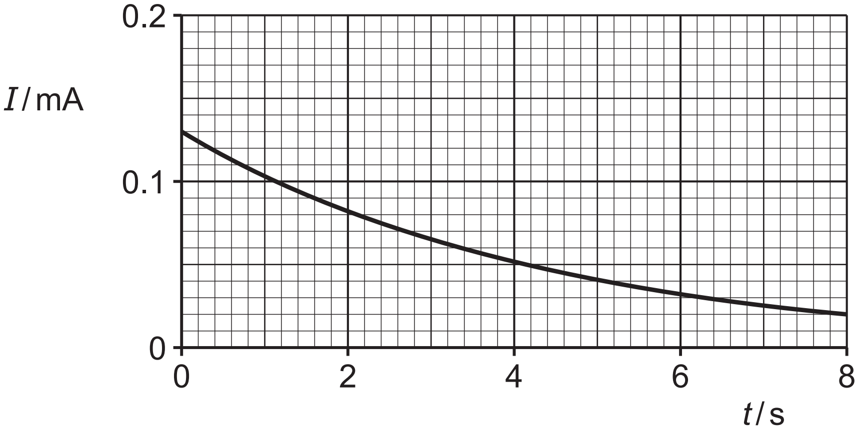
(a) Explain the shape of the line in Fig. 6.2. **[3]**

(b)(i) resistance R **[2]**

(b)(ii) the time constant τ of the circuit in Fig. 6.1. **[3]**

(c) Use your answers in (b) to determine capacitance C. **[2]**

Question 6



Solution 6

(a)

Mark scheme

- p.d. across capacitor proportional to charge on capacitor
- p.d. across capacitor = p.d. across resistor
- current in resistor proportional to p.d. across resistor
- current in resistor = rate of decrease of charge on capacitor

Any two points, 1 mark each

charge proportional to current so rate of decrease of current decreases as current decreases (therefore exponential shape)

Solution 6

(b)(i) Worked steps

Know. At the instant the switch closes, the capacitor still has its full 12 V across it, and that whole p.d. now appears across the resistor. So the first current on the graph gives the resistance directly.

Equation. $R = \dfrac{V}{I}$.

Substitute. $R = \dfrac{12}{0.13 \times 10^{-3}}$, reading $I = 0.13$ mA at $t = 0$.

Answer. $R = 9.2 \times 10^{4}\ \Omega$.

Solution 6

Check. The graph is in **milliamps**. Using 0.13 A gives $92\ \Omega$, a thousand times too small, and the later capacitance would be a thousand times too large.

Mark scheme

- $R = V/I$
- $= 12/(0.13 \times 10^{-3})$
- $= 9.2 \times 10^{4}\ \Omega$

Solution 6

(b)(ii)

Mark scheme

- correct read-off of at least one pair of values for I and t
- attempted read-off of t when $I = 0.048$ mA
- **or**
- substitution of a correct pair of values of I and t into $I = 0.13\exp(-t/\tau)$
- $\tau = 4.3$ s

Solution 6

(c) Worked steps

Know. For a capacitor discharging through a resistor, the time constant is the product of the two: $\tau = RC$. You now have τ from (b)(ii) and R from (b)(i).

Equation. $C = \dfrac{\tau}{R}$.

Substitute. $C = \dfrac{4.3}{9.2 \times 10^4}$.

Answer. $C = 4.7 \times 10^{-5}$ F.

Solution 6

Check. The units work out: an ohm times a farad is a second, so seconds divided by ohms gives farads. If your two earlier answers differ slightly from these, use **your own** values; the mark is for the method and for carrying your figures forward.

Mark scheme

- $\tau = RC$
- $C = \tau/R = 4.3/(9.2 \times 10^4)$
- $= 4.7 \times 10^{-5}$ F

Question 5

9702/43 J23 ▪ Q5 ▪ 9 marks

Part of an electric circuit is shown in Fig. 5.1.
The circuit is used to produce half-wave rectification of an alternating voltage of potential difference (p.d.) V_{IN}.
The output p.d. across the 14 kΩ resistor is V_{OUT}.

(a)(i) A component is missing from the circuit of Fig. 5.1.
Complete the circuit diagram in Fig. 5.1 by adding the circuit symbol for the missing component, correctly connected. **[1]**

(a)(ii) A capacitor C is shown in the circuit of Fig. 5.1.
State the effect on V_{OUT} of including the capacitor in the circuit. **[1]**

(b)(i) Determine the frequency of V_{IN}. **[1]**

(b)(ii) Show that the time constant τ for the discharge of the capacitor through the resistor is 0.038 s. **[2]**

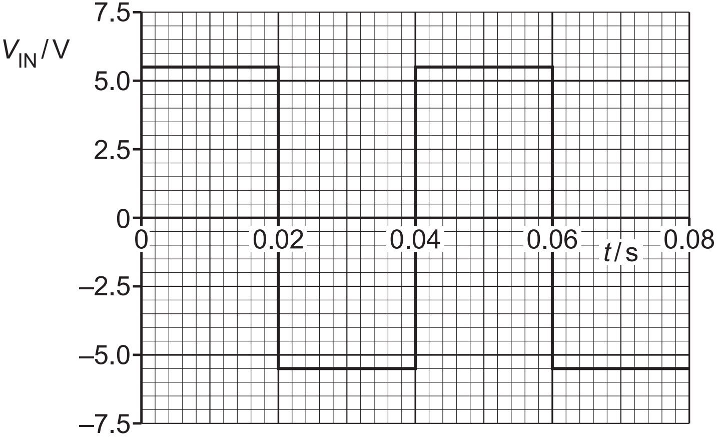
Question 5

(b)(iii) Calculate the capacitance of C. Give a unit with your answer. **[2]**

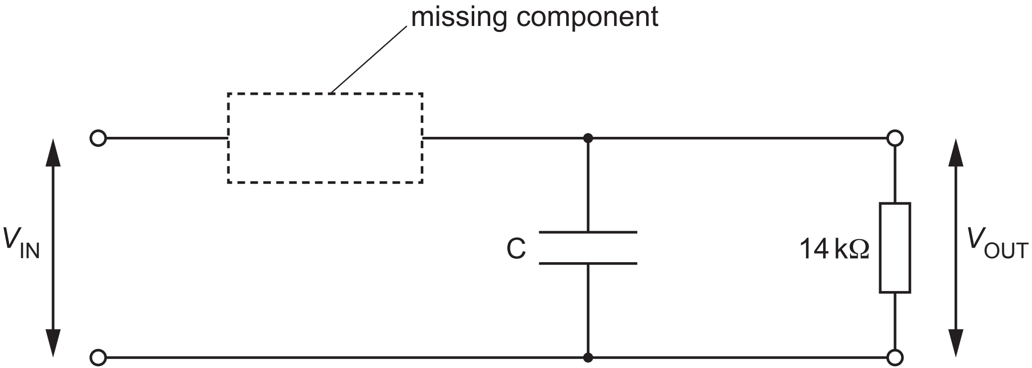
(c) The circuit of Fig. 5.1 is modified so that it produces full-wave rectification of an input voltage.

Suggest, with a reason, how V_{OUT} now varies with time when V_{IN} is as shown in Fig. 5.2. **[2]**

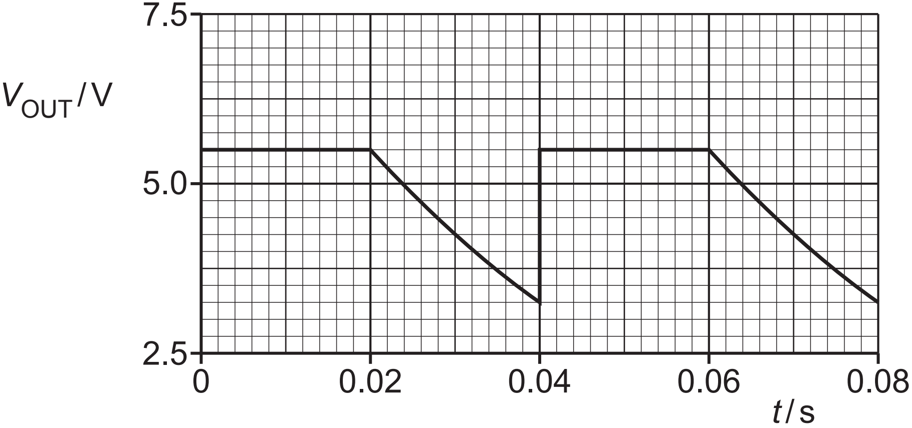
Question 5



Question 5



Question 5



Solution 5

(a)(i)

Mark scheme

- correct circuit symbol for a diode shown correctly connected in series with the wires leading into and out of the dotted box

(a)(ii)

Mark scheme

- smoothing / V_{OUT} is smoothed

Solution 5

(b)(i) Worked steps

Know. Frequency is one over the period, and the period is the time for one **complete** cycle of the input voltage.

Read the graph. One full cycle takes 0.04 s.

Equation. $f = \dfrac{1}{T}$.

Answer. $f = \dfrac{1}{0.04} = 25$ Hz.

Solution 5

Check. Measure a whole cycle, not a single hump. Reading the period as the width of one positive half gives 50 Hz, exactly double.

Mark scheme

- frequency $= 1/0.04$
- $= 25$ Hz

Solution 5

(b)(ii) Worked steps

Know. Between the peaks of the rectified voltage the capacitor discharges through the 14 kΩ resistor, so the sagging part of the output is an exponential decay.

Equation. $V = V_0 \exp\left(-\frac{t}{\tau}\right)$, with $\tau = RC$.

Read the graph. The output falls from 5.50 V to 3.25 V in 0.020 s.

Solution 5

Substitute. $3.25 = 5.50\exp\left(-\dfrac{0.020}{\tau}\right)$, so $\ln\left(\dfrac{3.25}{5.50}\right) = -\dfrac{0.020}{\tau}$.

Answer. $\tau = 0.038$ s, as required.

Solution 5

Check. For half-wave rectification the gap between peaks is a **whole** period, 0.040 s, but here the read-off is over 0.020 s of it. Always take Δt between the two points you actually read, not from the shape of the wave.

Mark scheme

- $V = V_0 \exp(-t/RC)$ and $\tau = RC$
- **or**
- $V = V_0 \exp(-t/\tau)$
- $3.25 = 5.50 \exp(-0.020/\tau)$ leading to $\tau = 0.038$ s

Solution 5

(b)(iii) Worked steps

Know. The time constant of an RC circuit is the product of the resistance and the capacitance, so dividing gives the capacitance.

Equation. $C = \dfrac{\tau}{R}$.

Substitute. $C = \dfrac{0.038}{14 \times 10^3}$.

Answer. $C = 2.7 \times 10^{-6}$ F, and the unit asked for is the **farad**.

Solution 5

Check. The resistance is given in kilohms, so multiply by 10^3 first. And answer the "give a unit" instruction: a bare number scores one mark out of two here, however correct it is.

Mark scheme

- $\tau = RC$
- capacitance $= 0.038/14000$
- $= 2.7 \times 10^{-6}$ F

Solution 5

(c)

Mark scheme

- V_{IN} has constant magnitude in both positive and negative directions
- (so) V_{OUT} is (now) constant / V_{OUT} does not vary with time