Exercise walk-through

Discharging a capacitor ▪ 19.3

eXercise

You must be able to

- use the **time constant** $\tau = RC$
- use $x = x_0 e^{-t/RC}$ for charge, p.d. or current
- interpret the discharge graphs

Method — Exponential decay

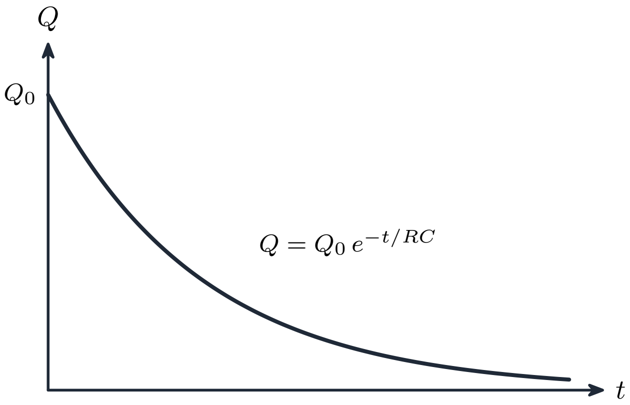
When a charged capacitor discharges through a resistor, the charge (and V, I) decays **exponentially**:

$$Q = Q_0 e^{-t/RC}, \quad V = V_0 e^{-t/RC}, \quad I = I_0 e^{-t/RC}.$$

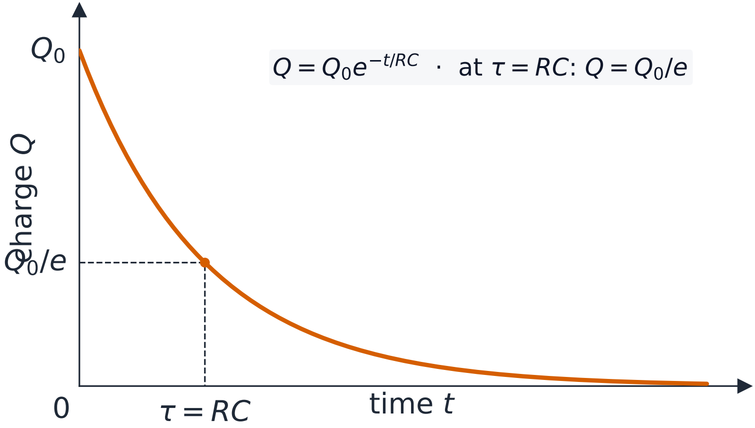
The **time constant** $\tau = RC$ is the time for the charge to fall to $\frac{1}{e} \approx 37\%$ of its initial value.

Worked example. 470 μF through 2.0 kΩ: $\tau = RC = (2000)(470 \times 10^{-6}) = 0.94$ s.

Method — Exponential decay



Method — Exponential decay



Exponential discharge of a capacitor

Method — Finding the time — take logs

To find the **time** for the charge (or V, or I) to fall to a given value, take the **natural log** of both sides of $V = V_0 e^{-t/RC}$:

$$\ln\frac{V}{V_0} = -\frac{t}{RC} \quad \Longrightarrow \quad t = -RC\ln\frac{V}{V_0}.$$

Worked example. For the capacitor above (charged to 6.0 V, $\tau = 0.94$ s), the time for the p.d. to fall to 2.0 V is $t = -0.94\ln\frac{2.0}{6.0} = -0.94 \times (-1.10) = 1.0$ s.

Practice 2.1 ▪ 1 mark

Write the formula for the **time constant**.

Solution 2.1

Method: Exponential decay

Worked steps

$\tau = RC$ **B1**.

Practice 2.2 ▪ 2 marks

A $100\ \mu$F capacitor discharges through a 10 kΩ resistor. Find the **time constant**.

Solution 2.2

Method: Exponential decay

Worked steps

$\tau = RC = (10 \times 10^3)(100 \times 10^{-6}) = 1.0$ s M1 A1.

Practice 2.3 ▪ 1 mark

State what happens to the **charge** as a capacitor discharges.

Solution 2.3

Method: Exponential decay

Worked steps

It **decreases exponentially** towards zero B1.

Practice 2.4 ▪ 1 mark

Write the discharge equation for **charge**.

Solution 2.4

Method: Exponential decay

Worked steps

$Q = Q_0 e^{-t/RC}$ **B1**.

Exam-style 3.1 ▪ 1 mark

The time constant RC is the time for the charge to fall to:

A zero

B half its initial value

C $1/e$ (about 37%) of its initial value

D $1/2e$ of its initial value

Solution 3.1

Method: Exponential decay

A zero

B half its initial value

C $1/e$ (about 37%) of its initial value ✓

D $1/2e$ of its initial value

Worked steps

C — $1/e$ (about 37%).

Exam-style 3.2 ▪ 4 marks

A $470\ \mu$F capacitor charged to 6.0 V discharges through a 2.0 kΩ resistor. Calculate (a) the time constant, (b) the p.d. after 1.0 s.

Solution 3.2

Method: Exponential decay

Worked steps

(a) $\tau = RC = (2.0 \times 10^3)(470 \times 10^{-6}) = 0.94$ s M1 A1. (b) $V = V_0 e^{-t/RC} = 6.0\, e^{-1.0/0.94}$ M1 $= 6.0 \times 0.345 = 2.1$ V A1.

Exam-style 3.3 ▪ 2 marks

Sketch a graph of **charge against time** for a discharging capacitor.

Solution 3.3

Method: Exponential decay

Worked steps

A curve starting at the initial charge and **decaying exponentially** towards zero B1, with the same shape (never quite reaching zero) B1.

Exam-style 3.4 ▪ 1 mark

State what happens to the time constant if the **resistance** is doubled.

Solution 3.4

Method: Exponential decay

Worked steps

It **doubles** ($\tau = RC$) **B1**.

Exam-style 3.5 ▪ 4 marks

A $220\ \mu$F capacitor charged to 9.0 V discharges through a 15 kΩ resistor. Calculate the **time** for the p.d. to fall to 3.0 V.

Solution 3.5

Method: Exponential decay

Worked steps

$\tau = RC = (15 \times 10^3)(220 \times 10^{-6}) = 3.3$ s **M1**; $t = -RC \ln \frac{V}{V_0} = -3.3 \ln \frac{3.0}{9.0}$ **M1** **M1** $= -3.3 \times (-1.10) = 3.6$ s **A1**.

Exam-style 3.6 ▪ 2 marks

A capacitor C is charged and then discharged through a resistor R. A data logger records the p.d. V_R across the resistor and the p.d. V_C across the capacitor. At $t = 0$ the p.d. across the capacitor is 9.0 V, and the initial current is 1.8 mA. The p.d. across the capacitor falls to 3.3 V after 12 s.

(a) **State** the relationship between V_C and V_R during the discharge, and give a reason.

(b) **Determine** the resistance R.

(c) **Determine** the time constant τ of the circuit.

(d) Hence **determine** the capacitance C, in μF.

(e) **Explain**, in terms of the current in the circuit, why the decay of the charge on the capacitor is **exponential**.

Solution 3.6

Method: Exponential decay

Worked steps

(a) They are **equal** at every instant, $V_C = V_R$ B1, because the capacitor and resistor form a single loop with no other source, so the p.d. driving the current is the capacitor's own p.d. B1.

(b) $R = \dfrac{V}{I} = \dfrac{9.0}{1.8 \times 10^{-3}}$ M1 $= 5.0 \times 10^3\ \Omega$ A1.

(c) $V = V_0 e^{-t/\tau}$, so $\dfrac{3.3}{9.0} = e^{-12/\tau}$ M1; $\ln(0.367) = -\dfrac{12}{\tau}$, giving $-1.003 = -\dfrac{12}{\tau}$ M1 and $\tau = 12$ s A1. (The p.d. has fallen to $1/e$ of its start, which is what the time constant means.)

Solution 3.6

(d) $\tau = RC$, so $C = \dfrac{12}{5.0 \times 10^3}$ M1 $= 2.4 \times 10^{-3}$ F $= 2.4 \times 10^3\ \mu$F A1.

(e) The current is $I = \dfrac{V_C}{R}$, and $V_C \propto Q$, so the current — the rate at which charge leaves — is **proportional to the charge still on the capacitor** B1. A quantity whose rate of decrease is proportional to itself decays exponentially B1; so as the charge falls the current falls with it, the discharge slows, and Q never quite reaches zero B1.