Exercise walk-through

Energy stored in a capacitor ▪ 19.2

eXercise

You must be able to

- find energy from the **area under the potential–charge graph**
- use $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \dfrac{Q^2}{2C}$

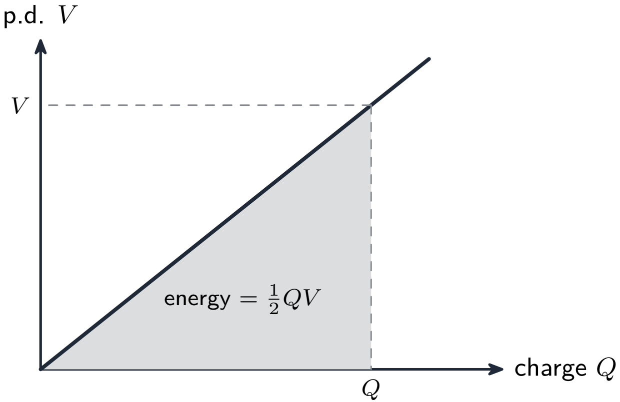
Method — Energy stored

The energy is the **area under the V–Q graph** — a triangle:

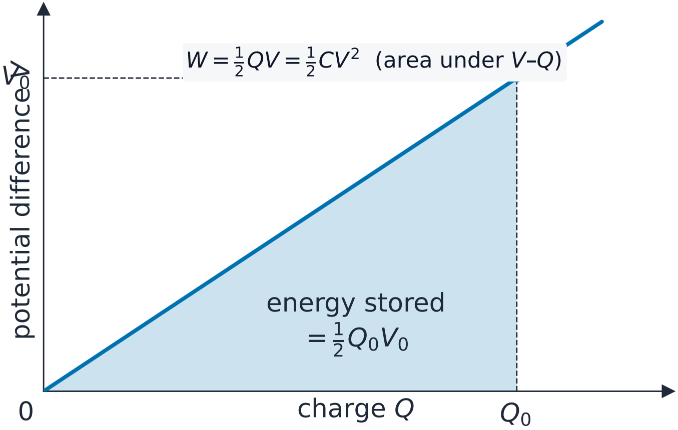
$$W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}.$$

The factor $\frac{1}{2}$ appears because the **average** p.d. during charging is $V/2$ (it grows from 0 to V).
Worked example. 100 μF at 12 V: $W = \frac{1}{2}CV^2 = \frac{1}{2}(100 \times 10^{-6})(12)^2 = 7.2 \times 10^{-3}$ J.

Method — Energy stored



Method — Energy stored



Energy stored is the area under the Q-V graph

Practice 2.1 ▪ 1 mark

Write the **three** forms of the capacitor energy formula.

Solution 2.1

Worked steps

$$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$ **B1**.

Practice 2.2 ▪ 2 marks

Find the energy stored in a $100\ \mu$F capacitor charged to 12 V.

Solution 2.2

Method: Energy stored

Worked steps

$W = \frac{1}{2}CV^2 = \frac{1}{2}(100 \times 10^{-6})(12)^2 = 7.2 \times 10^{-3}$ J **M1** **A1**.

Practice 2.3 ▪ 1 mark

State what the **area** under a potential–charge graph represents.

Solution 2.3

Method: Energy stored

Worked steps

The **energy stored** in the capacitor B1.

Exam-style 3.1 ▪ 1 mark

The energy stored in a capacitor is:

A QV

B $\frac{1}{2}QV$

C CV

D Q^2C

Solution 3.1

Method: Energy stored

A QV

B $\frac{1}{2}QV$ ✓

C CV

D Q^2C

Worked steps

B — $\frac{1}{2}QV$.

Exam-style 3.2 ▪ 3 marks

A $220\ \mu$F capacitor stores 4.0 mJ of energy. Calculate the **voltage** across it.

Solution 3.2

Method: Energy stored

Worked steps

$$W = \tfrac{1}{2}CV^2 \Rightarrow V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2(4.0 \times 10^{-3})}{220 \times 10^{-6}}} \ \boxed{\text{M1}}\boxed{\text{M1}} = 6.0\ \text{V} \ \boxed{\text{A1}}.$$

Exam-style 3.3 ▪ 2 marks

Explain why the energy stored is $\frac{1}{2}QV$ and **not** QV.

Solution 3.3

Method: Energy stored

Worked steps

As the capacitor charges, the p.d. rises from 0 to V **B1**; the **average** p.d. is $V/2$, so the work done is $Q \times (V/2) = \frac{1}{2}QV$, not QV **B1**.

Exam-style 3.5 ▪ 2 marks

A defibrillator stores energy in a $32\ \mu$F capacitor charged to 2.5 kV, and delivers it to a patient in a pulse lasting 8.0 ms.

(a) **Determine** the energy stored by the capacitor when fully charged.

(b) **Determine** the charge stored.

(c) In the pulse, 75% of the stored energy is delivered to the patient. **Determine** the average power delivered.

(d) The capacitor is charged instead to 1.8 kV. **Determine** the percentage by which the stored energy falls, and **explain** why it is not the same percentage as the fall in p.d.

(e) On the same axes, **sketch** how the energy stored varies with the p.d. across the capacitor, and how the **charge** stored varies with it. **Label** each line.

Solution 3.5

Method: Energy stored

Worked steps

(a) $E = \frac{1}{2}CV^2 = \frac{1}{2}(32 \times 10^{-6})(2500)^2$ M1 $= 100$ J A1.
(b) $Q = CV = 32 \times 10^{-6} \times 2500$ M1 $= 8.0 \times 10^{-2}$ C A1.
(c) Energy delivered $= 0.75 \times 100 = 75$ J M1; $P = \dfrac{E}{t} = \dfrac{75}{8.0 \times 10^{-3}}$ M1 $= 9.4 \times 10^3$ W A1. (About nine kilowatts, for eight milliseconds.)
(d) New energy $= \frac{1}{2}(32 \times 10^{-6})(1800)^2 = 51.8$ J M1, a fall of $\dfrac{100 - 51.8}{100} = 48\%$ A1 — while the p.d. fell by only 28%, because $E \propto V^2$, so the fractional change in energy is roughly **twice** the fractional change in p.d. B1.

Solution 3.5

(e) Charge: a **straight line through the origin**, since $Q = CV$ B1. Energy: a **curve through the origin** getting steeper, since $E \propto V^2$ B1. Both labelled, with axes labelled B1.