Past-paper walk-through

Capacitors and capacitance ▪ 19.1

eXercise

Questions in this set

- 9702/41 N25 Q6 ▪ 11 marks
- 9702/42 M25 Q5 ▪ 9 marks
- 9702/42 N24 Q7 ▪ 10 marks
- 9702/42 J24 Q6 ▪ 10 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 6

9702/41 N25 ▪ Q6 ▪ 11 marks

Two parallel plate capacitors C_1 and C_2 are connected to a supply that has a potential difference (p.d.) V_S. The capacitors may be connected in series or in parallel.

The supply provides charge Q_S and the plates of the two capacitors acquire charges Q_1 and Q_2 respectively. The p.d.s across the plates of the capacitors are V_1 and V_2 respectively.

(a) Complete Table 6.1 to indicate how Q_S, Q_1 and Q_2 relate to each other, and how V_S, V_1 and V_2 relate to each other, for series and parallel connections of the capacitors to the supply.

Table 6.1

	relationship between charges	relationship between p.d.s
series		
parallel		

Question 6

[4]

Question 6

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series		
parallel		

Question 6

9702/41 N25 ▪ Q6 ▪ 11 marks

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The supply provides charge Q_S and the plates of the two capacitors acquire charges Q_1 and Q_2 respectively. The p.d.s across the plates of the capacitors are V_1 and V_2 respectively.

(b)(i) Calculate the p.d. across the capacitor. **[2]**

(b)(ii) Calculate the charge on the capacitor. **[2]**

(b)(iii) The capacitor is now connected in parallel with a capacitor of capacitance $180\ \mu$F that is initially uncharged.

Determine the total energy, in mJ, now stored in the two capacitors. **[3]**

Solution 6

(a) Mark scheme

- series charges:
- $QS = Q1 = Q2$ **B1**
- series p.d.s:
- $VS = V1 + V2$ **B1**
- parallel charges: $QS = Q1 + Q2$ **B1**
- parallel p.d.s:
- $VS = V1 = V2$ **B1**

Solution 6

(b)(i) Worked steps

- $E = \frac{1}{2}CV^2$, so $V = \sqrt{\dfrac{2E}{C}}$.
- $V = \sqrt{\dfrac{2 \times 19 \times 10^{-3}}{470 \times 10^{-6}}}$
- $V = 9.0$ V

Solution 6

Mark scheme

- E = ½ CV2 **C1**
- p.d. = [(2 × 19 × 10–3) / (470 × 10–6)]½
- = 9.0 V **A1**

Solution 6

(b)(ii)

Worked steps

Now that the p.d. is known, the charge follows from the definition of capacitance.

- $Q = CV = 470 \times 10^{-6} \times 9.0$
- $Q = 4.2 \times 10^{-3}$ C

Or in one step from the energy: $Q = \sqrt{2EC}$, which avoids carrying a rounded V.

Mark scheme

- E = Q2 / 2C or C = Q / V **C1**
- Q = (19 × 10–3 × 2 × 470 × 10–6)½ or
- Q = 470 × 10–6 × 9.0
- Q = 4.2 × 10–3 C **A1**

Solution 6

(b)(iii) Worked steps

Connecting a second capacitor in **parallel** to a charged one shares the charge, and the total charge is conserved.

- Total capacitance: $C = (470 + 180) \times 10^{-6} = 650\ \mu$F.
- The charge is unchanged at $Q = 4.2 \times 10^{-3}$ C from part (b)(ii).
- $E = \dfrac{Q^2}{2C} = \dfrac{(4.23 \times 10^{-3})^2}{2 \times 650 \times 10^{-6}}$
- $E = 0.014$ J $= 14$ mJ

Solution 6

Energy fell from 19 mJ to 14 mJ even though no charge was lost — the missing 5 mJ is dissipated in the connecting wires as the charge redistributes, which is the point the question is testing.

Mark scheme

- total charge unchanged **C1**
- total capacitance $= (470 + 180) \times 10–6$ (F) **C1**
- $E = Q2 / 2C = (4.23 \times 10–3)2 / (2 \times 650 \times 10–6)$ $(= 0.014$ J$)$
- $E = 14$ mJ **A1**

Question 5

9702/42 M25 ▪ Q5 ▪ 9 marks

5 **(a)** A capacitor of capacitance C_1 is connected in series with a second capacitor of capacitance C_2.

Show that the combined capacitance C of the two capacitors is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}.$$

Question 5

[2]

(b) Three identical capacitors, each of capacitance *C*, are connected in a network as shown in Fig. 5.1.

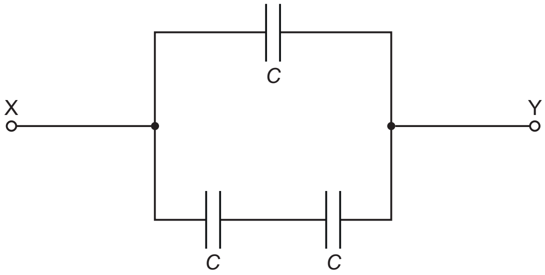


Fig. 5.1

Question 5

The variation of the charge Q with the potential difference (p.d.) V between the terminals X and Y is shown in Fig. 5.2.

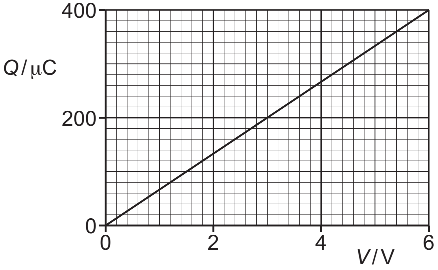


Fig. 5.2

Show that C is equal to 44 μF.

Question 5

[3]

(c) The capacitor network in Fig. 5.1 is charged and then connected to a resistor of resistance $54\,\mathrm{k\Omega}$. The capacitor network discharges through the resistor.

(i) Determine the time constant τ of the circuit. Give a unit with your answer.

Question 5

Question 5

(ii) Determine the time taken for the discharge current to reduce to 15% of the initial discharge current.

Question 5

[Total: 9]

Solution 5

(a) Worked steps

Start from the two physical facts about capacitors in **series**, state them, then substitute.

- The same charge sits on every capacitor in a series chain: $Q = Q_1 = Q_2$.
- The p.d.s add to the supply p.d.: $V = V_1 + V_2$.
- For any capacitor $V = \dfrac{Q}{C}$, so $\dfrac{Q}{C} = \dfrac{Q}{C_1} + \dfrac{Q}{C_2}$.
- Q cancels: $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

Solution 5

The first two lines are where the marks are, and the first is the one candidates leave out. “Same charge” is not obvious — it follows from charge conservation on the isolated plates between the two capacitors — and without stating it the substitution has no justification.

Mark scheme

- $Q = Q1 = Q2$ and $V = V1 + V2$
- $V = Q / C$ so:
- $Q / C = Q / C1 + Q / C2$ leading to $1 / C = 1 / C1 + 1 /$ **C2**

Solution 5

(b) Worked steps

Two steps, and neither is a formula you look up: work out the combination, then read the gradient.

- One capacitor of value C in **parallel** with two of value C in **series**. The series pair is $\frac{1}{2}C$ (equal capacitors in series halve), and parallel capacitances add, so the total is $C + \frac{1}{2}C = \frac{3}{2}C$.
- On a graph of Q against V, the gradient **is** the capacitance: gradient $= \dfrac{400 \times 10^{-6}}{6.0} = 6.7 \times 10^{-5}$ F.
- So $\frac{3}{2}C = 6.7 \times 10^{-5}$, giving $C = \dfrac{2 \times 400 \times 10^{-6}}{3 \times 6.0} = 44\ \mu$F.

Solution 5

Capacitors behave the opposite way round to resistors: **series** capacitances combine by reciprocals and **parallel** ones add. Getting that backwards gives $\frac{2}{3}C$ and an answer of 100 μF.

Mark scheme

- total capacitance $=$ C $+$ ½C $=$ (3 / 2)C
- total capacitance $=$ gradient
- $=$ 400 $\times$ 10–6 / 6.0
- either: C $=$ (2 $\times$ 400 $\times$ 10–6) / (3 $\times$ 6.0) $=$ 4.4 $\times$ 10–5 F $=$ 44 μF
- or:
- C $=$ (2 $\times$ 400) / (3 $\times$ 6.0) $=$ 44 μF

Solution 5

(c)(i) Worked steps

The time constant is the product of the resistance and the **total** capacitance of the circuit being discharged.

- $\tau = RC_{\text{total}} = 54 \times 10^3 \times \frac{3}{2} \times 44 \times 10^{-6}$
- $\tau = 3.6$ s

Solution 5

Use the combination, not the single capacitor — the whole network discharges through the resistor. And check the units: ohms times farads gives seconds, which is a quick way to catch a missing micro prefix.

Mark scheme

- $\tau = \mathrm{RC}$
- $= 54 \times 103 \times (3/2) \times 44 \times 10–6$
- $= 3.6\,\mathrm{s}$

Solution 5

(c)(ii) Worked steps

Exponential decay, with the fraction remaining given.

- $\dfrac{V}{V_0} = e^{-t/\tau}$, so $0.15 = e^{-t/3.6}$
- Take natural logs: $\ln 0.15 = -\dfrac{t}{3.6}$, and $\ln 0.15 = -1.90$.
- $t = 1.90 \times 3.6 = 6.8$ s

Solution 5

The same ratio applies to charge, p.d. and current, since all three decay with the same time constant — so it does not matter which quantity the 0.15 refers to.
Sense-check: 6.8 s is just under two time constants, and after two time constants about 14% remains. An answer near 0.54 s means the equation was used the wrong way up.

Mark scheme

- $0.15 = \exp(-t\,/\,3.6)$
- $t = 6.8\,\mathrm{s}$

Question 7

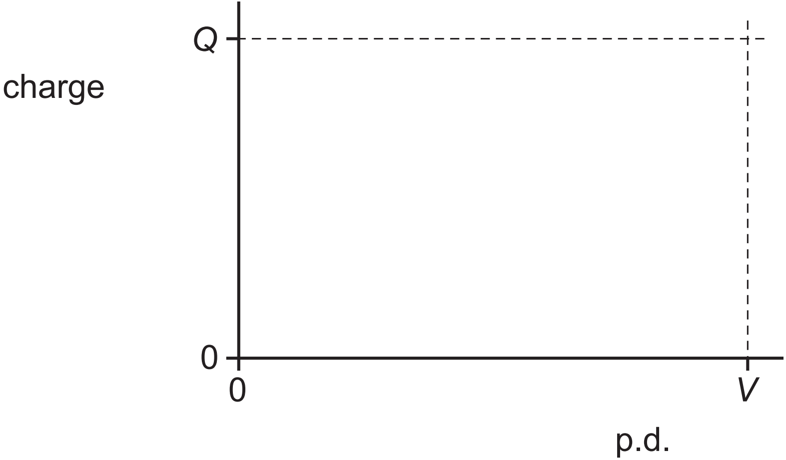
9702/42 N24 ▪ Q7 ▪ 10 marks

(a) Define the capacitance of a parallel-plate capacitor. **[2]**

Question 7

	X	Y
final p.d.		
final charge		

Question 7



Question 7

9702/42 N24 ▪ Q7 ▪ 10 marks

(b)(i) On Fig. 7.1, sketch the variation of charge with p.d. for capacitor X as the p.d. increases from 0 to V. **[2]**

(b)(ii) Determine an expression, in terms of Q and V, for the work W done on capacitor X during the charging process. Explain your reasoning. **[2]**

Question 7

9702/42 N24 ▪ Q7 ▪ 10 marks

(c)(i) Complete Table 7.1 to show expressions, in terms of Q and V, for the final p.d.s across, and the final charges on, the two capacitors.

Table 7.1

	X	Y
final p.d.		
final charge		

Question 7

[3]

(c)(ii) State whether the total energy stored in the two capacitors is less than, the same as, or greater than the energy initially stored in capacitor X. **[1]**

Solution 7

(a)

Mark scheme

- charge / potential (difference)
- charge is charge on one plate, and potential is p.d. between the plates

Solution 7

(b)(i)

Worked steps

Know. Capacitance is a constant for a given capacitor, so $Q = CV$ says the charge is **directly proportional** to the p.d. across the plates.
Draw. A single straight line. It starts at the **origin**, because an uncharged capacitor has no p.d. across it, and it rises with a constant positive gradient.
End point. The question follows the charging from 0 up to V, so stop the line at the point (V, Q).
Check. The gradient of this line is the capacitance itself. Drawing a curve would be claiming the capacitance changes as the capacitor fills, which it does not.

Mark scheme

- straight line starting at the origin
- line with positive gradient ending at (V, Q)

Solution 7

(b)(ii) Worked steps

Know. Work done in moving charge through a p.d. is $W = QV$ for a **constant** p.d. Here the p.d. is not constant: it climbs from 0 to V as the charge builds up. So add up the work in small steps, which is exactly what the **area under the graph** does.
Reasoning. The area under a charge-against-p.d. line is the work done on the capacitor. The graph is a triangle of base V and height Q.
Answer. $W = \frac{1}{2}QV$.

Solution 7

Check. Explaining *why* is worth as much as the formula here. The factor of $\frac{1}{2}$ appears because the average p.d. during charging is only half the final value: the first charge arrives with nothing to push against, the last with the full V opposing it.

Mark scheme

- work done is the area under the graph
- $W = \frac{1}{2}QV$

Solution 7

(c)(i) Mark scheme

final p.d. shown as $V/4$ for both capacitors
final charges add together to give Q
charge on Y $= 3\times$ charge on X (and both charges shown as a multiple of Q)
Fully correct answer:

	X	Y
final p.d.	$V/4$	$V/4$
final charge	$Q/4$	$3Q/4$

Solution 7

(c)(ii)

Mark scheme

- less than

Question 6

9702/42 J24 ▪ Q6 ▪ 10 marks

Two capacitors X and Y are connected in series to a power supply of voltage V.

(a) The capacitance of X is C_X and the capacitance of Y is C_Y.

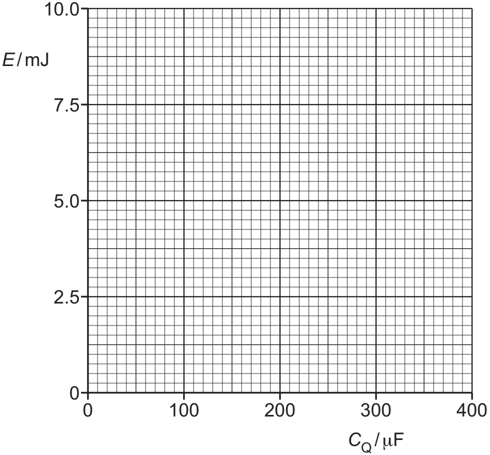
Derive an expression, in terms of C_X and C_Y, for the combined capacitance C_T of the capacitors in this circuit. Explain your reasoning. **[3]**

(b)(i) Show that the supply voltage V is 5.0 V. **[2]**

(b)(ii) Calculate the total energy, in mJ, stored in the capacitors when C_Q has its maximum value. **[3]**

(b)(iii) On Fig. 6.2, sketch the variation of the total energy E stored in the capacitors with C_Q, as C_Q varies from 0 to 400 μF. **[2]**

Question 6



Question 6

Solution 6

(a) Worked steps

Know. Capacitors in **series** carry the **same charge**. No charge can reach the middle plates from outside; it is only moved from one plate to its neighbour, so $Q_X = Q_Y = Q$.
Add the p.d.s. Going round the loop, the two p.d.s share the supply voltage:
$V_X + V_Y = V$.

Solution 6

Substitute. Each p.d. is $V = \dfrac{Q}{C}$, so $\dfrac{Q}{C_X} + \dfrac{Q}{C_Y} = \dfrac{Q}{C_T}$.

Answer. Cancel the common Q: $\dfrac{1}{C_T} = \dfrac{1}{C_X} + \dfrac{1}{C_Y}$.

Solution 6

Check. The two physical statements are the marks: equal charge, and p.d.s that add. Quoting the final formula alone earns nothing on a "derive and explain your reasoning" question.

Mark scheme

- equal charge on both capacitors
- $V_X + V_Y = V$
- $(Q/C_X) + (Q/C_Y) = (Q/C_T)$ leading to $(1/C_X) + (1/C_Y) = (1/C_T)$
- **or**
- $(V_X/Q) + (V_Y/Q) = (V/Q)$ leading to $(1/C_X) + (1/C_Y) = (1/C_T)$

Solution 6

(b)(i) Worked steps

Know. The stored energy fixes the supply voltage, because both the energy and the capacitance are given.

Equation. $E = \frac{1}{2}CV^2$, so $V = \sqrt{\frac{2E}{C}}$.

Solution 6

Substitute. $V = \sqrt{\dfrac{2 \times 2.5 \times 10^{-3}}{200 \times 10^{-6}}}$.

Answer. $V = 5.0$ V, as required.

Solution 6

Check. Convert the microfarads before dividing. Using 200 instead of 200×10^{-6} gives $V = 5.0 \times 10^{-3}$ V, and the power of ten is the only thing wrong.

Mark scheme

- $E = \frac{1}{2} C V^2$
- $V = \sqrt{\frac{2 \times 2.5 \times 10^{-3}}{200 \times 10^{-6}}} = 5.0$ V

Solution 6

(b)(ii) Worked steps

Know. With C_Q at its maximum the capacitance in this branch is at its largest, and the supply voltage stays at 5.0 V.
Combine. Total capacitance $= 200 + 400 = 600$ μF.
Equation. $E = \frac{1}{2}CV^2$.
Substitute. $E = \frac{1}{2} \times 600 \times 10^{-6} \times 5.0^2$.
Answer. $E = 7.5 \times 10^{-3}$ J $= 7.5$ mJ.

Solution 6

Check. The answer is asked for in **millijoules**, so convert. Writing 7.5×10^{-3} in a box labelled mJ says $7.5\ \mu$J, which is a thousand times too small.

Mark scheme

- total capacitance $= 600\ \mu$F
- $E = \frac{1}{2} \times 600 \times 10^{-6} \times 5.0^2$
- $(= 7.5 \times 10^{-3}$ J)
- $= 7.5$ mJ

Solution 6

(b)(iii) Worked steps

Know. With the supply voltage fixed, $E = \frac{1}{2}CV^2$ makes the energy **directly proportional** to the total capacitance. Adding C_Q adds to that total in equal steps, so the graph is a **straight line**.

Two points. At $C_Q = 0$ only the $200\ \mu$F is there, giving $E = \frac{1}{2} \times 200 \times 10^{-6} \times 5.0^2 = 2.5$ mJ. At $C_Q = 400\ \mu$F the answer to (b)(ii) applies: $E = 7.5$ mJ.

Draw. A straight line of positive gradient from $(0,\ 2.5)$ to $(400,\ 7.5)$.

Solution 6

Check. The line does **not** pass through the origin. Even with C_Q removed there is still a charged capacitor storing 2.5 mJ, and that is the intercept.

Mark scheme

- line with positive gradient starting at $(0, 2.5)$
- straight line passing through $(400, 7.5)$