Exercise walk-through

Damped and forced oscillations, resonance ▪ 17.3

eXercise

You must be able to

- describe **light**, **critical** and **heavy** damping and sketch their graphs
- explain **resonance** and the condition for it

Method — Damping

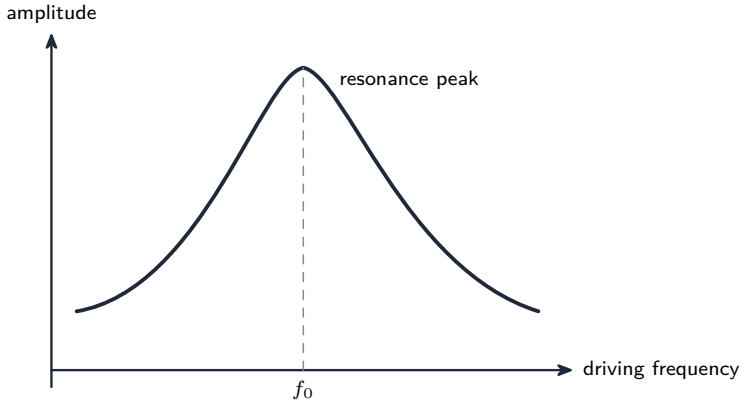
A **resistive force** removes energy, so the amplitude **shrinks** — this is **damping**:

- **light:** amplitude dies away slowly over many cycles;
- **critical:** returns to equilibrium **fastest** with **no overshoot**;
- **heavy:** returns slowly, no oscillation.

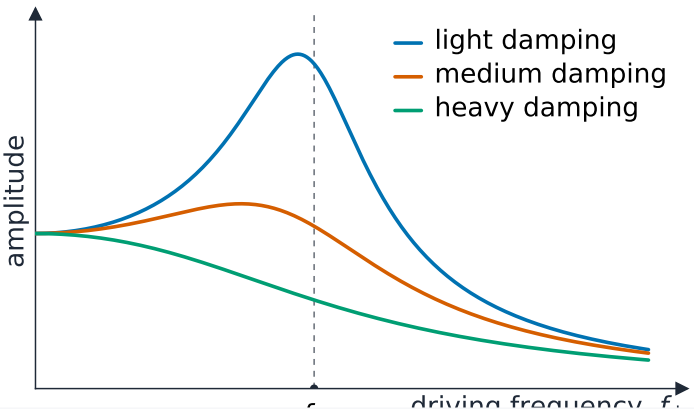
Method — Resonance

A system **forced** to oscillate has the largest amplitude when the **driving frequency equals its natural frequency** f_0 — this is **resonance**:
More damping makes the resonance peak **lower and broader**.

Method — Resonance



Method — Resonance



resonance: max amplitude near f_0 · lighter damping ⇒ sharper peak

The resonance curve and damping

Practice 2.1 ▪ 1 mark

State what causes **damping**.

Solution 2.1

Worked steps

A **resistive force** (friction / drag / air resistance) removing energy from the oscillator B1.

Practice 2.2 ▪ 2 marks

Name the **three** types of damping.

Solution 2.2

Worked steps

Light, **critical** and **heavy** damping B1 B1 *for two, for all three.*

Practice 2.3 ▪ 1 mark

State what is meant by **resonance**.

Solution 2.3

Worked steps

The condition where a forced oscillation reaches its **maximum amplitude** B1.

Practice 2.4 ▪ 1 mark

State the condition for resonance to occur.

Solution 2.4

Worked steps

The **driving frequency equals the natural frequency** of the system B1.

Exam-style 3.1 ▪ 1 mark

Critical damping is the damping that:

A lets the system oscillate forever

B returns the system to equilibrium fastest, without overshooting

C returns the system to equilibrium most slowly

D increases the amplitude

Solution 3.1

Method: Damping

A lets the system oscillate forever

B returns the system to equilibrium fastest, without overshooting ✓

C returns the system to equilibrium most slowly

D increases the amplitude

Worked steps

B — returns to equilibrium fastest without overshooting.

Exam-style 3.2 ▪ 3 marks

Describe how the **amplitude** of a forced oscillation changes as the driving frequency is increased through the natural frequency.

Solution 3.2

Method: Resonance

Worked steps

The amplitude **rises** as the driving frequency approaches the natural frequency B1, reaching a **maximum at** f_0 (resonance) B1, then **falls** again as the frequency increases past f_0 B1.

Exam-style 3.3 ▪ 2 marks

Give **one** example each of where **light** damping and **critical** damping are useful.

Solution 3.3

Method: Damping

Worked steps

Light damping — e.g. a pendulum/clock or musical instrument (sustained oscillation); critical damping — e.g. a galvanometer/voltmeter needle or car suspension/door closer (quick settle) B1 B1.

Exam-style 3.4 ▪ 2 marks

Explain what happens to the **resonance peak** when more damping is added.

Solution 3.4

Method: Resonance

Worked steps

The peak becomes **lower** (smaller maximum amplitude) B1 and **broader** (occurs over a wider range of frequencies) B1.

Exam-style 3.5 ▪ 2 marks

A machine part is mounted on rubber blocks. When the machine runs, the part is driven into oscillation, and a technician measures the amplitude of the part at each driving frequency.

(a) **State** the conditions needed for **resonance** to occur.

(b) The amplitude peaks sharply at a driving frequency of 24 Hz. **State** what this tells you about the part, and **determine** its natural period.

(c) The rubber blocks are replaced with stiffer ones, which **increase** the natural frequency of the part but also **increase** the damping. On one set of axes, **sketch** two curves of amplitude against driving frequency: the original mounting, labelled A, and the stiffer one, labelled B.

(d) **Explain** why an engineer would prefer the stiffer, more heavily damped mounting even though the machine's own running frequency cannot be changed.

Solution 3.5

Method: Resonance

Worked steps

(a) The system is driven by a **periodic force** whose frequency equals the **natural frequency** of the system B1, and there is a **maximum transfer of energy** to it, so its amplitude becomes maximum B1.

(b) The natural frequency of the part is 24 Hz B1; $T = \dfrac{1}{f} = \dfrac{1}{24} = 0.042$ s B1.

(c) Curve A: a sharp peak at 24 Hz B1. Curve B: its peak is at a **higher** frequency B1, is **lower** in amplitude B1, and is **broader** — heavier damping flattens and widens the resonance curve; both curves start at a non-zero amplitude at zero frequency and fall away at high frequency B1.

Solution 3.5

(d) The machine's running frequency is now **further from** the part's natural frequency, so the part is driven well away from resonance B1; and the extra damping lowers the amplitude at every frequency, so even at resonance the oscillation would be smaller and less likely to cause damage B1.