Past-paper walk-through

Energy in simple harmonic motion ▪ 17.2

eXercise

Questions in this set

- 9702/43 J25 Q5 ▪ 8 marks
- 9702/42 M25 Q4 ▪ 12 marks
- 9702/42 J23 Q4 ▪ 11 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 5

9702/43 J25 ▪ Q5 ▪ 8 marks

5 A cuboidal block floats in a liquid with its base horizontal, as shown in Fig. 5.1.

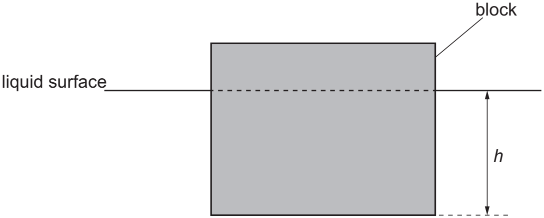


Fig. 5.1

The base of the block is at a depth *h* below the surface of the liquid.

The block is displaced downwards by a small distance and then released so that it oscillates

Question 5

The block is displaced downwards by a small distance and then released so that it oscillates.

Fig. 5.2 shows the variation with h of the acceleration a of the block.

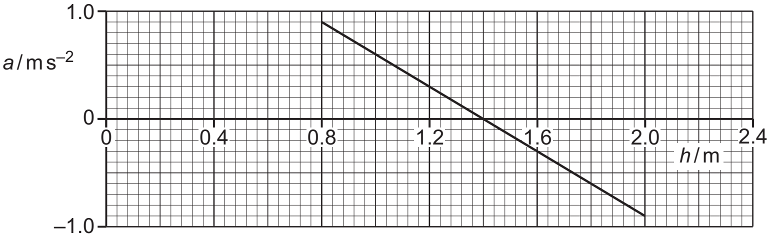


Fig. 5.2

Fig. 5.3 shows the variation with h of the kinetic energy E_K of the block.

Question 5

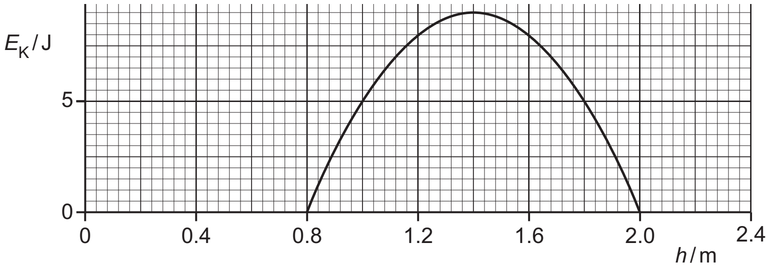


Fig. 5.3

(a) **(i)** Determine the amplitude of the oscillations.

Question 5

(ii) State what the line in Fig. 5.2 shows about the nature of the oscillations.

Question 5

(b) State **three** other quantitative conclusions that can be drawn from Fig. 5.2 and Fig. 5.3 about the block and its oscillations. Use the space for any working.

Question 5

[3]

(c) On Fig. 5.4, sketch the variation with h of the potential energy E_P of the oscillations.

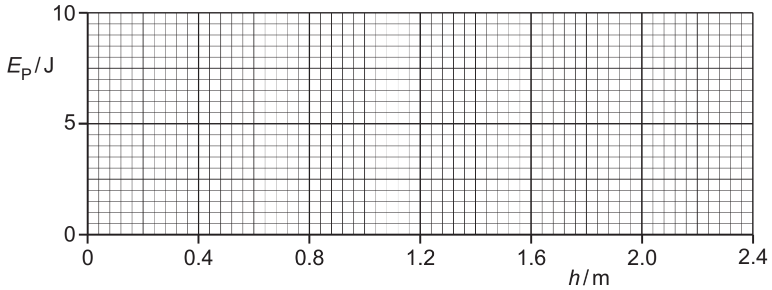


Fig. 5.4

[3]

Question 5

Question 5

[Total: 8]

Solution 5

(a)(i)

Worked steps

The amplitude is half the total travel, not the whole of it.

- The ball moves between $h = 0.8$ m and $h = 2.0$ m, a range of 1.2 m.
- amplitude $= \dfrac{1.2}{2} = 0.60$ m

Mark scheme

- amplitude $= 0.60\,\text{m}$

Solution 5

(a)(ii)

Mark scheme

- oscillations are simple harmonic

Solution 5

(b)

Worked steps

Three conclusions, each a stated quantity:

- The **equilibrium position** is at the centre of the motion, $h = 1.4$ m.
- The **total energy** of the oscillation is 9.0 J — the value the curve reaches at either extreme, where all of it is potential.
- The **angular frequency** is 1.2 rad s^{-1} (equivalently, a period of $2\pi/1.2 = 5.2$ s).

Mark scheme

- Any three points from:
- ▪
- mean / equilibrium position is at h = 1.4 m
- ▪
- total energy of oscillations = 9.0 J
- ▪

Solution 5

(c) Worked steps

Energy in SHM goes as displacement squared, so the curve is a parabola in h.

- Draw a **U-shape** with its minimum at the equilibrium position, $h = 1.4$ m, where the potential energy is **zero**.
- It rises symmetrically to 9.0 J at both ends, $h = 0.8$ and $h = 2.0$ m.
- The curve rests on the h axis at its minimum — it does not dip below, because energy measured from equilibrium cannot be negative.

Solution 5

Mark scheme

- U-shaped curve resting on h axis (with minimum at $EP = 0$)
- curve from $h = 0.8\,m$ to $h = 2.0\,m$, with minimum EP shown at $h = 1.4\,m$
- both end-points of curve shown at $EP = 9.0\,J$
- 9702/43
- May/June 2025

Question 4

9702/42 M25 ▪ Q4 ▪ 12 marks

4 A small crystal is made to vibrate with simple harmonic motion. The variation with time t of the displacement x of one surface of the crystal from its equilibrium position is shown in Fig. 4.1.

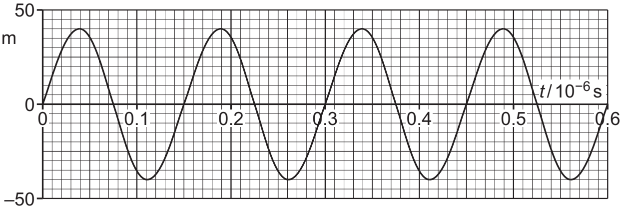


Fig. 4.1

(a) Show that the angular frequency of the vibration of the surface is $4.2 \times 10^7\ \text{rad s}^{-1}$.

Question 4

[2]

(b) Determine the maximum acceleration a_0 of the vibration of the surface.

Question 4

(c) The crystal may be modelled as a single mass of 2.4×10^{-4} kg that vibrates as shown in Fig. 4.1.

Calculate the total energy E of the vibrations.

Question 4

(d) The crystal generates ultrasound waves that are used to obtain diagnostic information about internal structures.

(i) The crystal is made from piezoelectric material.

Explain how the crystal is made to vibrate.

Question 4

(ii) A parallel beam of ultrasound waves is incident on a muscle-bone boundary. Data for muscle and bone are given in Table 4.1.

Table 4.1

material	density / $kg\,m^{-3}$	speed of sound / $m\,s^{-1}$
muscle	1100	1600
bone	1900	4100

Calculate the percentage of the intensity of the ultrasound beam that is transmitted at this boundary.

Question 4

Question 4

[Total: 12]

Solution 4

(a) Worked steps

Angular frequency comes straight from the period.

- $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.15 \times 10^{-6}}$
- $\omega = 4.2 \times 10^{7}\ \mathrm{rad\,s^{-1}}$

Solution 4

The period is in **microseconds**, so keep the 10^{-6}. Dropping it gives 42 rad s^{-1}, which is a factor of a million out and makes every later part wrong.

Mark scheme

- $\omega = 2\pi$ / T
- $= 2\pi$ / (0.15 $\times$ 10–6) $=$ 4.2 $\times$ 107 rad s–1

Solution 4

(b) Worked steps

For simple harmonic motion the acceleration is greatest at maximum displacement.

- $a_0 = \omega^2 x_0$
- $a_0 = (4.2 \times 10^7)^2 \times 40 \times 10^{-6}$
- $a_0 = 7.1 \times 10^{10}\ \mathrm{m\,s^{-2}}$

Square ω, not x_0 — this is the most common slip in the whole topic. And the amplitude is in micrometres, so it too carries a 10^{-6}.

Solution 4

An acceleration of 10^{10} m s^{-2} looks absurd until you notice the frequency: about 6.7 MHz, with the crystal face turning round more than six million times a second.

Mark scheme

- a0 $=$ ω2x0
- $=$ (4.2 $\times$ 107)2 $\times$ 40 $\times$ 10–6
- $=$ 7.1 $\times$ 1010 m s–2

Solution 4

(c) Worked steps

The total energy of an oscillator is its kinetic energy at maximum speed, and $v_0 = \omega x_0$.

- $E = \frac{1}{2} m\omega^2 x_0^2$
- $E = \frac{1}{2} \times 2.4 \times 10^{-4} \times (4.2 \times 10^7)^2 \times (40 \times 10^{-6})^2$
- $E = 340$ J

Solution 4

Both ω and x_0 are squared here, which is what distinguishes this formula from the acceleration one in (b). Carry the unrounded ω forward; using 4.2 rather than 4.19 moves the answer by about one per cent, which is inside the accepted range but only just.

Mark scheme

- $E = ½m\omega 2xo2$
- $= ½ \times 2.4 \times 10–4 \times (4.2 \times 107)2 \times (40 \times 10–6)2$
- $= 340$ J

Solution 4

(d)(i)

Mark scheme

- apply alternating p.d. (to / across crystal)
- applying p.d. to / across crystal causes it to distort

Solution 4

(d)(ii) Worked steps

Reflection at a boundary is decided by the two **specific acoustic impedances**, $Z = \rho c$.

- Muscle: $Z_{\mathrm{m}} = 1100 \times 1600 = 1.76 \times 10^{6}\ \mathrm{kg\,m^{-2}\,s^{-1}}$
- Bone: $Z_{\mathrm{b}} = 1900 \times 4100 = 7.79 \times 10^{6}\ \mathrm{kg\,m^{-2}\,s^{-1}}$
- $\dfrac{I_{\mathrm{r}}}{I_0} = \left(\dfrac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2 = \left(\dfrac{7.79 - 1.76}{7.79 + 1.76}\right)^2$
- $= 0.40$, that is 40%

Three things to check every time: the difference goes on top and the sum underneath, the whole bracket is squared, and the answer is a ratio with no unit. A coefficient greater than 1 means the fraction was inverted. **Mark scheme**

Solution 4

- $Z = \rho$ c
- Zm $= 1100 \times 1600$ ($= 1.76 \times 106$)
- Zb $= 1900 \times 4100$ ($= 7.79 \times 106$)
- intensity reflection co-efficient$=$ [(7.79 – 1.76) / (7.79 $+$ 1.76)]2
- $= 0.40$ or 40%
- percentage transmitted $=$ 60%
- 9702/42
- February/March 2025

Question 4

9702/42 J23 ▪ Q4 ▪ 11 marks

A small steel sphere is oscillating vertically on the end of a spring, as shown in Fig. 4.1.

(a)(i) Calculate the frequency of the oscillations. **[2]**

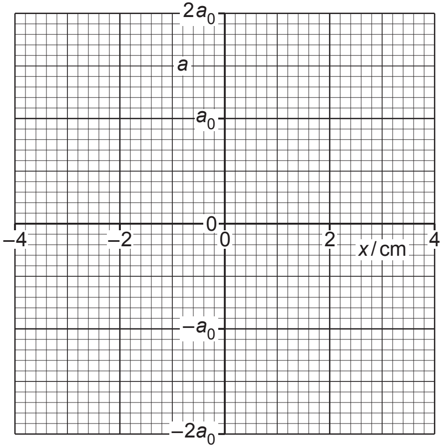
(a)(ii) Show that the amplitude of the oscillations is 3.4 cm. **[1]**

(a)(iii) Calculate the maximum acceleration a_0 of the sphere. **[2]**

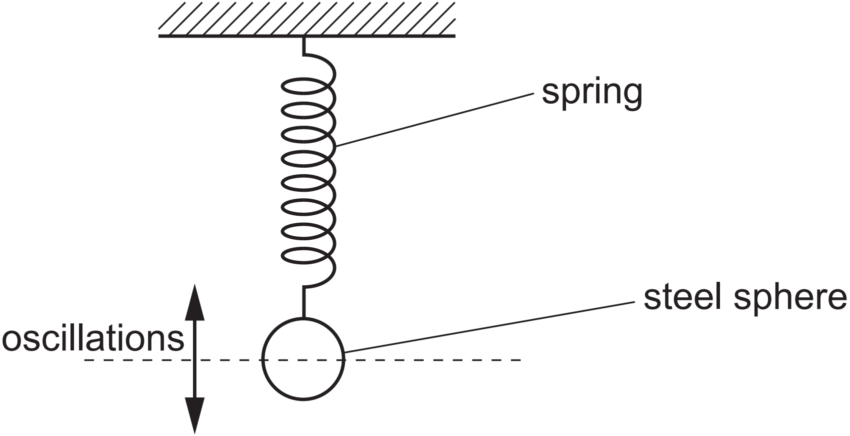
(b) On Fig. 4.2, sketch the variation with x of the acceleration a of the sphere. **[3]**

(c) Describe, without calculation, the interchange between the potential energy and the kinetic energy of the oscillations. **[3]**

Question 4



Question 4



Solution 4

(a)(i) Worked steps

Know. Angular frequency and frequency measure the same thing in different units: one counts radians per second, the other cycles per second.

Equation. $\omega = 2\pi f$, so $f = \dfrac{\omega}{2\pi}$.

Substitute. $f = \dfrac{9.7}{2\pi}$.

Answer. $f = 1.5$ Hz.

Solution 4

Check. The frequency must be **smaller** than ω, by a factor of about six. Getting 61 Hz means the 2π was multiplied rather than divided.

Mark scheme

- $\omega = 2\pi f$
- $f = 9.7/2\pi$
- $= 1.5$ Hz

Solution 4

(a)(ii) Worked steps

Know. The sphere is momentarily at rest at the extremes of its motion, so the graph reaches zero exactly where x^2 equals the amplitude squared.
Read. The line meets the x^2 axis at 11.6 cm^2.
Take the root. $x_0 = \sqrt{11.6}$.
Answer. $x_0 = 3.4$ cm, as required.

Solution 4

Check. The axis carries x^2, so the intercept is a squared length: 11.6 cm^2 is not a displacement. This is the same trick as reading a $\ln$ axis, and the same trap.

Mark scheme

- amplitude $= \sqrt{11.6} = 3.4$ cm

Solution 4

(a)(iii) Worked steps

Know. In simple harmonic motion the acceleration is largest at the extremes, where the displacement is largest.
Equation. $a_0 = \omega^2 x_0$.
Convert. The amplitude must be in metres: 3.4 cm $= 3.4 \times 10^{-2}$ m.
Substitute. $a_0 = (9.7)^2 \times (3.4 \times 10^{-2})$.
Answer. $a_0 = 3.2$ m s^{-2}.

Solution 4

Check. Use ω, not f: putting 1.5 in place of 9.7 gives 0.077 m s^{-2}, forty times too small. The 2π has to go back in before squaring.

Mark scheme

- $a_0 = \omega^2 x_0$
- $= 9.7^2 \times 3.4 \times 10^{-2}$
- $= 3.2$ m s^{-2}

Solution 4

(b) Worked steps

Know. The defining equation of simple harmonic motion is $a = -\omega^2 x$. That is a straight line through the origin, and the minus sign makes the gradient **negative**.
Draw. A straight line through $(0,\ 0)$ sloping downwards from left to right.
End points. The motion only reaches $x = \pm 3.4$ cm, so stop the line there: it passes through $(+3.4,\ -a_0)$ and $(-3.4,\ +a_0)$, with $a_0 = 3.2$ m s^{-2}.

Solution 4

Check. The negative gradient is the physics: the acceleration always points **back towards** the middle, so it is opposite in sign to the displacement. A line drawn through the origin with a positive gradient describes something that accelerates away and never oscillates. Drawing the line beyond ± 3.4 cm loses the third mark, because the sphere never goes there.

Mark scheme

- sketch: straight line through the origin with negative gradient
- line with negative gradient passing through $(+3.4, -a_0)$ and $(-3.4, +a_0)$
- line with ends at $x = \pm 3.4$ cm and $a = \pm a_0$

Solution 4

(c) Mark scheme

- sum of potential energy and kinetic energy is constant
- at maximum displacement, kinetic energy is zero
- **or**
- at maximum displacement, potential energy is maximum
- at zero displacement, kinetic energy is maximum
- **or**
- at zero displacement, potential energy is minimum