Exercise walk-through

Energy in simple harmonic motion ▪ 17.2

eXercise

You must be able to

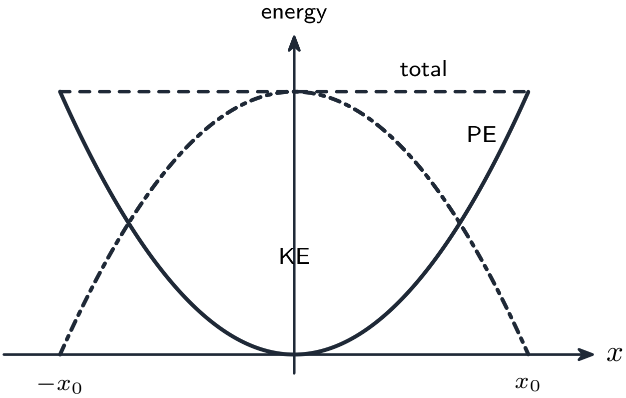
- describe the **KE** $\leftrightarrow$ **PE** interchange during SHM
- use $E = \frac{1}{2} m\omega^2 x_0^2$ for the total energy

Method — Energy interchange

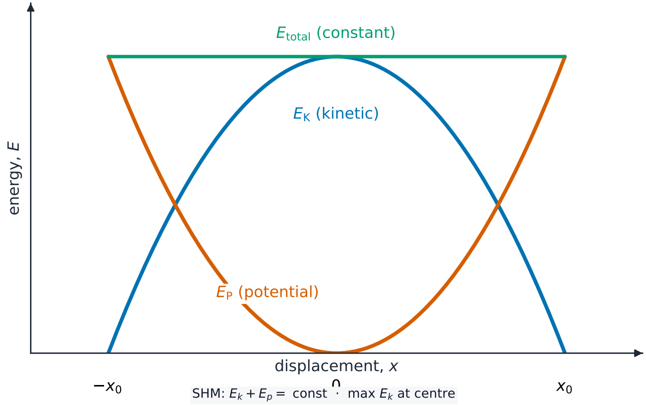
KE and PE swap as the oscillator moves; with no damping the **total energy is constant**:

- at $x = 0$: KE is **maximum**, PE minimum;
- at $x = \pm x_0$: KE $= 0$, PE **maximum**.

Method — Energy interchange



Method — Energy interchange



Energy interchange in simple harmonic motion

Method — Total energy

$$E = \frac{1}{2} m v_{\max}^2 = \frac{1}{2} m \omega^2 x_0^2.$$

So $E \propto x_0^2$ (doubling the amplitude gives **four times** the energy) and $E \propto \omega^2$.

Method — Energy varies at twice the frequency

In one full oscillation the object passes through the centre (KE maximum) **twice**, so the **KE and PE vary at twice** the frequency of the displacement. On a graph of KE against **time**, one repeat of the KE pattern takes **half** the oscillation period — so if the KE graph repeats every t_E, the oscillation period is $T = 2t_E$.

Practice 2.1 ▪ 2 marks

Describe the **energy interchange** during SHM.

Solution 2.1

Method: Energy interchange

Worked steps

Energy is continually transferred between **kinetic** and **potential** forms B1; with no damping the **total** stays constant B1.

Practice 2.2 ▪ 2 marks

State where the **kinetic energy** is maximum and where the **potential energy** is maximum.

Solution 2.2

Method: Energy interchange

Worked steps

KE is maximum at the **equilibrium** ($x = 0$); PE is maximum at the **extremes** ($x = \pm x_0$) B1 B1.

Practice 2.3 ▪ 1 mark

Write the formula for the **total energy** of a simple harmonic oscillator.

Solution 2.3

Method: Energy interchange

Worked steps

$E = \frac{1}{2} m\omega^2 x_0^2$ **B1**.

Exam-style 3.1 ▪ 1 mark

In SHM, the total energy is proportional to:

A the amplitude x_0

B the amplitude squared x_0^2

C $1/x_0$

D the period T

Solution 3.1

A the amplitude x_0

B the amplitude squared x_0^2 ✓

C $1/x_0$

D the period T

Worked steps

B — the amplitude squared.

Exam-style 3.2 ▪ 3 marks

A 0.20 kg mass oscillates with $\omega = 10$ rad s^{-1} and amplitude 0.050 m. Calculate the **total energy**.

Solution 3.2

Method: Total energy

Worked steps

$E = \frac{1}{2} m\omega^2 x_0^2 = \frac{1}{2}(0.20)(10)^2(0.050)^2$ M1 M1 $= 0.025$ J A1.

Exam-style 3.3 ▪ 2 marks

Sketch how the **kinetic energy** and **potential energy** vary with displacement in SHM.

Solution 3.3

Method: Energy interchange

Worked steps

KE: a **downward** parabola, maximum at $x = 0$, zero at $\pm x_0$ **B1**; PE: an **upward** parabola, zero at $x = 0$, maximum at $\pm x_0$; their sum is constant **B1**.

Exam-style 3.4 ▪ 1 mark

State what happens to the total energy if the **amplitude is doubled**.

Solution 3.4

Method: Total energy

Worked steps

It becomes **four times** larger ($E \propto x_0^2$) **B1**.

Exam-style 3.5 ▪ 2 marks

A graph of the **kinetic energy** of an oscillator against **time** repeats every 0.40 s.
(a) State the **period** of the oscillation, explaining your answer.
(b) Calculate the **angular frequency** ω of the oscillation.

Solution 3.5

Method: Energy varies at twice the frequency

Worked steps

(b) $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.80} = 7.9\ \text{rad s}^{-1}$ **M1** **A1**.

Exam-style 3.6 ▪ 2 marks

A mass hangs on a spring and oscillates vertically with simple harmonic motion. A data logger records its **kinetic energy** against time, giving a curve that repeats every 0.40 s. The mass is 0.25 kg and the amplitude of the oscillation is 4.0 cm.

(a) **Determine** the period of the oscillation of the **mass**, and explain why it is not 0.40 s.

(b) **Determine** the angular frequency of the oscillation.

(c) **Determine** the total energy of the oscillation.

(d) **Determine** the speed of the mass when its displacement is 2.0 cm.

(e) The oscillation is **lightly damped**. **State and explain** what happens to the time between successive maxima of the kinetic-energy curve, and to the height of those maxima.

Solution 3.6

Method: Energy varies at twice the frequency

Worked steps

(a) The kinetic energy is zero at **both** ends of the swing and maximum at the centre twice per cycle, so the energy curve repeats **twice** per oscillation **B1**; the period of the mass is therefore $2 \times 0.40 = 0.80$ s **B1**.
(b) $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.80}$ **M1** $= 7.9$ rad s^{-1} **A1**.
(c) The total energy equals the maximum kinetic energy, $E = \frac{1}{2}m\omega^2 x_0^2$ **M1**
$= \frac{1}{2}(0.25)(7.85)^2(0.040)^2$ **M1** $= 1.2 \times 10^{-2}$ J **A1**.
(d) $v = \omega\sqrt{x_0^2 - x^2}$ **M1** $= 7.85\sqrt{(0.040)^2 - (0.020)^2}$ **M1**
$= 7.85 \times 0.0346 = 0.27$ m s^{-1} **A1**.

Solution 3.6

(e) Light damping barely changes the period, so the time between maxima stays about 0.40 s B1. The maxima get **lower** with time B1, because the amplitude decays and the total energy is proportional to the **square** of the amplitude, so the energy falls faster than the amplitude does B1.