Past-paper walk-through

Simple harmonic oscillations ▪ 17.1

eXercise

Questions in this set

- 9702/42 N25 Q5 ▪ 10 marks
- 9702/44 N25 Q4 ▪ 10 marks
- 9702/42 J25 Q5 ▪ 9 marks
- 9702/43 J25 Q5 ▪ 8 marks
- 9702/44 J25 Q4 ▪ 8 marks
- 9702/42 M25 Q4 ▪ 12 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 5

9702/42 N25 ▪ Q5 ▪ 10 marks

A steel ball on the end of a thin string oscillates with small oscillations, as shown in Fig. 5.1.

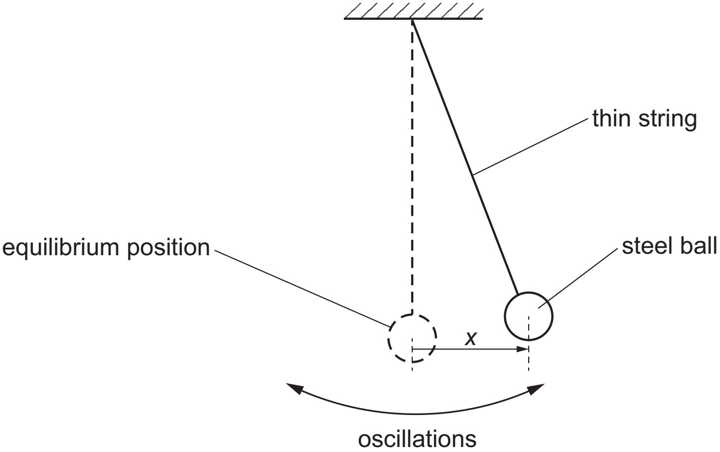
(a)(i) Explain how Fig. 5.2 shows that the oscillations of the ball are simple harmonic. **[2]**

(a)(ii) Determine the period T of the oscillations. **[3]**

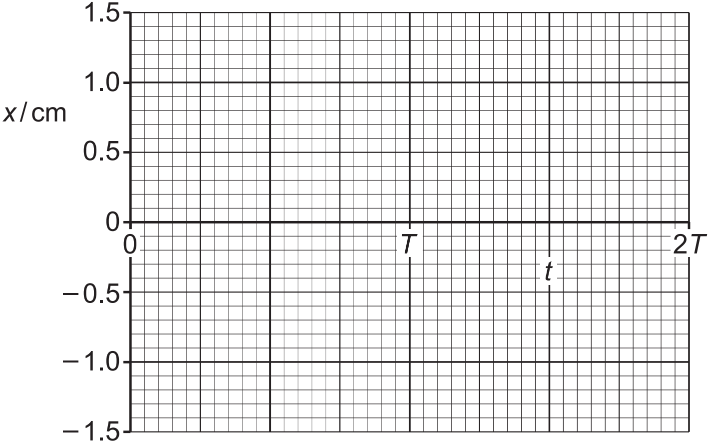
(b)(i) State what is meant by damping. **[2]**

(b)(ii) On Fig. 5.3, sketch a possible variation of the displacement x of the ball with t between $t = 0$ and $t = 2T$. **[3]**

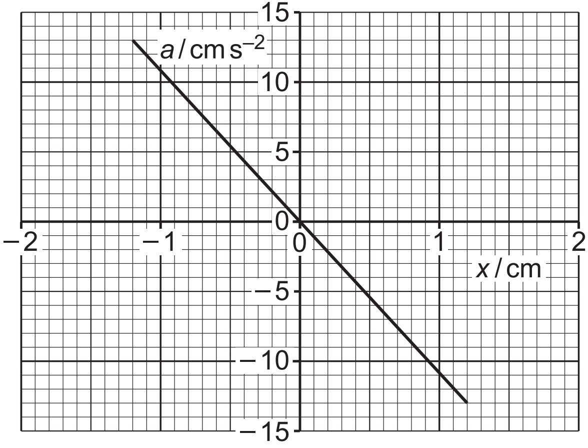
Question 5


thin string
equilibrium position
steel ball
x
oscillations

Question 5



Question 5



Solution 5

(a)(i)

Mark scheme

- straight line through the origin shows that a is proportional to x **B1**
- negative gradient shows that a is always in the opposite direction to x **B1**

Solution 5

(a)(ii) Worked steps

Use the relation that defines simple harmonic motion, applied to the two maxima.

- For SHM, $a = -\omega^2 x$, so the graph's gradient gives ω^2 and the maxima satisfy $a_0 = \omega^2 x_0$.
- Read x_0 and a_0 off Fig. 5.2 and rearrange: $\omega = \sqrt{\dfrac{a_0}{x_0}}$.
- Then $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{x_0}{a_0}}$.
- With the values from the graph, $T = 2\pi\sqrt{\dfrac{1.2}{13}} = 1.9$ s.

Solution 5

Keep x_0 and a_0 in **matching** units — the ratio is what matters, so reading both off the same graph in the units printed on its axes is safest.

Mark scheme

- a0 $=$ ω2x0 **C1**
- ω $=$ 2π / T **C1**
- T $=$ 2π $\sqrt{}$(x0 / a0)
- $=$ 2π $\sqrt{}$ (1.2 / 13)
- $=$ 1.9 s **A1**

Solution 5

(b)(i)

Mark scheme

- loss of energy of oscillations **B1**
- due to resistive force(s) **B1**

Solution 5

(b)(ii) Worked steps

Do not sketch a decaying wave here. Read the three things the mark scheme asks for, and each one rules an ordinary damped oscillation out:

1. The curve **starts at** $x = 1.2$ **cm** when $t = 0$ — the amplitude found in part (a), because the ball is released from its maximum displacement.
2. It stays **entirely on one side of the** t**-axis** for the whole interval 0 to $2T$ — so the ball never passes through the equilibrium position at all.
3. Both the **magnitude of** x **and the magnitude of the gradient decrease continuously** — it creeps back towards zero, more and more slowly, and never turns round.

Solution 5

A curve that never crosses the axis is not lightly damped: this is **heavy (over) damping**. So the sketch is a single falling curve from 1.2 cm that flattens towards the t-axis and approaches it without touching or crossing it.

Solution 5

Two ways to lose all three marks at once: drawing a sine wave inside a decaying envelope (that crosses the axis every half period), and starting the curve at the origin. The displacement is a maximum at $t = 0$, not zero.

Mark scheme

- line starting from x = ±1.2 cm at t = 0 **B1**
- line starting from non-zero value of x from t = 0 to t = 2T that is entirely either above or below the t-axis **B1**
- curve from t = 0 starting from non-zero x value, with both magnitude of x value and magnitude of gradient continuously decreasing **B1**

Question 4

9702/44 N25 ▪ Q4 ▪ 10 marks

(a) State what is meant by the frequency of the oscillations of an oscillating object. **[1]**

(b)(i) Explain how Fig. 4.2 shows that the period of the oscillations is 0.80 s. **[1]**

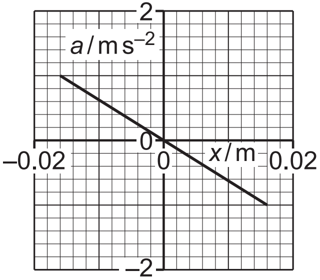
(b)(ii) Calculate the angular frequency ω of the oscillations. **[2]**

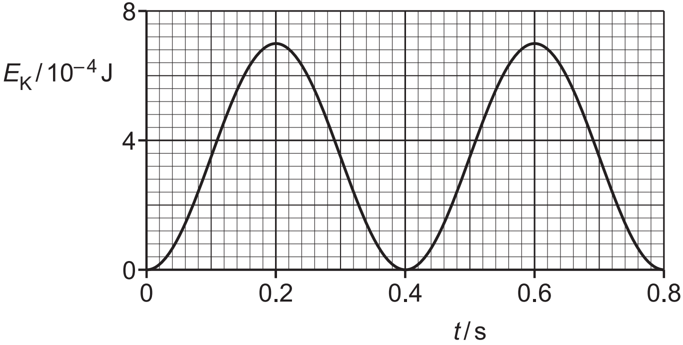
(b)(iii) Apart from the period, frequency and angular frequency of the oscillations, determine **three** other conclusions about the object and its oscillations that may be drawn from Fig. 4.1 and Fig. 4.2. The conclusions may be qualitative or quantitative. Use the space below for any working.

1. 2. 3. **[3]**

(b)(iv) Describe the interchange between kinetic energy and potential energy during the oscillations. Numerical values are **not** required. **[3]**

Question 4





Solution 4

(a)

Mark scheme

- number of oscillations per unit time **B1**

Solution 4

(b)(i)

Mark scheme

- kinetic energy (of object) reaches maximum / minimum / zero twice in a cycle **A1**

Solution 4

(b)(ii) Worked steps

- $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.80}$
- $\omega = 7.9\ \text{rad s}^{-1}$

Solution 4

Check it against the other data: $a_0 = \omega^2 x_0 = 7.9^2 \times 0.016 = 1.0$ m s^{-2}, which is the value given — so the two readings agree.

Mark scheme

- $\omega = 2\pi$ / T
- $= 2\pi$ / 0.80 or a0 $= \omega$2x0
- $\omega = \sqrt{}(1.0$ / $0.016)$ **C1**
- $\omega = 7.9$ rad s–1 **A1**

Solution 4

(b)(iii) Worked steps

The question asks for conclusions **other than** the period, frequency and angular frequency, so quote quantities you can compute from the graph.

- The oscillations are **simple harmonic** (the acceleration is proportional to displacement and oppositely directed).
- The **amplitude** is 0.016 m.
- The **maximum speed** is $v_0 = \omega x_0 = 7.9 \times 0.016 = 0.13$ m s^{-1}.
- The **total energy** is $\frac{1}{2} m\omega^2 x_0^2 = 7.0 \times 10^{-4}$ J.
- The **mass** follows from $a_0 = \dfrac{F_0}{m}$: $m = 0.087$ kg.

Solution 4

Any three earn the marks — but each must be a quantity with a value, not a restatement of the graph. **Mark scheme**

- Any three points from:
- ▪ oscillations are simple harmonic
- ▪ amplitude = 0.016 m
- ▪ maximum speed = 0.13 m s–1
- ▪ total energy of oscillations = 7.0 × 10–4 J
- ▪ mass of object = 0.087 kg
- ▪ maximum momentum = 0.011 kg m s–1 or 0.011 N s **B3**

Solution 4

(b)(iv)

Mark scheme

- Any two points from:
- ▪ kinetic energy is a maximum at zero displacement or kinetic energy is zero at maximum displacement
- ▪ potential energy is zero at zero displacement or potential energy is a maximum at maximum displacement
- ▪ kinetic energy is maximum when the potential energy is zero or potential energy is a maximum when the kinetic energy is zero **B2**
- kinetic energy + potential energy is constant **B1**

Question 5

9702/42 J25 ▪ Q5 ▪ 9 marks

(a) State what is meant by simple harmonic motion. **[2]**

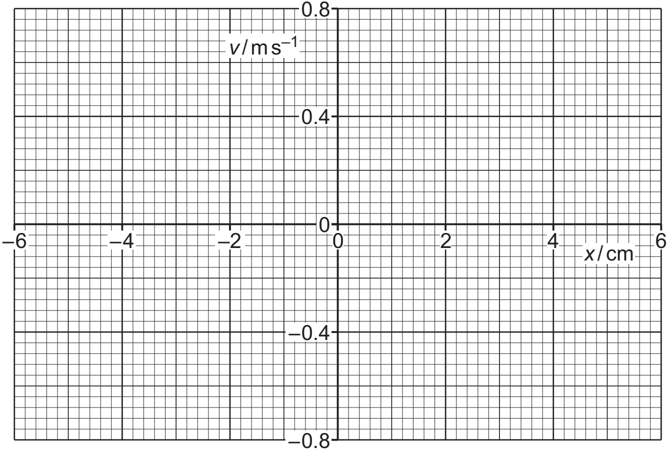
(b)(i) Calculate the period of the oscillation. **[1]**

(b)(ii) Determine the amplitude x_0 of the oscillation. **[2]**

(b)(iii) Use your answer in (b)(ii) to determine the equation for v in terms of the displacement x of the block, where v is in ms^{-1} and x is in m. **[1]**

(b)(iv) On Fig. 5.1, sketch the variation of v with x. **[3]**

Question 5



Solution 5

(a)

Mark scheme

- (motion in which) acceleration is (directly) proportional to displacement **B1**
- (motion in which) acceleration is (always) in the opposite direction to displacement or acceleration is (always) directed towards a fixed point **B1**

Solution 5

(b)(i)

Worked steps

- $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{16}$
- $T = 0.39$ s

Mark scheme

- period $= 2\pi$ / 16
- $= 0.39$ s **A1**

Solution 5

(b)(ii)

Worked steps

The maximum speed occurs at the equilibrium position, where $v_0 = \omega x_0$.

- $x_0 = \dfrac{v_0}{\omega} = \dfrac{0.56}{16}$
- $x_0 = 0.035$ m

Mark scheme

- v0 $= \omega$x0 or v0 $= \omega($
-)
- 2
- 2
- 0
- 0 x
- − C1

Solution 5

(b)(iii) Worked steps

The general SHM speed equation carries the amplitude you just found.

- $v = \pm\omega\sqrt{x_0^2 - x^2}$
- $v = \pm 16\sqrt{0.035^2 - x^2}$

Solution 5

Check it at the two ends: at $x = 0$ it gives ± 0.56 m s^{-1}, the maximum speed; at $x = x_0$ it gives zero, as it must at the extremes of the motion.

Mark scheme

- v $=$ $\pm$16 $\sqrt{}$(0.0352 – x2) **A1**

Solution 5

(b)(iv) Worked steps

A v–x graph for simple harmonic motion is an **ellipse**, not a wave, because $v = \pm\omega\sqrt{x_0^2 - x^2}$ — for every displacement there are two speeds, equal and opposite.
Draw it from the three conditions the mark scheme names:

1. A **closed loop around the origin** — the oscillator returns to every state, so the curve joins up.
2. It **crosses** $v = 0$ **at** $x = \pm 3.5$ **cm** — the amplitude, where the object is momentarily at rest before turning round.
3. It **crosses** $x = 0$ **at** $v = \pm 0.56$ $\mathbf{m\,s^{-1}}$ — the maximum speed, at the equilibrium position.

Solution 5

Those four intercepts fix the ellipse completely, so mark them first and then join them with a smooth curve.

The two ways to lose marks are drawing a sine curve (that is v against **time**, a different graph) and drawing only the upper half. The motion runs both ways, so the loop must be complete and symmetrical about both axes.

Mark scheme

- closed loop surrounding the origin **B1**
- loop crosses $v = 0$ at maximum values of x at $x = \pm\ 3.5$ cm **B1**
- loop crosses $x = 0$ at maximum values of v at $v = \pm\ 0.56$ m s–1 **B1**

Question 5

9702/43 J25 ▪ Q5 ▪ 8 marks

5 A cuboidal block floats in a liquid with its base horizontal, as shown in Fig. 5.1.

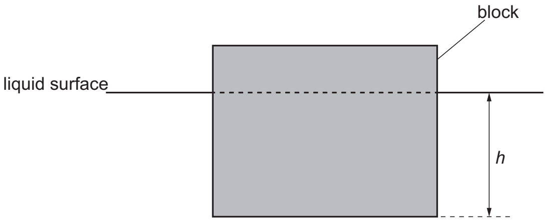


Fig. 5.1

The base of the block is at a depth *h* below the surface of the liquid.

The block is displaced downwards by a small distance and then released so that it oscillates

Question 5

The block is displaced downwards by a small distance and then released so that it oscillates.

Fig. 5.2 shows the variation with *h* of the acceleration *a* of the block.

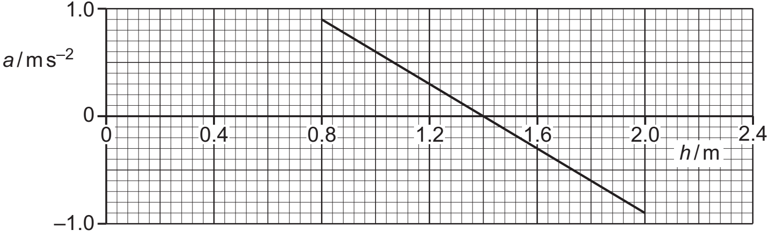


Fig. 5.2

Fig. 5.3 shows the variation with *h* of the kinetic energy E_K of the block.

Question 5

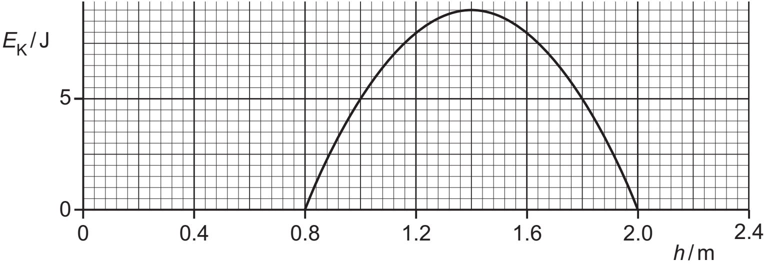


Fig. 5.3

(a) **(i)** Determine the amplitude of the oscillations.

Question 5

(ii) State what the line in Fig. 5.2 shows about the nature of the oscillations.

Question 5

(b) State **three** other quantitative conclusions that can be drawn from Fig. 5.2 and Fig. 5.3 about the block and its oscillations. Use the space for any working.

[3]

(c) On Fig. 5.4, sketch the variation with h of the potential energy E_P of the oscillations.

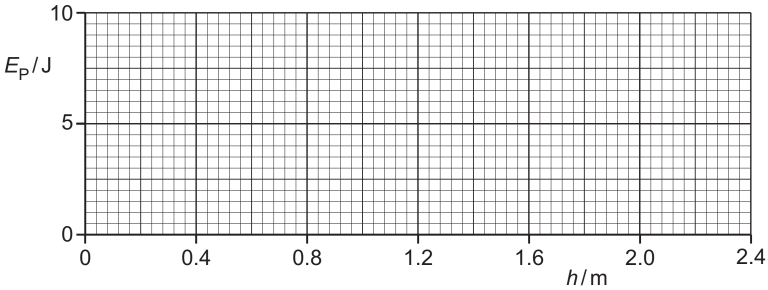


Fig. 5.4

[3]

Question 5

Question 5

[Total: 8]

Solution 5

(a)(i)

Worked steps

The amplitude is half the total travel, not the whole of it.

- The ball moves between $h = 0.8$ m and $h = 2.0$ m, a range of 1.2 m.
- amplitude $= \dfrac{1.2}{2} = 0.60$ m

Mark scheme

- amplitude = 0.60 m

Solution 5

(a)(ii)

Mark scheme

- oscillations are simple harmonic

Solution 5

(b)

Worked steps

Three conclusions, each a stated quantity:

- The **equilibrium position** is at the centre of the motion, $h = 1.4$ m.
- The **total energy** of the oscillation is 9.0 J — the value the curve reaches at either extreme, where all of it is potential.
- The **angular frequency** is 1.2 rad s^{-1} (equivalently, a period of $2\pi/1.2 = 5.2$ s).

Mark scheme

- Any three points from:
- ▪
- mean / equilibrium position is at h = 1.4 m
- ▪
- total energy of oscillations = 9.0 J
- ▪

Solution 5

(c) Worked steps

Energy in SHM goes as displacement squared, so the curve is a parabola in h.

- Draw a **U-shape** with its minimum at the equilibrium position, $h = 1.4$ m, where the potential energy is **zero**.
- It rises symmetrically to 9.0 J at both ends, $h = 0.8$ and $h = 2.0$ m.
- The curve rests on the h axis at its minimum — it does not dip below, because energy measured from equilibrium cannot be negative.

Solution 5

Mark scheme

- U-shaped curve resting on h axis (with minimum at $EP = 0$)
- curve from $h = 0.8\,m$ to $h = 2.0\,m$, with minimum EP shown at $h = 1.4\,m$
- both end-points of curve shown at $EP = 9.0\,J$
- 9702/43
- May/June 2025

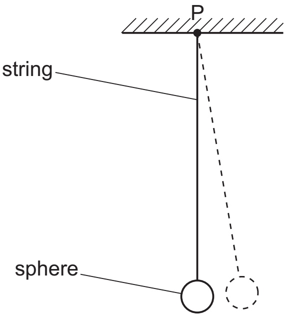
Question 4

9702/44 J25 ▪ Q4 ▪ 8 marks

4 **(a)** State what is meant by simple harmonic motion.

Question 4

(b) A small sphere is suspended from a fixed point P by a string of negligible mass, as shown in Fig. 4.1.

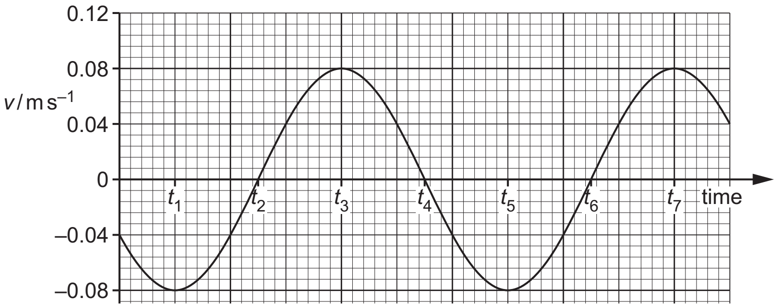


Question 4

Fig. 4.1

The sphere is given a small horizontal displacement and is then released.

The variation with time of the horizontal velocity v of the sphere is shown in Fig. 4.2.



Question 4

Question 4

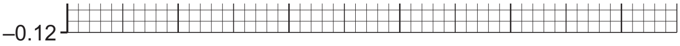


Fig. 4.2

(i) State two times at which the sphere is passing in the same direction through the equilibrium position.

Question 4

(ii) The time interval between t_1 and t_6 is 2.2 s.

Calculate the frequency of oscillation of the sphere.

Question 4

(c) The sphere in **(b)** is undergoing simple harmonic motion.

Use your answer in **(b)(ii)** and data from Fig. 4.2 to determine the maximum displacement of the sphere from its equilibrium position.

Question 4

[Total: 8]

Solution 4

(a)

Mark scheme

- (motion in which) acceleration is (directly) proportional to displacement
- (motion in which)
- acceleration is (always) in the opposite direction to displacement
- or
- acceleration is (always) directed towards a fixed point

Solution 4

(b)(i)

Mark scheme

- t1 and t5
- or
- t3 and t7

Solution 4

(b)(ii) Worked steps

Get the period from the trace first, then invert it.

- The period is $\dfrac{2.2}{1.25}$ in the units of the trace, so $f = \dfrac{1}{T} = \dfrac{1.25}{2.2}$
- $f = 0.57$ Hz

Solution 4

Read the period over **several** cycles and divide, rather than measuring one — that is what the 1.25 is doing, and it is where the accuracy comes from.

Mark scheme

- $f = 1 / T$
- period $= 2.2 / 1.25$
- $f = 1.25 / 2.2$
- $= 0.57$ Hz

Solution 4

(c) Worked steps

Maximum speed, angular frequency and amplitude are linked by one relation.

- $v_0 = \omega x_0$, and $\omega = 2\pi f$, so $v_0 = 2\pi f x_0$.
- $0.080 = 2\pi \times 0.57 \times x_0$
- $x_0 = \dfrac{0.080}{3.58} = 0.022$ m

Use ω, not f: forgetting the 2π gives 0.14 m, six times too large. And carry your own frequency forward from (b)(ii) — the mark is for the method.

Solution 4

A 22 mm amplitude at 0.57 Hz is a slow, small oscillation, which is consistent with a maximum speed of only 8 cm s ^{1}.

Mark scheme

- $v0 = \omega x0$ and $\omega = 2\pi f$
- $0.080 = 2\pi \times 0.57 \times x0$
- $x0 = 0.022\,\mathrm{m}$
- 9702/44
- May/June 2025

Question 4

9702/42 M25 ▪ Q4 ▪ 12 marks

4 A small crystal is made to vibrate with simple harmonic motion. The variation with time t of the displacement x of one surface of the crystal from its equilibrium position is shown in Fig. 4.1.

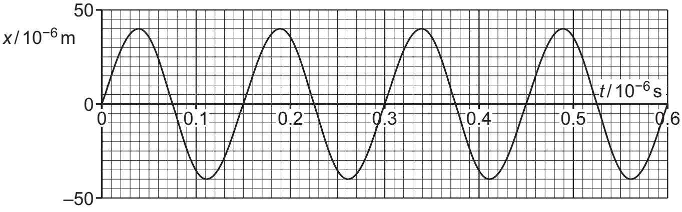


Fig. 4.1

(a) Show that the angular frequency of the vibration of the surface is 4.2×10^{7} rad s^{-1}.

Question 4

[2]

(b) Determine the maximum acceleration a_0 of the vibration of the surface.

Question 4

(c) The crystal may be modelled as a single mass of 2.4×10^{-4} kg that vibrates as shown in Fig. 4.1.

Calculate the total energy E of the vibrations.

Question 4

(d) The crystal generates ultrasound waves that are used to obtain diagnostic information about internal structures.

(i) The crystal is made from piezoelectric material.

Explain how the crystal is made to vibrate.

Question 4

(ii) A parallel beam of ultrasound waves is incident on a muscle-bone boundary. Data for muscle and bone are given in Table 4.1.

Table 4.1

material	density / $kg\,m^{-3}$	speed of sound / $m\,s^{-1}$
muscle	1100	1600
bone	1900	4100

Calculate the percentage of the intensity of the ultrasound beam that is transmitted at this boundary.

Question 4

Question 4

[Total: 12]

Solution 4

(a) Worked steps

Angular frequency comes straight from the period.

- $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.15 \times 10^{-6}}$
- $\omega = 4.2 \times 10^{7}\ \mathrm{rad\,s^{-1}}$

Solution 4

The period is in **microseconds**, so keep the 10^{-6}. Dropping it gives 42 rad s ^{1}, which is a factor of a million out and makes every later part wrong.

Mark scheme

- ω = 2π / T
- = 2π / (0.15 × 10–6) = 4.2 × 107 rad s–1

Solution 4

(b) Worked steps

For simple harmonic motion the acceleration is greatest at maximum displacement.

- $a_0 = \omega^2 x_0$
- $a_0 = (4.2 \times 10^7)^2 \times 40 \times 10^{-6}$
- $a_0 = 7.1 \times 10^{10}\ \mathrm{m\,s^{-2}}$

Square ω, not x_0 — this is the most common slip in the whole topic. And the amplitude is in micrometres, so it too carries a 10^{-6}.

Solution 4

An acceleration of 10^{10} m s^{-2} looks absurd until you notice the frequency: about 6.7 MHz, with the crystal face turning round more than six million times a second.

Mark scheme

- a0 $=$ ω2x0
- $=$ (4.2 $\times$ 107)2 $\times$ 40 $\times$ 10–6
- $=$ 7.1 $\times$ 1010 m s–2

Solution 4

(c) Worked steps

The total energy of an oscillator is its kinetic energy at maximum speed, and $v_0 = \omega x_0$.

- $E = \frac{1}{2} m\omega^2 x_0^2$
- $E = \frac{1}{2} \times 2.4 \times 10^{-4} \times (4.2 \times 10^7)^2 \times (40 \times 10^{-6})^2$
- $E = 340$ J

Solution 4

Both ω and x_0 are squared here, which is what distinguishes this formula from the acceleration one in (b). Carry the unrounded ω forward; using 4.2 rather than 4.19 moves the answer by about one per cent, which is inside the accepted range but only just.

Mark scheme

- E = ½mω2xo2
- = ½ × 2.4 × 10–4 × (4.2 × 107)2 × (40 × 10–6)2
- = 340 J

Solution 4

(d)(i)

Mark scheme

- apply alternating p.d. (to / across crystal)
- applying p.d. to / across crystal causes it to distort

Solution 4

(d)(ii) Worked steps

Reflection at a boundary is decided by the two **specific acoustic impedances**, $Z = \rho c$.

- Muscle: $Z_\mathrm{m} = 1100 \times 1600 = 1.76 \times 10^6\ \mathrm{kg\,m^{-2}\,s^{-1}}$
- Bone: $Z_\mathrm{b} = 1900 \times 4100 = 7.79 \times 10^6\ \mathrm{kg\,m^{-2}\,s^{-1}}$
- $\dfrac{I_\mathrm{r}}{I_0} = \left(\dfrac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2 = \left(\dfrac{7.79 - 1.76}{7.79 + 1.76}\right)^2$
- $= 0.40$, that is 40%

Three things to check every time: the difference goes on top and the sum underneath, the whole bracket is squared, and the answer is a ratio with no unit. A coefficient greater than 1 means the fraction was inverted. Mark scheme

Solution 4

- $Z = \rho$ c
- Zm $=$ 1100 $\times$ 1600 ($=$ 1.76 $\times$ 106)
- Zb $=$ 1900 $\times$ 4100 ($=$ 7.79 $\times$ 106)
- intensity reflection co-efficient$=$ [(7.79 – 1.76) / (7.79 $+$ 1.76)]2
- $=$ 0.40 or 40%
- percentage transmitted $=$ 60%
- 9702/42
- February/March 2025