Past-paper walk-through

Equation of state ▪ 15.2

eXercise

Questions in this set

- 9702/42 J25 Q4 ▪ 8 marks
- 9702/44 J25 Q3 ▪ 10 marks
- 9702/43 J23 Q4 ▪ 12 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

9702/42 J25 ▪ Q4 ▪ 8 marks

(a)(i) State the meaning, in the equation, of the symbol T. **[1]**

(a)(ii) Identify the constant B. **[1]**

(b)(i) State the meanings, in this equation, of the symbols m and $\langle c^2 \rangle$.

$\langle c^2 \rangle$: **[2]**

(b)(ii) Use the equations in (a) and (b) to derive an expression, in terms of A, B and T, for the mean kinetic energy E_K of a molecule of the gas. **[2]**

(c) On Fig. 4.1, sketch the variation with T of the root-mean-square (r.m.s.) speed of the molecules of an ideal gas. **[2]**

Solution 4

(a)(i)

Mark scheme

- thermodynamic temperature **B1**

(a)(ii)

Mark scheme

- molar gas constant **B1**

Solution 4

(b)(i)

Mark scheme

- m:
- mass of one molecule (of the gas) **B1**
- $\langle c2 \rangle$: mean-square speed (of molecules) **B1**

Solution 4

(b)(ii) Worked steps

This is a *derive* question, so nothing is substituted — the two equations from (a) and (b) are set equal and rearranged.

- The kinetic-theory result: $pV = \frac{1}{3}Nm\langle c^2\rangle$.
- The gas law in the question's constants: $pV = \dfrac{NBT}{A}$.
- The left-hand sides are the same pV, so $\dfrac{NBT}{A} = \frac{1}{3}Nm\langle c^2\rangle$.
- N cancels, leaving $m\langle c^2\rangle = \dfrac{3BT}{A}$.
- Mean kinetic energy is $E_K = \frac{1}{2}m\langle c^2\rangle$, so $E_K = \dfrac{3BT}{2A}$.

Solution 4

Both marks are for the route, not the answer: one for equating the two expressions for pV, one for bringing in $E_K = \frac{1}{2}m\langle c^2\rangle$ and finishing the algebra. Write both lines out.

Solution 4

Note what the result says: the mean kinetic energy depends only on T, not on the mass of the molecule — the same statement as $E_K = \frac{3}{2}kT$, with B/A playing the part of k.

Mark scheme

- NBT / A = 1
- 3 Nm⟨c2⟩ **M1**
- clear use of EK = 1
- 2 m⟨c2⟩ leading to EK = 3BT / 2A **A1**

Solution 4

(c) Worked steps

Get the shape from the algebra, not from memory.

- $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$, so $\langle c^2\rangle \propto T$ and $c_{\text{r.m.s.}} \propto \sqrt{T}$.

A square-root graph gives the two features the mark scheme asks for:

Solution 4

1. It **passes through the origin** with a positive gradient — at $T = 0$ the molecules have no kinetic energy, so the r.m.s. speed is zero.
2. It is a **smooth curve whose gradient decreases** — doubling the speed needs four times the temperature, so the curve flattens as T rises.

Solution 4

A straight line loses the second mark, and a curve that starts above the origin loses the first. Both come from sketching $c_{\mathrm{r.m.s.}}$ against T when the proportionality is really to $\sqrt{T}$.

Mark scheme

- line with positive gradient passing through the origin **B1**
- smooth curve with decreasing positive gradient **B1**

Question 3

9702/44 J25 ▪ Q3 ▪ 10 marks

3 **(a)** State what is meant by an ideal gas.

Question 3

(b) An ideal gas at a pressure of 1.6×10^5 Pa has a density of $1.9\,\text{kg}\,\text{m}^{-3}$.

(i) Show that the root-mean-square (r.m.s.) speed of molecules of this gas is approximately $500\,\text{m}\,\text{s}^{-1}$.

Question 3

[3]

(ii) One molecule of the gas has a mass of 4.7×10^{-26} kg.

Determine the thermodynamic temperature of the gas.

Question 3

(c) Calculate the internal energy U of 6.0 mol of the gas in **(b)**. Explain your reasoning.

Question 3

[Total: 10]

Solution 3

(a)

Mark scheme

- (gas that obeys) $pV \propto T$ (at all values of p, V and T)
- where T is thermodynamic temperature

Solution 3

(b)(i) Worked steps

The kinetic-theory equation can be written in terms of **density** instead of the number of molecules, and that is what makes this a one-line calculation.

- $pV = \frac{1}{3}Nm\langle c^2\rangle$, and the density is $\rho = \dfrac{Nm}{V}$ — total mass over volume.
- Dividing through by V: $p = \frac{1}{3}\rho\langle c^2\rangle$.
- Rearranged, $c_{\text{r.m.s.}} = \sqrt{\langle c^2\rangle} = \sqrt{\dfrac{3p}{\rho}}$.
- $c_{\text{r.m.s.}} = \sqrt{\dfrac{3 \times 1.6 \times 10^5}{1.9}} = 500\ \text{m s}^{-1}$

Solution 3

Three marks for three things: the kinetic-theory equation, the substitution $\rho = Nm/V$, and the arithmetic. Show the middle step — it is what turns a formula with N and m in it, neither of which you were given, into one you can evaluate.

Mark scheme

- $\mathrm{pV} = \frac{1}{3}\,\mathrm{Nm}\langle \mathrm{c2}\rangle$ and $\mathrm{Nm} / \mathrm{V} = \rho$
- $(\mathrm{p} = \frac{1}{3}\,\rho\,\langle \mathrm{c2}\rangle\;)$
- r.m.s. speed $= \surd\langle \mathrm{c2}\rangle$
- $1.6 \times 105 = \frac{1}{3} \times 1.9 \times \langle \mathrm{c2}\rangle$ leading to r.m.s. speed $= 500\,\mathrm{m\,s{-}1}$
- or
- r.m.s. speed $= \surd(3 \times 1.6 \times 105 / 1.9) = 500\,\mathrm{m\,s{-}1}$

Solution 3

(b)(ii) Worked steps

Set the two expressions for pV equal to get the temperature.

- $pV = \frac{1}{3}Nm\langle c^2\rangle$ and $pV = NkT$, so $\frac{1}{3}m\langle c^2\rangle = kT$, i.e. $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$.
- $\frac{1}{2} \times 4.7 \times 10^{-26} \times 503^2 = \frac{3}{2} \times 1.38 \times 10^{-23} \times T$
- $T = 290$ K

Use the mass of **one molecule** here, not the density — the relation $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$ is per molecule, which is why it pairs with k rather than with R.
Carry the unrounded speed forward (503, not 500), and answer in **kelvin**: 290 K is about 17 °C, which is a sensible room temperature and a good check that nothing was dropped. **Mark scheme**

Solution 3

- $(pV =)\ \frac{1}{3} \mathrm{Nm}\langle c2\rangle = \mathrm{NkT}$
- (so) $½\ m\langle c2\rangle = (3 / 2)\, kT$
- $½ \times 4.7 \times 10–26 \times 5032 = (3 / 2) \times 1.38 \times 10–23 \times T$
- $T = 290\,K$
- or
- $pV = NkT$ and $Nm / V = \rho$
- (so) $T = pm / \rho k$
- (C1)
- $T = 1.6 \times 105 \times 4.7 \times 10–26 / (1.9 \times 1.38 \times 10–23)$
- $= 290\,K$

Solution 3

- (A1)
- 9702/44
- May/June 2025

Solution 3

(c) Worked steps

For an **ideal** gas the internal energy is entirely kinetic: the molecules have no forces between them, so there is no potential energy to add. Say that first — it is one of the three marks.

- $U = N \times \frac{1}{2} m\langle c^2 \rangle$, or equivalently $U = N \times \frac{3}{2} kT$, or per mole $U = n \times \frac{3}{2} RT$.
- Using the molar form with 6.0 mol at 287 K: $U = 6.0 \times \frac{3}{2} \times 8.31 \times 287$
- $U = 21\,000$ J

Solution 3

All three routes give the same number, so pick whichever matches the quantities you already have — moles and temperature is the shortest here.
The statement about potential energy is not decoration. It is the whole reason the internal energy of an ideal gas depends on temperature alone, and a solution that goes straight to the arithmetic loses that mark. **Mark scheme**

- potential energy (of molecules) is zero
- $U = N \times ½\ m\langle c2\rangle = 6.0 \times 6.02 \times 1023 \times ½ \times 4.7 \times 10–26 \times 5032$
- or
- $U = N \times (3 / 2)\ kT = 6.0 \times 6.02 \times 1023 \times (3 / 2) \times 1.38 \times 10–23 \times 287$
- or

Solution 3

- $U = n \times (3/2)\,RT = 6.0 \times (3/2) \times 8.31 \times 287$
- $U = 21000\,J$
- 9702/44
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Question 4

9702/43 J23 ▪ Q4 ▪ 12 marks

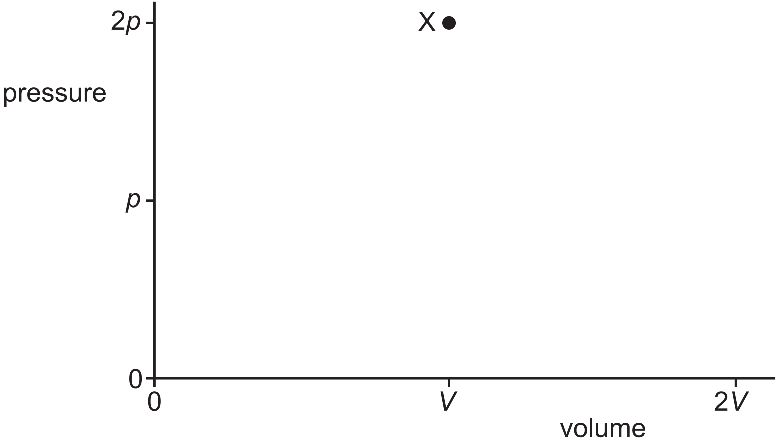
(a) State **two** of the basic assumptions of the kinetic theory of gases.

1. 2. **[2]**

Question 4

change	increase in internal energy of gas	thermal energy transferred to gas	work done on gas
X to Y			
Y to Z	$+U$		
Z to X			$+W$

Question 4



Question 4

9702/43 J23 ▪ Q4 ▪ 12 marks

(b)(i) Determine an expression for the temperature T of the gas in state X, in terms of n, p and V. Identify any other symbols that you use. **[2]**

(b)(ii) On Fig. 4.1, sketch the variation with volume of pressure for the gas as the gas undergoes the three changes. The state X is labelled. Label states Y and Z. **[3]**

(b)(iii) During the change of state from Y to Z, the increase in internal energy of the gas is U. During the change of state from Z to X, the work done on the gas is W.