Past-paper walk-through

Thermal equilibrium ▪ 14.1

eXercise

Questions in this set

- 9702/44 J25 Q2 ▪ 7 marks
- 9702/42 M25 Q3 ▪ 12 marks
- 9702/43 J23 Q3 ▪ 11 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

9702/44 J25 ▪ Q2 ▪ 7 marks

2 **(a)** State what is meant by two objects being in thermal equilibrium.

Question 2

(b) A mass X of ice at $0\,°C$ is placed in a beaker containing a mass M of water at Celsius temperature t. The beaker is perfectly insulated and has negligible heat capacity. After some time, the ice that was added reaches thermal equilibrium with the original water in the beaker.

The specific latent heat of fusion of water is L. The specific heat capacity of water is c.
The final Celsius temperature of the system is θ.

Give expressions, in terms of some or all of X, M, t, θ, L and c, for the thermal energy:

(i) E_1, gained by the ice as it melts to become water at $0\,°C$

Question 2

(ii) E_2, lost by the water as its Celsius temperature decreases from t to θ

Question 2

(iii) E_3, gained by the melted ice as its Celsius temperature increases from 0 °C to θ.

Question 2

(c) Use your answers in **(b)** to show that the final Celsius temperature θ of the system is given by

$$\theta = \frac{Mct - XL}{c(M + X)}.$$

[2]

[Total: 7]

Solution 2

(a)

Mark scheme

- same temperature (as each other)
- no net transfer of thermal energy (between them)

Solution 2

(b)(i)

Worked steps

Melting takes energy but no temperature change, so the specific **latent heat** applies and there is no $\Delta\theta$ in the expression.

- $E_1 = XL$

X is the mass of ice and L its specific latent heat of fusion. Every gram must melt before any of it can warm up, which is why this term stands on its own.

Mark scheme

- E1 = XL

Solution 2

(b)(ii)

Worked steps

The warm water **cools**, so its temperature change is from its starting value t down to the final value θ.

- $E_2 = Mc(t - \theta)$

Write the change as $(t - \theta)$, in that order: the water is losing energy, and this expression is the positive amount it loses. Reversing the bracket makes every later line change sign.

Mark scheme

- E2 = Mc(t – θ)

Solution 2

(b)(iii)

Worked steps

Once melted, the ice-water starts at 0 °C and has to be warmed to the same final temperature as everything else.

- $E_3 = Xc\theta$

The mass is X — the melt-water is the ice, now liquid — and the temperature change is $\theta - 0 = \theta$. This is the term most often left out of the energy balance in part (c), and writing it here separately is what the question is setting up.

Mark scheme

- E3 $=$ Xcθ

Solution 2

(c) Worked steps

Nothing is substituted here — this is a derivation, so name the three energy transfers, write the conservation statement, and rearrange.

- E_1: energy to **melt** the ice, XL.
- E_2: energy **lost by the warm water** as it cools from t to the final temperature θ, that is $Mc(t-\theta)$.
- E_3: energy to **warm the melt-water** from 0 °C to θ, that is $Xc\theta$.

Energy is conserved, and the water supplies both of the other two:

- $E_2 = E_1 + E_3$, so $Mc(t-\theta) = XL + Xc\theta$.

Solution 2

Now collect the θ terms on one side:

- $Mct - Mc\theta = XL + Xc\theta$
- $Mct - XL = Xc\theta + Mc\theta = c\theta(M + X)$
- $\theta = \dfrac{Mct - XL}{c(M + X)}$

Both marks are for the route: one for the energy statement with all three terms present, one for finishing the algebra. The term most often dropped is $Xc\theta$ — the ice does not stop at melting, it then has to be warmed to the same final temperature as everything else.

Solution 2

A check on the final expression: $c(M + X)$ in the denominator is the heat capacity of **all** the water once the ice has melted, which is what the mixture ends up as.

Mark scheme

- E2 = E1 + E3
- Mc(t – θ) = XL + Xcθ
- and
- completion of algebra to reach θ = (Mct – XL) / c(M + X)
- 9702/44
- May/June 2025

Question 3

9702/42 M25 ▪ Q3 ▪ 12 marks

3 **(a)** Two metal cuboids P and Q are in thermal contact with each other.

(i) P and Q are in thermal equilibrium.

State what is meant by the term thermal equilibrium.

Question 3

(ii) Data for P and Q are given in Table 3.1.

Table 3.1

	P	Q
specific heat capacity / $J\,kg^{-1}\,K^{-1}$	390	910
mass / kg	0.54	0.37

P and Q are initially both at the same temperature.

P is supplied with 24 kJ of thermal energy. After some time, P and Q are once again both at the same temperature as each other.

P and Q are perfectly insulated from the surroundings.

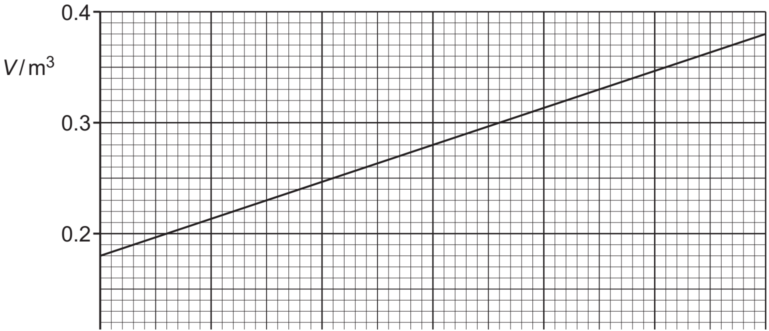
Determine the change in temperature ΔT of Q.

Question 3

Question 3

(b) Nitrogen may be assumed to be an ideal gas. A fixed amount of nitrogen gas is contained at a constant pressure of 1.6×10^5 Pa.

The variation of the volume V of the gas with the temperature θ of the gas is shown in Fig. 3.1.



Question 3

Question 3

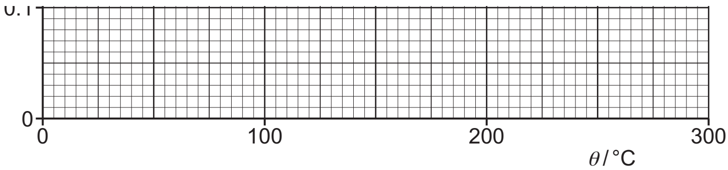


Fig. 3.1

(i) The temperature of the nitrogen gas is increased from 0 °C to 210 °C.
Determine the work done on the gas.

Question 3

Question 3

Question 3

(ii) Determine the number N of molecules of nitrogen gas.

Question 3

(iii) The mass of a nitrogen molecule is 4.7×10^{-26} kg.

Calculate the root-mean-square (r.m.s.) speed of a nitrogen molecule at 210 °C.

Question 3

[Total: 12]

Solution 3

(a)(i)

Mark scheme

- (P and Q are at the) same temperature
- no net transfer of thermal energy (between P and Q)

Solution 3

(a)(ii) Worked steps

Two materials are heated together, so they share the same temperature rise and the energy supplied is split between them.

- $Q = mc\Delta T$ for each, added: $Q = (m_1c_1 + m_2c_2)\,\Delta T$.
- $24 \times 10^3 = (0.54 \times 390 \times \Delta T) + (0.37 \times 910 \times \Delta T)$
- The bracket is $210.6 + 336.7 = 547.3$ J K^{-1}.
- $\Delta T = \dfrac{24 \times 10^3}{547.3} = 44$ K

Solution 3

Both masses take the same ΔT because they are in **thermal equilibrium** — that is the physics the question is testing, and it is why the two terms factorise. Solving them separately and adding the answers is the usual mistake.

Mark scheme

- $Q = mc\Delta T$
- $24 \times 103 = (0.54 \times 390 \times \Delta T) + (0.37 \times 910 \times \Delta T)$
- $\Delta T = 44\,K$

Solution 3

(b)(i) Worked steps

At constant pressure the work done is pressure times the change in volume, and the sign follows the direction of that change.

- $W = p\Delta V$
- $\Delta V = 0.18 - 0.32 = -0.14$ m^3
- $W = 1.6 \times 10^5 \times (-0.14) = -2.2 \times 10^4$ J

Solution 3

Subtract in the order **final minus initial**. The gas was compressed, so ΔV is negative and the work done *by* the gas is negative — work was done *on* it. Quoting a positive number here throws away the sign the question is asking about.

Mark scheme

- work done $= p\Delta V$
- $= (1.6 \times 105) \times (0.18 – 0.32)$
- $= –2.2 \times 104$ J

Solution 3

(b)(ii) Worked steps

Use the equation of state written per molecule, since the question asks for a number of molecules rather than moles.

- $pV = NkT$, so $N = \dfrac{pV}{kT}$
- $N = \dfrac{1.6 \times 10^5 \times 0.18}{1.38 \times 10^{-23} \times 273}$
- $N = 7.6 \times 10^{24}$

Solution 3

Two traps: use the state that goes with the volume you substitute (here the 0.18 m^3 one), and put the temperature in **kelvin**. Using R instead of k answers a different question — it gives moles, and would need multiplying by the Avogadro constant.

Mark scheme

- pV = NkT
- N = (1.6 × 105 × 0.18) / (1.38 × 10–23 × 273)
- = 7.6 × 1024

Solution 3

(b)(iii) Worked steps

Mean kinetic energy connects to temperature, and from it to the r.m.s. speed.

- $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$, so $c_{\text{r.m.s.}} = \sqrt{\dfrac{3kT}{m}}$
- $T = 210 + 273 = 483$ K
- $c_{\text{r.m.s.}} = \sqrt{\dfrac{3 \times 1.38 \times 10^{-23} \times 483}{4.7 \times 10^{-26}}}$
- $c_{\text{r.m.s.}} = 650\ \text{m s}^{-1}$

Solution 3

The mass in this formula is the mass of **one molecule**, not of the gas — the units are the check: $\mathrm{J\,K^{-1}} \times \mathrm{K}$ over kg gives $\mathrm{m^2\,s^{-2}}$, and the square root is a speed. And convert 210 °C to kelvin; using 210 gives 430 m $\mathrm{s^{-1}}$, which is the marked wrong answer for this question.

Mark scheme

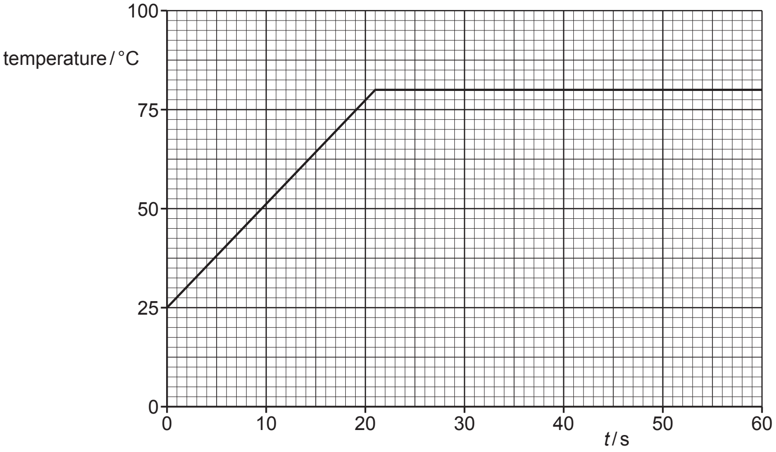
- ½m<c2> = (3 / 2)kT
- r.m.s. speed = $\sqrt{}$[(3 × 1.38 × 10–23 × (210 + 273) / (4.7 × 10–26)]
- = 650 m s–1
- 9702/42
- February/March 2025

Question 3

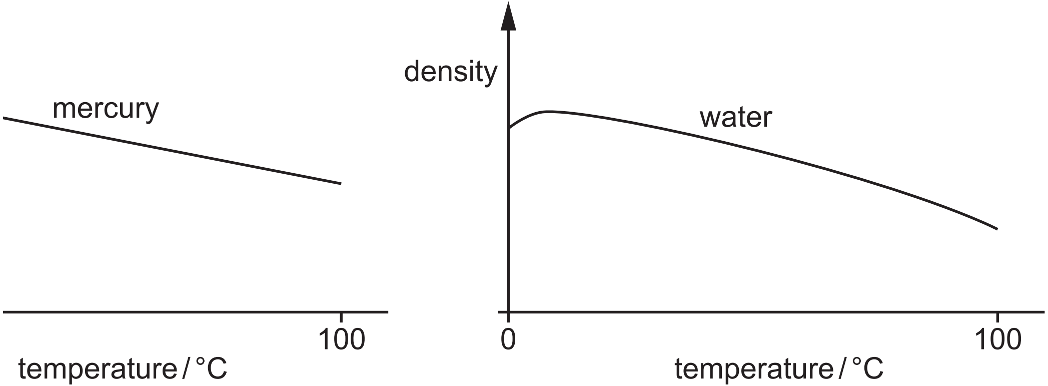
9702/43 J23 ▪ Q3 ▪ 11 marks

(a) State the reason why two objects that are at the same temperature are described as being in thermal equilibrium. **[1]**

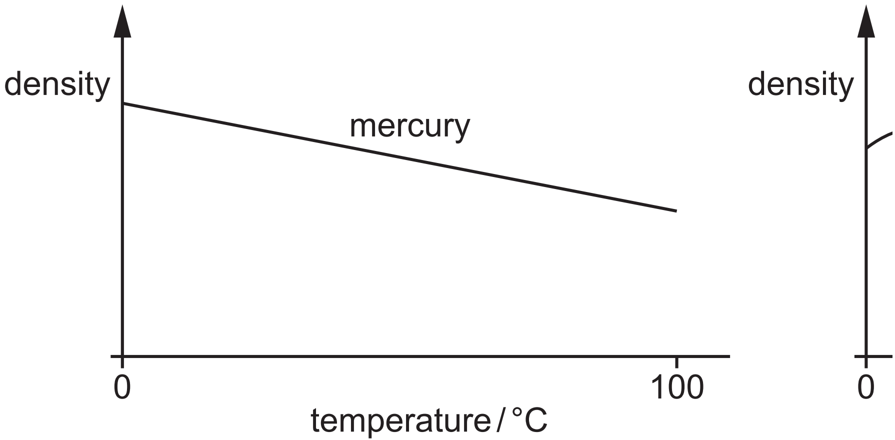
Question 3



Question 3


mercury
100
temperature / °C
density
water
0
100
temperature / °C

Question 3



Question 3

9702/43 J23 ▪ Q3 ▪ 11 marks

(b)(i) mercury is a suitable liquid **[1]**

(b)(ii) water is not a suitable liquid. **[2]**

Question 3

9702/43 J23 ▪ Q3 ▪ 11 marks

(c)(i) State the boiling temperature, in °C, of the liquid. **[1]**

(c)(ii) Determine the specific heat capacity, in $\mathrm{J\,g^{-1}\,K^{-1}}$, of the liquid. **[4]**

Question 3

9702/43 J23 ▪ Q3 ▪ 11 marks

(d) The experiment in **(c)** is repeated using water instead of the liquid in **(c)**. The mass of liquid used, the power supplied, and the initial temperature are all unchanged. The specific heat capacity of water is approximately twice that of the liquid in **(c)**. The boiling temperature of water is $100°$C.

On Fig. 3.2, sketch the variation with time t of the temperature of the water between $t = 0$ and $t = 60$ s. Numerical calculations are not required. **[2]**

Solution 3

(a)

Mark scheme

- no net thermal energy is transferred (between them)

Solution 3

(b)(i)

Mark scheme

- variation (of density with temperature) is linear
- or
- each temperature has a unique value of density

Solution 3

(b)(ii)

Mark scheme

- variation (of density with temperature) is not linear
- region where the density does not vary with temperature
- different temperatures have the same density

Any two points, 1 mark each

Solution 3

(c)(i)

Mark scheme

- boiling point $= 80\ ^\circ\mathrm{C}$

Solution 3

(c)(ii) Worked steps

Know. The heater supplies energy to **both** the liquid and the beaker holding it. Only what is left after the beaker has taken its share belongs to the liquid.

Energy supplied. $Q = Pt = 810 \times 21 = 17\,000$ J, reading the heating time as 21 s from the graph.

Energy into the beaker. $mc\Delta\theta = 42 \times 0.84 \times (80 - 25) = 1940$ J.

Energy into the liquid. $17\,010 - 1940 = 15\,070$ J.

Equation. $c = \dfrac{Q}{m\Delta\theta} = \dfrac{15\,070}{120 \times (80 - 25)}$.

Answer. $c = 2.3$ J g^{-1} K^{-1}.

Solution 3

Check. The unit asked for uses **grams**, so leave both masses in grams and do not convert. Forgetting the beaker gives 2.6 J g^{-1} K^{-1}: close enough to look right, which is why the beaker term carries its own mark. **Mark scheme**

- $Q = Pt$ and $t = 21$ s
- (thermal energy supplied $= 810 \times 21 = 17000$ J)
- $c = Q/m\Delta\theta$
- thermal energy absorbed by beaker $= 42 \times 0.84 \times (80 - 25)$
- ($= 1940$ J)
- s.h.c. of liquid $= [(810 \times 21) - (42 \times 0.84 \times (80 - 25))]/[120 \times (80 - 25)]$

Solution 3

- $= 2.3\ \mathrm{J\ g^{-1}\ K^{-1}}$

Solution 3

(d) Worked steps

Know. With the same power and the same mass, doubling the specific heat capacity halves the rate of temperature rise. Once the water boils, the temperature stops rising even though energy is still going in.

First part. A straight line from $25\ ^\circ$C, rising at about **half** the gradient of the original line.

Second part. When the line reaches $100\ ^\circ$C, it becomes **horizontal** and stays there to $t = 60$ s.

Solution 3

Check. The plateau is the mark most often lost. Energy continues to arrive during boiling, but it goes into the latent heat of vaporisation, separating the molecules rather than speeding them up, so the thermometer reading holds steady.

Mark scheme

- sketch: straight diagonal line from 25 °C to 100 °C and then horizontal at 100 °C
- straight diagonal line starting at 25 °C with gradient approximately half that of the original line