Exercise walk-through

Gravitational field of a point mass ▪ 13.3

eXercise

You must be able to

- derive and use $g = \frac{GM}{r^2}$
- describe how g varies with distance
- explain why g is nearly constant near the Earth's surface

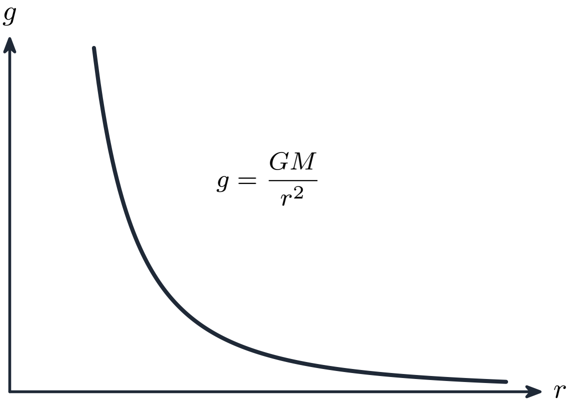
Method — Field of a point mass

Put $F = \dfrac{GMm}{r^2}$ into $g = F/m$:

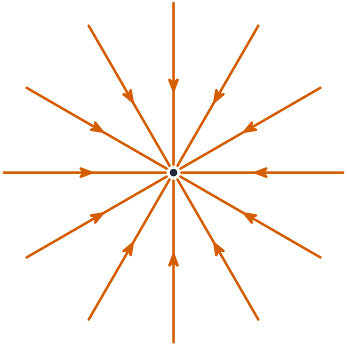
$$g = \frac{GM}{r^2}.$$

So g falls off as an **inverse square** of distance:

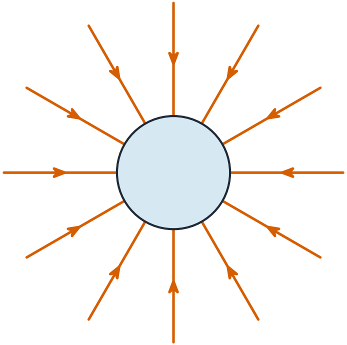
Method — Field of a point mass



Method — Field of a point mass



point mass



uniform sphere

Field lines around a point mass

Method — Nearly constant near the surface

Rising a height $h \ll R$ changes r from R to $R + h$, but $(R + h)/R \approx 1$, so g barely changes — it is effectively constant in a lab.

Practice 2.1 ▪ 1 mark

Write the equation for the gravitational field strength g due to a point mass.

Solution 2.1

Method: Field of a point mass

Worked steps

$$g = \frac{GM}{r^2} \quad \boxed{\mathbf{B1}}.$$

Practice 2.2 ▪ 1 mark

State how g varies with distance r from a point mass.

Solution 2.2

Method: Field of a point mass

Worked steps

$g \propto \dfrac{1}{r^2}$ (inverse-square) B1.

Practice 2.3 ▪ 2 marks

At the Earth's surface $g = 9.8$ N kg^{-1}. Find g at a distance of $2R$ from the centre (a height R above the surface).

Solution 2.3

Method: Nearly constant near the surface

Worked steps

At $2R$, g is $\dfrac{1}{2^2} = \dfrac{1}{4}$ of the surface value: $g = 9.8/4 = 2.45$ N kg^{-1} **M1** **A1**.

Practice 2.4 ▪ 1 mark

Explain why g is **nearly constant** near the Earth's surface.

Solution 2.4

Method: Nearly constant near the surface

Worked steps

A small height change is **tiny compared with the Earth's radius**, so r (and hence $g \propto 1/r^2$) barely changes B1.

Exam-style 3.1 ▪ 1 mark

At **twice** the distance from a point mass, g becomes:

A twice as large

B half as large

C one quarter as large

D four times as large

Solution 3.1

Method: Field of a point mass

A twice as large

B half as large

C one quarter as large ✓

D four times as large

Worked steps

C — one quarter.

Exam-style 3.2 ▪ 3 marks

The Moon has mass 7.3×10^{22} kg and radius 1.7×10^{6} m. Calculate g at its surface. ($G = 6.67 \times 10^{-11}$.)

Solution 3.2

Worked steps

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(7.3 \times 10^{22})}{(1.7 \times 10^{6})^{2}} \quad \text{M1} \; \text{M1} \quad = 1.7\ \text{N kg}^{-1} \quad \text{A1}.$$

Exam-style 3.3 ▪ 2 marks

Derive $g = \frac{GM}{r^2}$ from Newton's law of gravitation and $g = F/m$.

Solution 3.3

Worked steps

Force on a test mass m: $F = \dfrac{GMm}{r^2}$ **M1**; field strength $g = \dfrac{F}{m} = \dfrac{GM}{r^2}$ **A1**.

Exam-style 3.4 ▪ 1 mark

A planet has the same mass as Earth but **twice** the radius. State its surface g as a fraction of Earth's.

Solution 3.4

Method: Field of a point mass

Worked steps

$\dfrac{1}{4}$ of Earth's (since $g \propto 1/R^2$) B1.

Exam-style 3.5 ▪ 2 marks

On the axes, **sketch** how the gravitational field strength g of a planet varies with distance r from its centre, for r from the surface ($r = R$) out to $r = 4R$. Mark the surface value g_0.

Solution 3.5

Method: Field of a point mass

Worked steps

A curve **starting at** g_0 at $r = R$ and **falling** as an inverse square — to $g_0/4$ at $2R$, $g_0/9$ at $3R$, $g_0/16$ at $4R$ — approaching (but never reaching) zero **B1** **B1** *for a falling curve from g_0, for the correct inverse-square shape, e.g. $\approx g_0/4$ at $2R$.*

Exam-style 3.6 ▪ 1 mark

A planet of mass M and radius R has a gravitational field strength of g_0 at its surface. A satellite of mass m orbits it in a circle of radius r, where $r > R$.

(a) **State** why the gravitational field strength inside a **uniform** sphere behaves differently from the field outside it.

(b) **Show that** the orbital speed of the satellite is $v = \sqrt{\dfrac{GM}{r}}$, and hence that it does **not** depend on the mass of the satellite.

(c) **Derive** an expression for the period T of the orbit in terms of G, M and r, and state the name of the law it expresses.

(d) A geostationary satellite orbits the Earth once every 24 hours. Taking $GM = 4.00 \times 10^{14}$ m^3 s^{-2}, **determine** the radius of its orbit.

(e) **State** two further conditions a geostationary orbit must satisfy, beyond having the right period.

Solution 3.6

Method: Field of a point mass

Worked steps

(a) Outside the sphere all its mass may be treated as concentrated at the centre, giving $g \propto \frac{1}{x^2}$; inside, only the mass **closer to the centre** than the point contributes, so the field falls towards zero at the centre instead of rising **B1**.

(b) The gravitational force provides the centripetal force: $\frac{GMm}{r^2} = \frac{mv^2}{r}$ **M1**. The satellite mass m appears on both sides and **cancels** **M1**, leaving $v^2 = \frac{GM}{r}$, so $v = \sqrt{\frac{GM}{r}}$ **A1**.

Solution 3.6

(c) $T = \dfrac{2\pi r}{v} = 2\pi r\sqrt{\dfrac{r}{GM}}$ M1, so $T^2 = \dfrac{4\pi^2 r^3}{GM}$ A1 — that is $T^2 \propto r^3$, **Kepler's third law** B1.

(d) $T = 24 \times 3600 = 86\,400$ s M1;

$r^3 = \dfrac{GMT^2}{4\pi^2} = \dfrac{4.00 \times 10^{14} \times (86\,400)^2}{4\pi^2} = 7.56 \times 10^{22}$ M1; $r = 4.2 \times 10^7$ m A1.

(e) It must orbit **above the equator**, in the plane of the equator B1, and travel **west to east**, the same direction as the Earth's rotation B1.