Past-paper walk-through

Gravitational force between point masses ▪ 13.2

eXercise

Questions in this set

- 9702/41 J25 Q1 ▪ 9 marks
- 9702/43 J25 Q1 ▪ 9 marks
- 9702/43 N24 Q1 ▪ 12 marks
- 9702/42 J23 Q1 ▪ 11 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

9702/41 J25 ▪ Q1 ▪ 9 marks

(a) Define gravitational potential at a point. **[2]**

(b)(i) Show that the mass of Mars is 6.4×10^{23} kg. **[3]**

(b)(ii) Calculate the gravitational potential ϕ at the surface of Mars. Give a unit with your answer. **[2]**

(c)(i) The orbit has a period of 25 hours.

State what can be deduced from this about the rotation of Mars on its axis. **[1]**

(c)(ii) State **one** other feature of this orbit. **[1]**

Solution 1

(a)

Mark scheme

- work done per unit mass **B1**
- work (done in) moving mass from infinity (to the point) **B1**

Solution 1

(b)(i) Worked steps

The orbital radius is measured from the **centre** of Mars, so add the planet's radius to the orbit's height before anything else.

- $r = 3.4 \times 10^6 + 1.7 \times 10^6 = 5.1 \times 10^6$ m
- For a circular orbit, gravity provides the centripetal force: $\dfrac{GMm}{r^2} = mr\omega^2$, so $GM = r^3\omega^2$ with $\omega = \dfrac{2\pi}{T}$.
- Rearranged: $M = \dfrac{4\pi^2 r^3}{GT^2}$.
- Substituting the values gives $M = 6.4 \times 10^{23}$ kg, as required.

Solution 1

Forgetting to add the radius is the single commonest error here, and it changes the answer by a factor of about two.

Mark scheme

- evidence of addition of 3.4×106 to 1.7×106 or 6.8×106 **C1**
- $GM \times 122 / (5.1 \times 106)$ or $GM \times 122 / (10.2 \times 106)$ **C1**
- $6.67 \times 10–11 \times M \times 122 \times [(5.1 \times 106)–1 – (10.2 \times 106)–1] = 5.1 \times 108$ leading to $M = 6.4 \times 1023$ kg **A1**

Solution 1

(b)(ii) Worked steps

- $\phi = -\dfrac{GM}{r}$, taken at the **surface**, so $r = 3.4 \times 10^6$ m.
- $\phi = -\dfrac{6.67 \times 10^{-11} \times 6.4 \times 10^{23}}{3.4 \times 10^6}$
- $\phi = -1.3 \times 10^7 \text{ J kg}^{-1}$

Solution 1

The unit is asked for, and the **minus sign** is part of the answer: potential is zero at infinity and negative everywhere else.

Mark scheme

- ϕ = (–) (6.67 × 10–11 × 6.4 × 1023) / (3.4 × 106) **C1**
- = –1.3 × 107 J kg–1 **A1**

Solution 1

(c)(i)

Mark scheme

- Mars takes (just under) 25 hours to rotate once on its axis **B1**

(c)(ii)

Mark scheme

- orbit is equatorial or orbit is in same direction as direction of rotation of Mars **B1**

Question 1

9702/43 J25 ▪ Q1 ▪ 9 marks

1 **(a)** Define gravitational potential at a point.

Question 1

(b) Mars is a planet that may be considered to be an isolated uniform sphere of radius 3.4×10^6 m.

A satellite of mass 122 kg is in orbit around Mars at a constant height of 1.7×10^6 m above the surface of the planet.

The height of the orbit is increased to 6.8×10^6 m above the surface. This increases the gravitational potential energy of the satellite by 5.1×10^8 J.

(i) Show that the mass of Mars is 6.4×10^{23} kg.

Question 1

[3]

(ii) Calculate the gravitational potential ϕ at the surface of Mars. Give a unit with your answer.

Question 1

(c) The satellite in **(b)** is moved to an orbit in which the satellite remains at the same point above the surface of Mars.

(i) The orbit has a period of 25 hours.

State what can be deduced from this about the rotation of Mars on its axis.

Question 1

(ii) State **one** other feature of this orbit.

Question 1

[Total: 9]

Solution 1

(a)

Mark scheme

- work done per unit mass
- work (done in) moving mass from infinity (to the point)

Solution 1

(b)(i) Worked steps

The orbital radius is measured from the **centre**, so add the planet's radius first.

- $r = 3.4 \times 10^6 + 1.7 \times 10^6 = 5.1 \times 10^6$ m
- Gravity supplies the centripetal force: $\dfrac{GMm}{r^2} = mr\omega^2$, so $GM = r^3\omega^2$ with $\omega = \dfrac{2\pi}{T}$.
- $M = \dfrac{4\pi^2 r^3}{GT^2} = 6.4 \times 10^{23}$ kg, as required.

Solution 1

Mark scheme

- evidence of addition of 3.4×106 to 1.7×106 or 6.8×106
- $GM \times 122 / (5.1 \times 106)$ or $GM \times 122 / (10.2 \times 106)$
- $6.67 \times 10–11 \times M \times 122 \times [(5.1 \times 106)–1 – (10.2 \times 106)–1] = 5.1 \times 108$
- leading to $M = 6.4 \times 1023$ kg

Solution 1

(b)(ii) Worked steps

- $\phi = -\dfrac{GM}{r}$ at the **surface**, so $r = 3.4 \times 10^6$ m.
- $\phi = -\dfrac{6.67 \times 10^{-11} \times 6.4 \times 10^{23}}{3.4 \times 10^6}$
- $\phi = -1.3 \times 10^7$ J kg^{-1}

Solution 1

The unit is asked for, and the sign is part of the answer: potential is zero at infinity, so everywhere else it is negative.

Mark scheme

- $\phi =$ (–) (6.67 $\times$ 10–11 $\times$ 6.4 $\times$ 1023) / (3.4 $\times$ 106)
- $=$ –1.3 $\times$ 107 J kg–1

Solution 1

(c)(i)

Mark scheme

- Mars takes (just under) 25 hours to rotate once on its axis

Solution 1

(c)(ii) Worked steps

A satellite that stays above one point on the surface has to match the planet's own rotation in **two** ways, and the question asks for the condition beyond the period.

- Its orbit must be **equatorial** — an orbit inclined to the equator carries the satellite north and south of its starting latitude, so it traces a figure of eight rather than staying put.
- It must travel **in the same direction as the planet's rotation**, west to east for Mars as for the Earth.

Solution 1

Either statement earns the mark. Both follow from the same requirement: the satellite's angular velocity must equal the planet's as a **vector**, not just in magnitude — same direction and same axis, and the period alone fixes only the magnitude.

Mark scheme

- orbit is equatorial
- or
- orbit is in same direction as direction of rotation of Mars
- 9702/43
- May/June 2025

Question 1

9702/43 N24 ▪ Q1 ▪ 12 marks

(a) State Newton's law of gravitation. **[2]**

(b)(i) By reference to forces, explain why the orbit of the satellite is circular. **[2]**

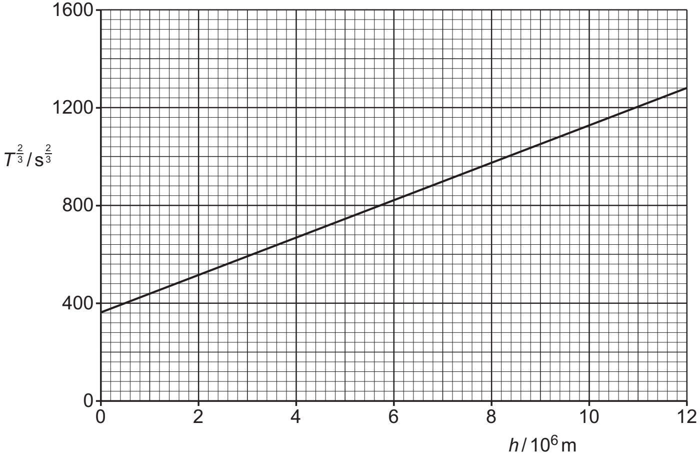
(b)(ii) Use Newton's law of gravitation to show that h and T are related by

$$(h + B)^3 = \frac{GA}{4\pi^2}T^2$$

where G is the gravitational constant and A and B are constants that depend on the properties of the planet. **[3]**

(b)(iii) Use the gradient and intercept of the line in Fig. 1.1 to determine values for A and B. Give units with your answers. **[5]**

Question 1



Solution 1

(a)

Mark scheme

- (gravitational) force is (directly) proportional to product of masses
- force (between point masses) is inversely proportional to the square of their separation

Solution 1

(b)(i)

Mark scheme

- (gravitational) force acts perpendicular to direction of motion
- gravitational force provides centripetal acceleration

Solution 1

(b)(ii) Worked steps

Know. A satellite in a circular orbit is held there by gravity alone, so the gravitational force **is** the centripetal force. Write both, set them equal, and turn the angular speed into a period.

Two forces. $\dfrac{GMm}{x^2} = mx\omega^2$, where x is the radius of the **orbit**, measured from the **centre** of the planet.

Solution 1

Bring in the period. $\omega = \dfrac{2\pi}{T}$, so $\dfrac{GMm}{x^2} = \dfrac{4\pi^2 mx}{T^2}$.

Rearrange. The satellite mass m cancels, leaving $x^3 = \dfrac{GMT^2}{4\pi^2}$.

Match the symbols. The orbit radius is the height above the surface plus the planet's own radius, so $x = h + B$ with B the **radius of the planet**, and A is the **mass of the planet**. That gives $(h + B)^3 = \dfrac{GA}{4\pi^2} T^2$.

Solution 1

Check. Saying which constant is which is worth a mark on its own. And m cancelling is the physics: every satellite at a given height takes the same time, whatever its mass.

Mark scheme

- $(F=)GMm/x^2 = mx\omega^2$ and $\omega = 2\pi/T$
- or
- $GMm/x^2 = 4\pi^2 mx/T^2$
- completion of algebra leading to $x^3 = GMT^2/4\pi^2$
- clear indication that $B =$ radius of planet and that $A =$ mass (of planet)

Solution 1

(b)(iii) Worked steps

Know. Take the cube root of the relationship so that it becomes a straight line. From $(h+B)^3 = \dfrac{GA}{4\pi^2} T^2$,

$$T^{\frac{2}{3}} = \sqrt[3]{\frac{4\pi^2}{GA}}\,(h+B).$$

So a graph of $T^{2/3}$ against h is a straight line of gradient $\sqrt[3]{\dfrac{4\pi^2}{GA}}$ and intercept gradient $\times$ B.

Gradient. From the line, e.g. $\dfrac{1280-360}{12\times 10^6} = 7.7\times 10^{-5}$.

Solution 1

Find A. Cube the gradient and rearrange: $A = \dfrac{4\pi^2}{G \times (\text{gradient})^3}$, giving $A = 1.3 \times 10^{24}$ kg.
Find B. The intercept on the $T^{2/3}$ axis is gradient $\times$ B, so $360 = 7.7 \times 10^{-5} \times B$ and $B = 4.7 \times 10^{6}$ m.
Check. The units are asked for, and they identify the constants: A is a **mass** in kg and B is a **length** in m. Both answers are sensible for a planet a little smaller than Earth, which is a quick way to spot a cube taken in the wrong direction. **Mark scheme**

- gradient $= \sqrt[3]{4\pi^2/GA}$
- e.g. $(1280 - 360)/(12 \times 10^6) = \sqrt[3]{4\pi^2/[6.67 \times 10^{-11} \times A]}$
- $A = 1.3 \times 10^{24}$ kg

Solution 1

- intercept $=$ gradient $\times$ B
- e.g. $360 = ((1280 - 360) \times B)/(12 \times 10^6)$
- $B = 4.7 \times 10^6$ m

Question 1

9702/42 J23 ▪ Q1 ▪ 11 marks

(a) State Newton's law of gravitation. **[2]**

Question 1

9702/42 J23 ▪ Q1 ▪ 11 marks

(b) A satellite is in a circular orbit around a planet. The radius of the orbit is R and the period of the orbit is T. The planet is a uniform sphere.
Use Newton's law of gravitation to show that R and T are related by

$$4\pi^2 R^3 = GMT^2$$

where M is the mass of the planet and G is the gravitational constant. **[2]**

Question 1

9702/42 J23 ▪ Q1 ▪ 11 marks

(c) The Earth may be considered to be a uniform sphere of mass 5.98×10^{24} kg and radius 6.37×10^{6} m.
A geostationary satellite is in orbit around the Earth.
Use the expression in **(b)** to determine the height of the satellite above the Earth's surface. **[3]**

Question 1

9702/42 J23 ▪ Q1 ▪ 11 marks

(d)(i) Calculate the angular speed of the satellite in this orbit. Give a unit with your answer. **[2]**

(d)(ii) Despite having the same orbital period, the orbit of this satellite is not geostationary.

Suggest **two** ways in which the orbit of this satellite could be different from the orbit of the satellite in (c).

1. 2. **[2]**

Solution 1

(a)

Mark scheme

- (gravitational) force is (directly) proportional to product of masses
- force (between point masses) is inversely proportional to the square of their separation

Solution 1

(b) Worked steps

Know. For a circular orbit, gravity provides **all** of the centripetal force. Nothing else acts on the satellite.

Set them equal. $\dfrac{GMm}{R^2} = mR\omega^2$.

Bring in the period. $\omega = \dfrac{2\pi}{T}$, so $\dfrac{GMm}{R^2} = \dfrac{4\pi^2 mR}{T^2}$.

Solution 1

Rearrange. Cancel m and cross-multiply: $4\pi^2 R^3 = GMT^2$, as required.

Solution 1

Check. Starting from $\dfrac{mv^2}{R}$ with $v = \dfrac{2\pi R}{T}$ reaches the same place and earns the same marks. What must appear either way is the statement that the two forces are equal, and the substitution that removes ω or v.

Mark scheme

- $GMm/R^2 = mR\omega^2$
- $\omega = 2\pi/T$ and algebra leading to $4\pi^2 R^3 = GMT^2$
- **or**
- $GMm/R^2 = mv^2/R$
- $v = 2\pi R/T$ and algebra leading to $4\pi^2 R^3 = GMT^2$

Solution 1

(c) Worked steps

Know. Geostationary means the satellite keeps pace with the Earth's rotation, so its period is **one day**: $T = 24 \times 60 \times 60 = 86\,400$ s.

Equation. $R^3 = \dfrac{GMT^2}{4\pi^2}$, from part (b).

Substitute. $R^3 = \dfrac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(86\,400)^2}{4\pi^2}$, so $R = 4.22 \times 10^7$ m.

Subtract. The question asks for the height **above the surface**, so take off the Earth's radius: $h = (4.22 \times 10^7) - (6.37 \times 10^6)$.

Solution 1

Answer. $h = 3.6 \times 10^7$ m.
Check. R in the formula is measured from the **centre** of the Earth, and the answer wanted is measured from the **surface**. Quoting 4.22×10^7 m loses the last mark even though the hard work is done.

Mark scheme

- $4\pi^2 \times R^3 = 6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times (24 \times 60 \times 60)^2$
- $(R = 4.22 \times 10^7 \text{ m})$
- $h = R - (6.37 \times 10^6)$
- $h = (4.22 \times 10^7) - (6.37 \times 10^6)$
- $= 3.6 \times 10^7$ m

Solution 1

(d)(i) Worked steps

Know. Angular speed depends only on how long one revolution takes, not on the radius.

Equation. $\omega = \dfrac{2\pi}{T}$.

Substitute. $\omega = \dfrac{2\pi}{24 \times 60 \times 60}$.

Answer. $\omega = 7.3 \times 10^{-5}$ rad s^{-1}, and the unit is asked for.

Solution 1

Check. Radians, not degrees: ω is defined with 2π for a full turn. Using 360 gives a number four hundred times too large and no marks.

Mark scheme

- $\omega = 2\pi/T$
- $= 2\pi/(24 \times 60 \times 60)$
- $= 7.3 \times 10^{-5}$ rad s^{-1}

Solution 1

(d)(ii)

Mark scheme

- orbit is from east to west
- orbit is not equatorial / orbit is polar