Exercise walk-through

Gravitational force between point masses ▪ 13.2

eXercise

You must be able to

- use $F = \dfrac{Gm_1 m_2}{r^2}$ and treat a sphere as a point mass
- analyse **circular orbits** (gravity provides the centripetal force)
- describe a **geostationary orbit**

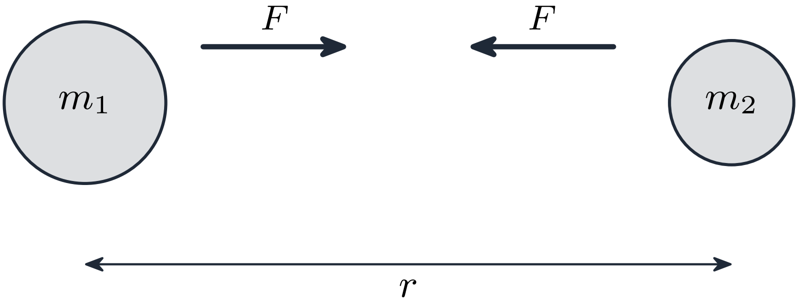
Method — Newton's law of gravitation

Two point masses attract with **equal, opposite** forces along the line joining them:

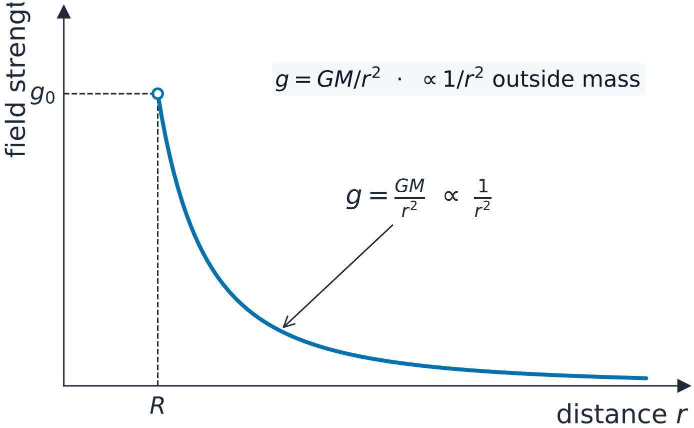
$$F = \frac{Gm_1 m_2}{r^2}, \qquad G = 6.67 \times 10^{-11}\ \mathrm{N\ m^2\,kg^{-2}}.$$

A uniform sphere acts as a **point mass at its centre** for points outside it.

Method — Newton's law of gravitation



Method — Newton's law of gravitation



Gravitational field strength against distance

Method — Circular orbits

For a satellite, gravity **is** the centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} = mr\omega^2 \Rightarrow T^2 = \frac{4\pi^2 r^3}{GM}.$$

A **geostationary** satellite: period **24 h**, orbits **west→east** above the **equator**, staying over one point on Earth.

Practice 2.1 ▪ 2 marks

State **Newton's law of gravitation**.

Solution 2.1

Method: Newton's law of gravitation

Worked steps

The gravitational force between two **point masses** is **proportional to the product of the masses** and **inversely proportional to the square of their separation** **B1** **B1**.

Practice 2.2 ▪ 1 mark

State the value and unit of the gravitational constant G.

Solution 2.2

Worked steps

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} **B1**.

Practice 2.3 ▪ 2 marks

Two 1000 kg masses are 2.0 m apart. Find the gravitational force between them.

Solution 2.3

Method: Newton's law of gravitation

Worked steps

$$F = \frac{Gm_1 m_2}{r^2} = \frac{(6.67 \times 10^{-11})(1000)(1000)}{2.0^2} = 1.67 \times 10^{-5}\ \mathrm{N} \quad \boxed{\textbf{M1}}\boxed{\textbf{A1}}.$$

Practice 2.4 ▪ 2 marks

State **two** features of a **geostationary** orbit.

Solution 2.4

Worked steps

Any two: period of **24 h**; orbits **west to east**; stays **above the equator**; appears fixed over one point on Earth **B1** **B1**.

Exam-style 3.1 ▪ 1 mark

If the distance between two masses is **doubled**, the gravitational force becomes:

A twice as large

B half as large

C one quarter as large

D four times as large

Solution 3.1

Method: Newton's law of gravitation

A twice as large

B half as large

C one quarter as large ✓

D four times as large

Worked steps

C — one quarter ($F \propto 1/r^2$).

Exam-style 3.2 ▪ 3 marks

Calculate the gravitational field strength at the Earth's surface. ($M = 6.0 \times 10^{24}$ kg, $R = 6.4 \times 10^{6}$ m, $G = 6.67 \times 10^{-11}$.)

Solution 3.2

Method: Newton's law of gravitation

Worked steps

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{(6.4 \times 10^6)^2} \quad \text{M1} \; \text{M1} \quad = 9.8 \text{ N kg}^{-1} \quad \text{A1}.$$

Exam-style 3.3 ▪ 4 marks

A satellite moves in a circular orbit of radius r around a planet of mass M. Show that gravity provides the centripetal force, and derive an expression linking the orbital **period** T to r.

Solution 3.3

Method: Circular orbits

Worked steps

Gravity provides the centripetal force: $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$ **M1**; with $v = \dfrac{2\pi r}{T}$ **M1**:
$\dfrac{GM}{r^2} = \dfrac{4\pi^2 r}{T^2}$ **M1**, so $T^2 = \dfrac{4\pi^2 r^3}{GM}$ **A1**.

Exam-style 3.4 ▪ 1 mark

State why a **geostationary** satellite must orbit directly above the **equator**.

Solution 3.4

Method: Circular orbits

Worked steps

Its orbital plane must contain the **Earth's centre**; only an equatorial orbit keeps it over a **fixed point** as the Earth spins **B1**.