Exercise walk-through

Kinematics of uniform circular motion ▪ 12.1

eXercise

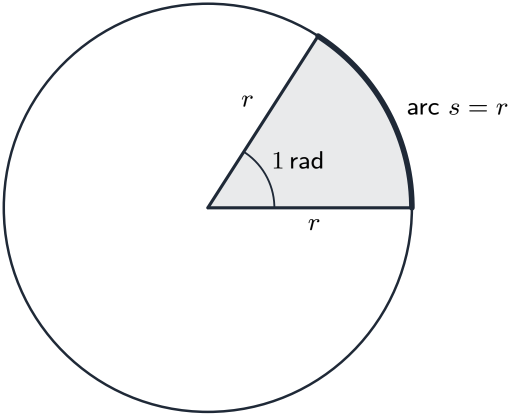
You must be able to

- define the **radian** and convert between radians and degrees
- use **angular speed** $\omega = \theta/t = 2\pi/T = 2\pi f$ and $v = r\omega$
- handle **connected wheels** (same axle $\rightarrow$ same ω; linked by a belt/chain $\rightarrow$ same rim speed)

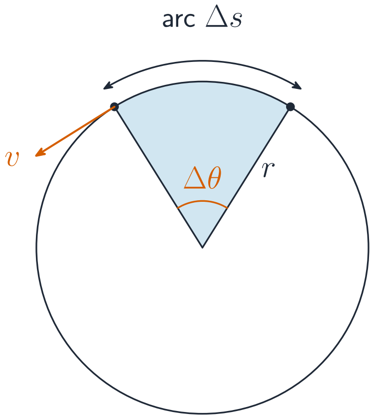
Method — Radians

One **radian** is the angle whose arc length equals the radius: $\theta = s/r$.
A full circle $= 2\pi$ rad; 1 rad $= 180°/\pi \approx 57.3°$. **Use radian mode** on your calculator.

Method — Radians



Method — Radians



Angular speed in uniform circular motion

Method — Angular speed

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi f \ \ (\text{rad s}^{-1}), \qquad v = r\omega.$$

Worked example. A point on the equator (radius 6.4×10^6 m, period 24 h): $\omega = \dfrac{2\pi}{24 \times 3600} = 7.3 \times 10^{-5}$ rad s^{-1}; $v = r\omega = 465$ m s^{-1}.

Method — Connected wheels

Exam questions often link wheels, cogs or pulleys. Two rules:

- **Fixed on the same axle** $\rightarrow$ they turn with the **same** ω (but different rim speeds if the radii differ).
- **Linked by a belt or chain** (no slipping) $\rightarrow$ the **rim speeds are equal**: $r_1\omega_1 = r_2\omega_2$.

Worked example. A belt links pulley A (radius 0.10 m, $\omega_A = 30$ rad s^{-1}) to pulley B (radius 0.050 m). Rim speed $v = r_A\omega_A = 0.10 \times 30 = 3.0$ m s^{-1}; so $\omega_B = \dfrac{v}{r_B} = \dfrac{3.0}{0.050} = 60$ rad s^{-1} (the smaller pulley spins faster).

Practice 2.1 ▪ 1 mark

Define the **radian**.

Solution 2.1

Worked steps

The angle subtended at the centre of a circle by an arc whose **length equals the radius** B1.

Practice 2.2 ▪ 1 mark

Convert $90°$ to **radians**.

Solution 2.2

Worked steps

$90^\circ = \frac{\pi}{2}$ rad ≈ 1.57 rad **B1**.

Practice 2.3 ▪ 2 marks

A wheel turns at 2.0 revolutions per second. Find its **angular speed**.

Solution 2.3

Worked steps

$\omega = 2\pi f = 2\pi \times 2.0 = 4\pi = 12.6$ rad s^{-1} **M1** **A1**.

Practice 2.4 ▪ 1 mark

Two pulleys are linked by a belt. State what is **equal** for the two pulleys.

Solution 2.4

Method: Connected wheels

Worked steps

Their **rim speeds** ($v = r\omega$) are equal (the belt does not slip) B1.

Exam-style 3.1 ▪ 1 mark

A full circle, in radians, is:

A π

B 2π

C 360

D 1

Solution 3.1

Method: Radians

A π

B 2π ✓

C 360

D 1

Worked steps

B — 2π.

Exam-style 3.2 ▪ 2 marks

A car travels around a circular track of radius 50 m at 20 m s^{-1}. Find its **angular speed**.

Solution 3.2

Method: Radians

Worked steps

$\omega = v/r = 20/50 = 0.40$ rad s^{-1} **M1** **A1**.

Exam-style 3.3 ▪ 2 marks

In a workshop, a motor turns a pulley of radius 25 mm at 1500 revolutions per minute. A tight belt runs from that pulley to a larger pulley of radius 75 mm on the shaft of a fan.

(a) Determine the angular speed of the **motor pulley**, in rad s^{-1}.

(b) The belt does not slip. Determine the angular speed of the **fan**, and state what stays the same for both pulleys.

Solution 3.3

Method: Connected wheels

Worked steps

(b) The belt gives both pulleys the same **rim speed** $v = r\omega$ B1: $v = 0.025 \times 157 = 3.93$ m s^{-1} M1, so the fan turns at $\omega = \frac{3.93}{0.075} = 52$ rad s^{-1} — a third of the motor's, because its radius is three times larger A1.

Exam-style 3.4 ▪ 1 mark

Convert 1.5 rad to **degrees**.

Solution 3.4

Worked steps

$$1.5 \times \frac{180}{\pi} = 86^\circ \quad \boxed{\textbf{B1}}.$$