Exercise walk-through

Kirchhoff's laws ▪ 10.2

eXercise

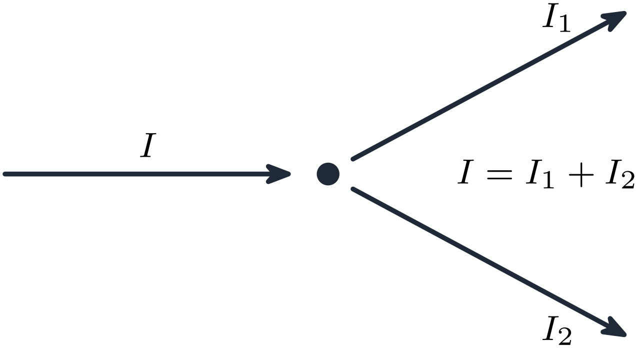
You must be able to

- state both **Kirchhoff's laws** and the conservation law each follows from
- combine resistors in **series** and **parallel**
- analyse a **combined** circuit: total resistance, total current, then each **branch** current and p.d.

Method — Kirchhoff's laws

First law (junction): current **in = current out** — from conservation of **charge**:
Second law (loop): around any closed loop, the **total e.m.f. = total p.d.** — from conservation of **energy**.

Method — Kirchhoff's laws



Method — Combining resistors

- **Series** (same current): $R = R_1 + R_2 + \dots$
- **Parallel** (same p.d.): $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dots$

Worked example. $3.0\ \Omega$ and $6.0\ \Omega$ in parallel: $\dfrac{1}{R} = \dfrac{1}{3.0} + \dfrac{1}{6.0} = \dfrac{1}{2.0}$, so $R = 2.0\ \Omega$.

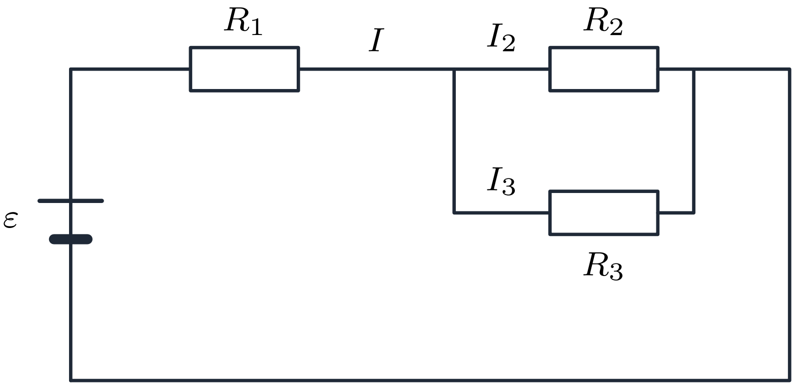
Method — Analysing a combined circuit

Real exam circuits mix series and parallel. Work in **steps**: collapse the parallel part, find the total current, then split it.

Worked example. $\varepsilon = 12$ V (negligible internal resistance), $R_1 = 4.0\ \Omega$ in series with $R_2 = 6.0\ \Omega$ and $R_3 = 3.0\ \Omega$ in parallel.

1. Parallel pair: $\dfrac{1}{R_\mathrm{p}} = \dfrac{1}{6.0} + \dfrac{1}{3.0} = \dfrac{1}{2.0}$, so $R_\mathrm{p} = 2.0\ \Omega$.
2. Total: $R = R_1 + R_\mathrm{p} = 4.0 + 2.0 = 6.0\ \Omega$; current $I = \dfrac{\varepsilon}{R} = \dfrac{12}{6.0} = 2.0$ A.
3. P.d. across the parallel part: $V_\mathrm{p} = IR_\mathrm{p} = 2.0 \times 2.0 = 4.0$ V.
4. Branch currents: $I_2 = \dfrac{V_\mathrm{p}}{R_2} = \dfrac{4.0}{6.0} = 0.67$ A, $I_3 = \dfrac{V_\mathrm{p}}{R_3} = \dfrac{4.0}{3.0} = 1.33$ A (check: $0.67 + 1.33 = 2.0$ A ✓).

Method — Analysing a combined circuit



Practice 2.1 ▪ 2 marks

State **Kirchhoff's first law** and which conservation law it follows from.

Solution 2.1

Method: Kirchhoff's laws

Worked steps

The **total current into a junction equals the total current out**; it follows from conservation of **charge** **B1** **B1**.

Practice 2.2 ▪ 2 marks

State **Kirchhoff's second law** and which conservation law it follows from.

Solution 2.2

Method: Kirchhoff's laws

Worked steps

Around any closed loop, the **total e.m.f. equals the total p.d.**; it follows from conservation of **energy** B1 B1.

Practice 2.3 ▪ 1 mark

Find the combined resistance of $4.0\ \Omega$ and $6.0\ \Omega$ in **series**.

Solution 2.3

Method: Analysing a combined circuit

Worked steps

$R = 4.0 + 6.0 = 10\ \Omega$ **B1**.

Practice 2.4 ▪ 2 marks

Find the combined resistance of two $4.0\ \Omega$ resistors in **parallel**.

Solution 2.4

Method: Analysing a combined circuit

Worked steps

$$\frac{1}{R} = \frac{1}{4.0} + \frac{1}{4.0} = \frac{1}{2.0}$$ **M1**, so $R = 2.0\ \Omega$ **A1**.

Exam-style 3.1 ▪ 1 mark

Three $6.0\ \Omega$ resistors in **parallel** have a combined resistance of:

A $2.0\ \Omega$

B $6.0\ \Omega$

C $18\ \Omega$

D $0.50\ \Omega$

Solution 3.1

Method: Analysing a combined circuit

A $2.0\ \Omega$ ✓

B $6.0\ \Omega$

C $18\ \Omega$

D $0.50\ \Omega$

Worked steps

A — $6.0/3 = 2.0\ \Omega$.

Exam-style 3.2 ▪ 4 marks

A 12 V battery drives current through a $3.0\ \Omega$ and a $6.0\ \Omega$ resistor in **series**. Calculate the current and the p.d. across each resistor.

Solution 3.2

Method: Analysing a combined circuit

Worked steps

$R = 3.0 + 6.0 = 9.0\ \Omega$ M1; $I = 12/9.0 = 1.33$ A A1; $V_3 = IR = 1.33 \times 3.0 = 4.0$ V, $V_6 = 1.33 \times 6.0 = 8.0$ V A1 A1.

Exam-style 3.3 ▪ 3 marks

A battery of e.m.f. 6.0 V and negligible internal resistance is connected to $R_1 = 2.0\ \Omega$ in series with two resistors $R_2 = 4.0\ \Omega$ and $R_3 = 4.0\ \Omega$ in **parallel** (as in the worked example).

(a) Calculate the total resistance and the current from the battery.

(b) Calculate the current in **each** of R_2 and R_3.

Solution 3.3

Method: Analysing a combined circuit

Worked steps

(b) $V_{\mathrm{p}} = IR_{\mathrm{p}} = 1.5 \times 2.0 = 3.0$ V **M1**; each branch $I = \dfrac{3.0}{4.0} = 0.75$ A **A1**, and the two are equal because $R_2 = R_3$ **A1**.

Exam-style 3.4 ▪ 1 mark

At a junction, 5.0 A enters and one branch carries 2.0 A away. Find the current in the other branch.

Solution 3.4

Method: Analysing a combined circuit

Worked steps

$5.0 - 2.0 = 3.0$ A **B1**.