Exercise walk-through

10.2.1

Kirchhoff's laws, deriving the resistor formulae and solving networks

A-Level Physics

You must be able to

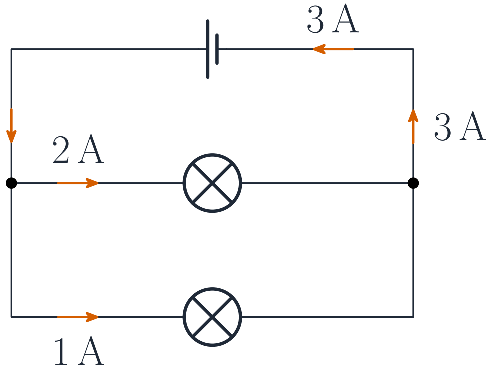
- recall **Kirchhoff's first law 基尔霍夫第一定律** and understand that it is a consequence of **conservation of charge 电荷守恒**
- recall **Kirchhoff's second law 基尔霍夫第二定律** and understand that it is a consequence of **conservation of energy 能量守恒**
- derive, using Kirchhoff's laws, a formula for the combined resistance of two or more resistors in **series**
- use the formula for the combined resistance of two or more resistors in series
- derive, using Kirchhoff's laws, a formula for the combined resistance of two or more resistors in **parallel**
- use the formula for the combined resistance of two or more resistors in parallel
- use Kirchhoff's laws to solve simple circuit problems

1 The method

Method — The two laws, and what each one conserves

- **First law.** At any **junction 节点**, the total current in equals the total current out. Charge cannot pile up at a point, so this is conservation of charge.
- **Second law.** Round any closed loop, the total **electromotive force 电动势** equals the total p.d. across the components. A unit of charge gives up exactly the energy it gained, so this is conservation of energy.
- The pairing is examined almost every session, and the two are easy to swap. First law $\rightarrow$ charge. Second law $\rightarrow$ energy.

Method — The two laws, and what each one conserves



at each junction: $3\,\mathrm{A} = 2\,\mathrm{A} + 1\,\mathrm{A}$

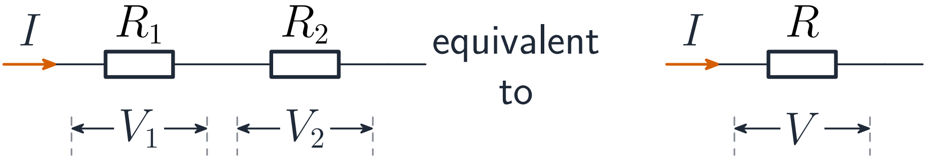
Method — Deriving the series formula

A derivation must name the law at each step; that is what earns the marks.

1. There is no junction between the resistors, so by the **first law** the same current I is in both.
2. By the **second law**, the p.d. across the combination is the sum of the p.d.s: $V = V_1 + V_2$.
3. Each p.d. is IR, so $IR = IR_1 + IR_2$.
4. Divide every term by I: $R = R_1 + R_2$.

Method — Deriving the series formula

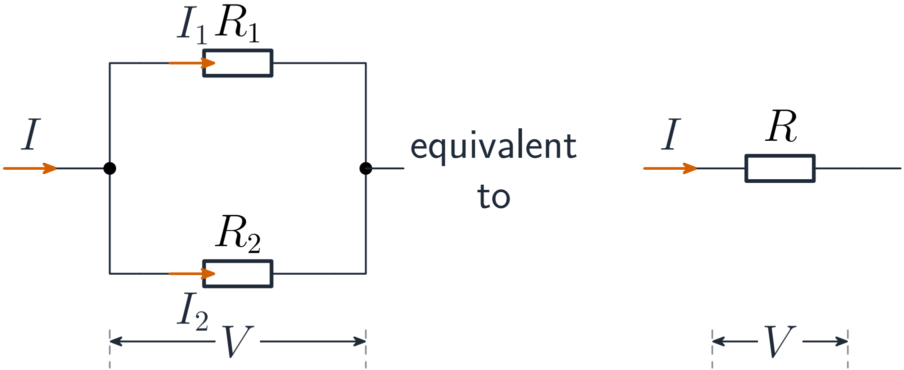
A derivation must name the law at each step; that is what earns the marks



Method — Deriving the parallel formula

1. Each resistor forms its own loop with the source, so by the **second law** the same p.d. V is across both.
2. By the **first law** the current arriving at the junction is shared: $I = I_1 + I_2$.
3. Each current is V/R, so $\dfrac{V}{R} = \dfrac{V}{R_1} + \dfrac{V}{R_2}$.
4. Divide every term by V: $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$.

Method — Deriving the parallel formula



Method — Reducing a network one step at a time

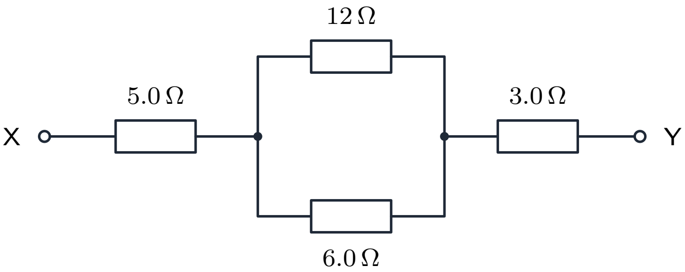
Never try to see the whole network at once. Replace one group, redraw, repeat.

1. The parallel pair: $\dfrac{1}{R_\mathrm{p}} = \dfrac{1}{12} + \dfrac{1}{6.0} = \dfrac{1}{12} + \dfrac{2}{12} = \dfrac{3}{12}$, so $R_\mathrm{p} = 4.0\ \Omega$.
2. What is left is three resistances in series: $5.0 + 4.0 + 3.0$.
3. $R_{XY} = \mathbf{12}\ \Omega$.

- For **two** resistors only, $R_\mathrm{p} = \dfrac{R_1 R_2}{R_1 + R_2}$ is quicker. It is wrong for three.
- A parallel combination is always **smaller** than its smallest resistor; a series one is always larger than its largest. Use that to check every answer.

Method — Reducing a network one step at a time

Never try to see the whole network at once.



Method — Which component dissipates the most power

Two rules settle this with no arithmetic at all, and which one applies depends on what the components share.

- **Same current** (in series): $P = I^2R$, so the **larger** resistance dissipates more.
- **Same p.d.** (in parallel): $P = \dfrac{V^2}{R}$, so the **smaller** resistance dissipates more.

A $4.0\ \Omega$ and an $8.0\ \Omega$ resistor are joined to a 12 V supply of negligible internal resistance.

1. **In series:** $R = 12\ \Omega$, so $I = 1.0$ A in both. $P_4 = (1.0)^2(4.0) = 4.0$ W and $P_8 = (1.0)^2(8.0) = 8.0$ W – the larger resistor wins.
2. **In parallel:** both have 12 V across them. $P_4 = \dfrac{12^2}{4.0} = 36$ W and $P_8 = \dfrac{12^2}{8.0} = 18$ W – now the smaller resistor wins.

Method — Which component dissipates the most power

- In a mixed circuit, the resistor carrying the **whole** current usually dissipates the most, because every parallel branch carries only a share of it.

Method — A loop with two cells that oppose

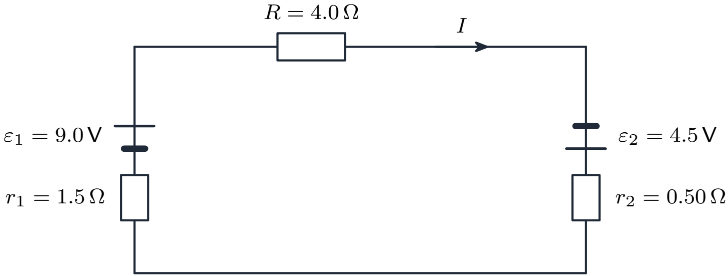
The second law handles this with no extra ideas: go once round the loop and add the e.m.f.s with their signs.

1. The cells are connected the opposite way round, so the net e.m.f. is $9.0 - 4.5 = 4.5$ V.
2. The two **internal resistances 内阻** are in series with the resistor: $R_{\text{total}} = 1.5 + 0.50 + 4.0 = 6.0\ \Omega$.
3. $I = \dfrac{4.5}{6.0} = \mathbf{0.75}$ A, driven the way the larger e.m.f. pushes.
4. The terminal p.d. of the 9.0 V cell is $\varepsilon - Ir = 9.0 - (0.75)(1.5) = \mathbf{7.9}$ V.

- The smaller cell is being driven backwards, so it is charging; its terminal p.d. is $4.5 + (0.75)(0.50)$, which is *larger* than its e.m.f.

Method — A loop with two cells that oppose

The second law handles this with no extra ideas: go once round the loop and add the e.m.f.s with their signs



2 Practice

Practice 2.1 ▪ 2 marks

Calculate the combined resistance of $2.0\ \Omega$, $3.0\ \Omega$ and $6.0\ \Omega$ connected in parallel.

Solution 2.1

Worked steps

- $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$ **M1**

$$\frac{1}{R} = \frac{1}{2.0} + \frac{1}{3.0} + \frac{1}{6.0} = \frac{3}{6.0} + \frac{2}{6.0} + \frac{1}{6.0} = 1.0 \Rightarrow R = \mathbf{1.0}\ \Omega$$ **A1**

- the answer is smaller than the smallest resistor, as it must be

Practice 2.2 ▪ 3 marks

For the network between X and Y in the worked example above, the $12\ \Omega$ resistor is replaced by a second $6.0\ \Omega$ resistor. Calculate the new resistance between X and Y.

Solution 2.2

Worked steps

- the parallel pair is now two equal resistors, so $R_{\mathrm{p}} = \dfrac{6.0}{2} = 3.0\ \Omega$ M1 A1
- in series with the others: $R_{XY} = 5.0 + 3.0 + 3.0 = \mathbf{11}\ \Omega$ A1

Practice 2.3 ▪ 2 marks

Explain why connecting a second resistor in parallel with the first always **decreases** the resistance between the same two points.

Solution 2.3

Worked steps

- the second resistor gives the charge an extra path, so for the same p.d. the total current is larger **B1**
- $R = V/I$, so a larger current for the same p.d. means a smaller resistance **B1**

Practice 2.4 ▪ 2 marks

A $4.0\ \Omega$ and an $8.0\ \Omega$ resistor are connected in **series** across a supply. State which dissipates the greater power, and by what factor.

Solution 2.4

Method: Which component dissipates the most power — p.12

Worked steps

- in series the current is the same in both, and $P = I^2R$ **B1**
- the $\mathbf{8.0\ \Omega}$ resistor, dissipating **twice** the power of the $4.0\ \Omega$ **B1**

Practice 2.5 ▪ 2 marks

The same two resistors are now connected in **parallel** across the same supply. State which dissipates the greater power, and explain why the answer has changed.

Solution 2.5

Method: Which component dissipates the most power — p.12

Worked steps

- in parallel the p.d. is the same across both, and $P = V^2/R$, so the $\mathbf{4.0}\ \Omega$ resistor dissipates the greater power **B1**
- the answer changes because the resistors now share the p.d. instead of the current **B1**

3

Exam-style

Exam-style 3.1 ▪ 1 mark

A $6.0\ \Omega$ resistor and a $3.0\ \Omega$ resistor are connected in parallel. This combination is in series with a $4.0\ \Omega$ resistor across a supply of e.m.f. 12 V and negligible internal resistance. What is the current from the supply?

A 0.50 A

B 1.0 A

C 2.0 A

D 3.0 A

Solution 3.1

Method: Which component dissipates the most power — p.12

A 0.50 A

B 1.0 A

C 2.0 A ✓

D 3.0 A

Worked steps

C

- parallel pair: $\frac{1}{R_\mathrm{p}} = \frac{1}{6.0} + \frac{1}{3.0} = \frac{1}{2.0}$, so $R_\mathrm{p} = 2.0\ \Omega$ **B1**

Solution 3.1

- total $= 4.0 + 2.0 = 6.0\ \Omega$, so $I = \dfrac{12}{6.0} = 2.0$ A

Exam-style 3.2 ▪ 4 marks

Two resistors of resistances R_1 and R_2 are connected in parallel. Use Kirchhoff's laws to show that the combined resistance R is given by

$$R = \frac{R_1 R_2}{R_1 + R_2}.$$

Name the law used at each step.

Solution 3.2

Method: Reducing a network one step at a time — p.10

Worked steps

- by **Kirchhoff's second law** each resistor is in its own loop with the source, so the same p.d. V is across both **B1**
- by **Kirchhoff's first law** the currents at the junction add: $I = I_1 + I_2$ **B1**
- substitute $I = V/R$ for each: $\dfrac{V}{R} = \dfrac{V}{R_1} + \dfrac{V}{R_2}$, so $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ **M1**
- put the right-hand side over a common denominator and invert:

$$\frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2} \Rightarrow R = \frac{R_1 R_2}{R_1 + R_2}$$

A1

Exam-style 3.3 ▪ 9 marks

A battery of e.m.f. 18 V and negligible internal resistance is connected to a $6.0\ \Omega$ resistor in series with a parallel combination of a $9.0\ \Omega$ resistor and an $18\ \Omega$ resistor.

Exam-style 3.3 (a) ▪ 2 marks

Show that the resistance of the parallel combination is $6.0\ \Omega$.

Solution 3.3 (a)

> Method: Which component dissipates the most power — p.12

Worked steps

$$\frac{1}{R_\mathrm{p}} = \frac{1}{9.0} + \frac{1}{18} = \frac{2}{18} + \frac{1}{18} = \frac{3}{18}$$ **M1**

- $R_\mathrm{p} = \dfrac{18}{3} = \mathbf{6.0\ \Omega}$ **A1**

Exam-style 3.3 (b) ▪ 2 marks

Calculate the current in the battery.

Solution 3.3 (b)

Worked steps

total resistance $= 6.0 + 6.0 = 12\ \Omega$ **M1**

$$I = \frac{18}{12} = \mathbf{1.5}\ \text{A}$$ **A1**

Exam-style 3.3 (c) ▪ 3 marks

Calculate the p.d. across the parallel combination and the current in each of its branches.

Solution 3.3 (c)

Worked steps

$V_{\mathsf{p}} = IR_{\mathsf{p}} = (1.5)(6.0) = \mathbf{9.0}$ V **M1** **A1**

- $I_9 = \dfrac{9.0}{9.0} = \mathbf{1.0}$ A and $I_{18} = \dfrac{9.0}{18} = \mathbf{0.50}$ A **A1**
- check: $1.0 + 0.50 = 1.5$ A, the current in the battery

Exam-style 3.3 (d) ▪ 2 marks

State, with a reason, which of the three resistors dissipates the greatest power.

Solution 3.3 (d)

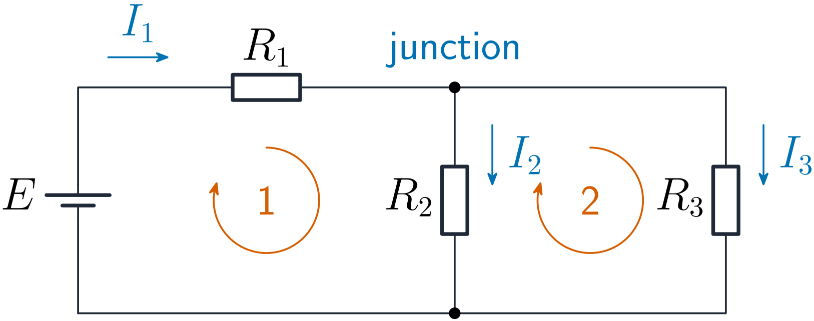
Worked steps

the $\mathbf{6.0}$ Ω resistor **B1**

- it carries the whole 1.5 A while each branch carries only part of it, and $P = I^2R$: 13.5 W against 9.0 W and 4.5 W **B1**

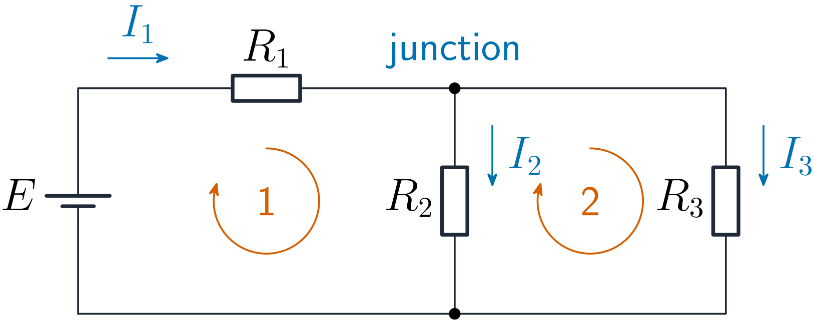
Exam-style 3.4 ▪ 10 marks

A cell of e.m.f. 12 V and negligible internal resistance is connected to $R_1 = 3.0\ \Omega$ in series with a parallel combination of $R_2 = 12\ \Omega$ and $R_3 = 4.0\ \Omega$, as shown.



Exam-style 3.4 (a) ▪ 1 mark

Write down the equation that Kirchhoff's first law gives for the junction.



Solution 3.4 (a)

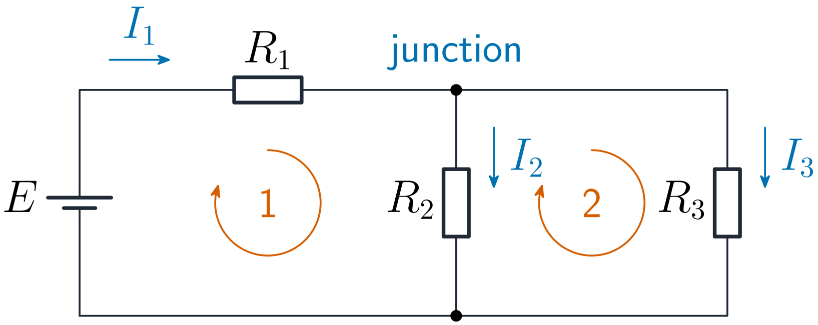
Method: The two laws, and what each one conserves — p.4

Worked steps

$I_1 = I_2 + I_3$ **B1**

Exam-style 3.4 (b) ▪ 4 marks

Calculate the current I_1 in the cell and the currents I_2 and I_3 in the two branches.



Solution 3.4 (b)

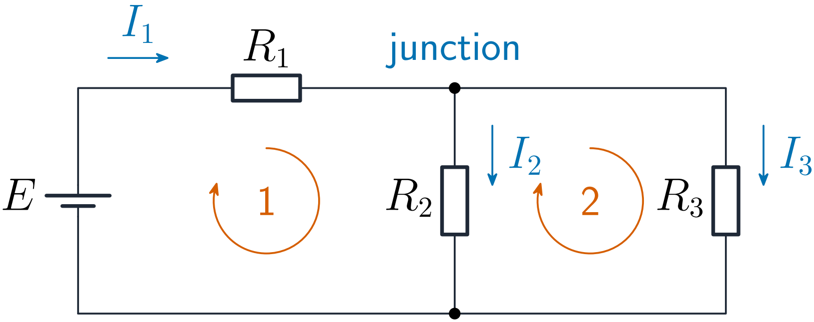
Worked steps

parallel pair: $\dfrac{1}{R_\mathrm{p}} = \dfrac{1}{12} + \dfrac{1}{4.0} = \dfrac{1}{12} + \dfrac{3}{12} = \dfrac{4}{12}$, so $R_\mathrm{p} = 3.0\ \Omega$ M1

- total $= 3.0 + 3.0 = 6.0\ \Omega$, so $I_1 = \dfrac{12}{6.0} = \mathbf{2.0}$ A A1
- p.d. across the pair $= (2.0)(3.0) = 6.0$ V M1
- $I_2 = \dfrac{6.0}{12} = \mathbf{0.50}$ A and $I_3 = \dfrac{6.0}{4.0} = \mathbf{1.5}$ A A1

Exam-style 3.4 (c) ▪ 2 marks

Calculate the p.d. across R_1.



Solution 3.4 (c)

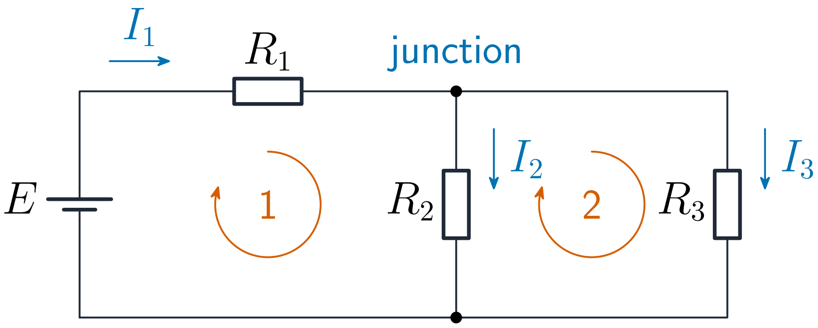
Worked steps

$V_1 = I_1 R_1$ **M1**

$$V_1 = (2.0)(3.0) = \mathbf{6.0}\ \mathrm{V}$$ **A1**

Exam-style 3.4 (d) ▪ 3 marks

Show that the total power dissipated in the three resistors equals the power supplied by the cell, and state which conservation law this illustrates.



Solution 3.4 (d)

Worked steps

$P = I^2 R$ for each resistor M1

- $P_1 = (2.0)^2(3.0) = 12$ W, $P_2 = (0.50)^2(12) = 3.0$ W,
  $P_3 = (1.5)^2(4.0) = 9.0$ W, total $= \mathbf{24}$ W A1
- the cell supplies $\varepsilon I_1 = (12)(2.0) = 24$ W, the same, illustrating **conservation of energy** – which is what Kirchhoff's second law expresses B1