Exercise walk-through

10.1.1

e.m.f., internal resistance and the V-I graph

A-Level Physics

You must be able to

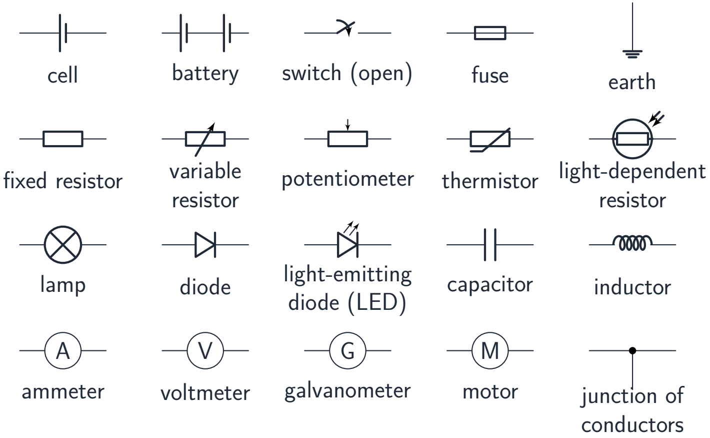
- recall, draw and interpret the standard **circuit symbols 电路符号** and circuit diagrams
- define the e.m.f. of a source as the **energy transferred per unit charge** in driving charge round a complete circuit
- distinguish between e.m.f. and potential difference in terms of energy
- understand the effect of the internal resistance of a source on the terminal potential difference

1 The method

Method — The symbols, and how a diagram is marked

- Draw with a **ruler**, with straight wires and square corners; a wire that simply ends is a lost mark.
- An **ammeter** goes **in series** with the component whose current you want; a **voltmeter** goes **in parallel** across the component whose p.d. you want. Swapping them is the commonest diagram error.
- Junctions that are connected get a **dot**; wires that cross without connecting do not.

Method — The symbols, and how a diagram is marked



Method — Two definitions, and they differ only in direction

	definition	where the energy goes
electromotive force (e.m.f.) E	the energy transferred **to** each unit of charge, from some other form into electrical, in driving it around a complete circuit	chemical $\rightarrow$ electrical, inside the source
p.d. V	the energy transferred **from** each unit of charge into another form, in a component	electrical $\rightarrow$ thermal, light, kinetic, in the component

- Both are **joules per coulomb**, so both are measured in **volts**. That is why the distinction has to be made in words about **energy**, and why "e.m.f. is the voltage of the battery" earns nothing.
- E.m.f. is **not a force**, despite its name.

Method — A real cell: an ideal source with a resistance in series

Follow one coulomb round the circuit:

$$E = V + Ir \quad \iff \quad V = E - Ir \quad \iff \quad E = I(R + r)$$

- It is given E joules by the source.
- It gives up Ir joules **inside** the cell, warming it – these are sometimes called the "lost volts".
- It gives up V joules **outside**, in the rest of the circuit. V is the **terminal p.d.**: what the circuit actually receives.

Two limiting cases the exam likes:

situation	what happens
no current drawn ($I = 0$)	$V = E$: the terminal p.d. **equals** the e.m.f., because nothing is lost inside
terminals shorted ($R = 0$)	$I = \dfrac{E}{r}$, the **largest current** the cell can deliver, and all of the energy is wasted inside it

Method — A real cell: an ideal source with a resistance in series

Worked example. A cell of e.m.f. 1.5 *V and internal resistance* $0.50\ \Omega$ *is connected to a* $2.5\ \Omega$ *resistor. Find the current and the terminal p.d.*

- the e.m.f. drives the current through **both** resistances

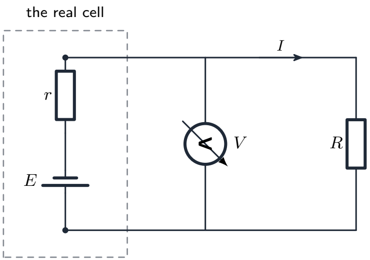
$$I = \frac{E}{R + r} = \frac{1.5\ \text{V}}{2.5\ \Omega + 0.50\ \Omega} = 0.50\ \text{A}$$

- then the terminal p.d. is what is left after the internal drop

$$V = E - Ir = 1.5\ \text{V} - 0.50\ \text{A} \times 0.50\ \Omega = \mathbf{1.25}\ \text{V}$$

Method — A real cell: an ideal source with a resistance in series

Find the current and the terminal p.d



V is the **terminal p.d.**: what the rest of the circuit actually receives

Each coulomb is given E joules by the source, gives up Ir joules inside the cell and V joules outside it, so $E = V + Ir$, or $V = E - Ir$. Nothing is lost – the missing energy is dissipated *inside* the cell, which is why a battery gets warm.

Method — The experiment: one graph, two answers

Vary the external resistance, record V and I each time, and plot V against I. Since

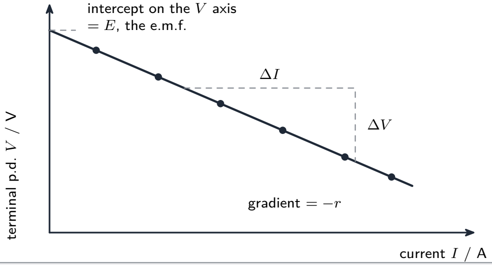
$$V = E - rI$$

is $y = c + mx$:

- the **intercept** on the V axis is the **e.m.f.** E;
- the **gradient** is $-r$, so the internal resistance is **minus** the gradient, and it is positive.
- This is how an e.m.f. is measured, because a real voltmeter always draws a **small current**, so the reading straight across the cell is always slightly **less** than E. The graph extrapolates to the $I = 0$ case that cannot quite be reached.
- **Power:** the source supplies EI in total, of which I^2r is wasted inside and VI is delivered outside.

Method — The experiment: one graph, two answers

Vary the external resistance, record ... and ... each time, and plot ... against ... .



Vary the external resistance, plot V against I, and $V = E - Ir$ gives a straight line: the **intercept** is the e.m.f. and the **gradient** is minus the internal resistance. Extrapolating back to $I = 0$ is how you measure an e.m.f. you can never read directly, because any real voltmeter draws some current.

2 Practice

Practice 2.1 ▪ 2 marks

Define the **e.m.f.** of a source.

Solution 2.1

Worked steps

- the **energy transferred to each unit of charge** from another form into electrical form **B1**
- in driving that charge **around a complete circuit** **B1**

Practice 2.2 ▪ 2 marks

Explain, in terms of energy, the difference between the e.m.f. of a cell and the potential difference across a resistor.

Solution 2.2

Method: A real cell: an ideal source with a resistance in series — p.7

Worked steps

- the **e.m.f.** is the energy given **to** each coulomb by the source, as chemical energy becomes electrical **B1**
- the **p.d.** is the energy given **up** by each coulomb in the component, as electrical energy becomes thermal or another form **B1**

Practice 2.3 ▪ 2 marks

A cell of e.m.f. 1.5 V and internal resistance 0.40 Ω delivers a current of 0.60 A. Calculate the terminal p.d.

Solution 2.3

Method: A real cell: an ideal source with a resistance in series — p.7

Worked steps

- use $V = E - Ir$ M1

$$V = 1.5\ \mathrm{V} - 0.60\ \mathrm{A} \times 0.40\ \Omega = \mathbf{1.26}\ \mathrm{V}$$ A1

Practice 2.4 ▪ 2 marks

Explain why the terminal p.d. of a cell falls as the current drawn from it increases.

Solution 2.4

Method: A real cell: an ideal source with a resistance in series — p.7

Worked steps

- a larger current means a larger p.d. Ir across the **internal resistance** **B1**
- the e.m.f. is fixed, so with more of it used inside the cell, less is left for the external circuit and $V = E - Ir$ falls **B1**

Practice 2.5 ▪ 2 marks

State the terminal p.d. of a cell when **no** current is drawn from it, and explain your answer.

Solution 2.5

Method: A real cell: an ideal source with a resistance in series — p.7

Worked steps

- the terminal p.d. **equals the e.m.f.** B1
- with $I = 0$ there is no p.d. across the internal resistance, so nothing is lost inside the cell B1

Practice 2.6 ▪ 2 marks

Calculate the largest current the cell in **2.3** could deliver, and state the condition under which this happens.

Solution 2.6

Method: A real cell: an ideal source with a resistance in series — p.7

Worked steps

- the largest current flows when the external resistance is **zero** – the terminals are short-circuited **B1**

$$I_{max} = \frac{E}{r} = \frac{1.5\ \text{V}}{0.40\ \Omega} = \mathbf{3.75}\ \text{A}$$ **B1**

3 Exam-style

Exam-style 3.1 ▪ 1 mark

As the current drawn from a cell increases, the terminal potential difference

A increases, because more energy is transferred each second.

B stays the same, because the e.m.f. of the cell is fixed.

C decreases, because more energy is dissipated in the internal resistance.

D decreases, because the e.m.f. of the cell falls.

Solution 3.1

Method: A real cell: an ideal source with a resistance in series — p.7

A increases, because more energy is transferred each second.

B stays the same, because the e.m.f. of the cell is fixed.

C decreases, because more energy is dissipated in the internal resistance. ✓

D decreases, because the e.m.f. of the cell falls.

Worked steps

C

- the internal resistance is unchanged, but a larger current gives a larger Ir inside the cell, so less of the fixed e.m.f. is left as terminal p.d. **B1**

Exam-style 3.2 ▪ 6 marks

A battery of e.m.f. 12 V and internal resistance $0.50\ \Omega$ is connected to a $2.5\ \Omega$ resistor in series with a parallel combination of a $6.0\ \Omega$ and a $3.0\ \Omega$ resistor.

Exam-style 3.2 (a) ▪ 2 marks

Draw a circuit diagram of this arrangement, including an ammeter to measure the current from the battery and a voltmeter to measure the terminal p.d.

Solution 3.2 (a)

Method: A real cell: an ideal source with a resistance in series — p.7

Worked steps

- the battery, the $2.5\ \Omega$ resistor and the parallel pair correctly in **series**, with the $6.0\ \Omega$ and $3.0\ \Omega$ in **parallel** with each other **B1**
- the **ammeter in series** with the battery and the **voltmeter in parallel** across its terminals, with correct symbols **B1**

Exam-style 3.2 (b) ▪ 2 marks

Calculate the total resistance of the external circuit.

Solution 3.2 (b)

Worked steps

- combine the parallel pair first **M1**

$$\frac{1}{R_\mathrm{p}} = \frac{1}{6.0\ \Omega} + \frac{1}{3.0\ \Omega} = \frac{1}{2.0\ \Omega}, \qquad R_\mathrm{p} = 2.0\ \Omega$$

$$R_\mathrm{ext} = 2.5\ \Omega + 2.0\ \Omega = \mathbf{4.5}\ \Omega$$

A1

Exam-style 3.2 (c) ▪ 2 marks

Calculate the current delivered by the battery.

Solution 3.2 (c)

Worked steps

- the e.m.f. drives the current through the external resistance **and** the internal resistance M1

$$I = \frac{E}{R_{\text{ext}} + r} = \frac{12\ \text{V}}{4.5\ \Omega + 0.50\ \Omega} = \mathbf{2.4}\ \text{A}$$ A1

Exam-style 3.3 ▪ 11 marks

A student varied the resistance of the external circuit and recorded the terminal p.d. V and the current I.

I / A	**0.20**	**0.45**	**0.70**	**0.95**	**1.20**
V / V	5.72	5.45	5.18	4.91	4.64

Exam-style 3.3 (a) ▪ 3 marks

State the equation relating V and I for this circuit, and state what the gradient and the intercept of a graph of V against I represent.

Solution 3.3 (a)

Method: The experiment: one graph, two answers — p.10

Worked steps

- $V = E - Ir$ B1
- the **intercept** on the V axis is the **e.m.f.** E B1
- the **gradient** is $-r$, so the internal resistance is minus the gradient B1

Exam-style 3.3 (b) ▪ 3 marks

Use the first and last pairs of readings to calculate the internal resistance of the source.

Solution 3.3 (b)

Worked steps

- the internal resistance is minus the gradient of V against I **M1**

$$\text{gradient} = \frac{4.64\ \text{V} - 5.72\ \text{V}}{1.20\ \text{A} - 0.20\ \text{A}} = -1.08\ \Omega$$ **M1**

$$r = \mathbf{1.08}\ \Omega$$ **A1**

Exam-style 3.3 (c) ▪ 2 marks

Calculate the e.m.f. of the source.

Solution 3.3 (c)

Worked steps

- substitute one pair of readings into $E = V + Ir$ **M1**

$$E = 5.72\ \text{V} + 0.20\ \text{A} \times 1.08\ \Omega = \mathbf{5.94}\ \text{V}$$ **A1**

Exam-style 3.3 (d) ▪ 2 marks

Calculate the power dissipated inside the source when the current is 1.20 A.

Solution 3.3 (d)

Worked steps

- the power dissipated inside the source is $I^2 r$ **M1**

$$P = (1.20\ \mathrm{A})^2 \times 1.08\ \Omega = \mathbf{1.6}\ \mathrm{W}$$ **A1**

Exam-style 3.3 (e) ▪ 1 mark

Explain why connecting a voltmeter directly across the terminals of the source does not give exactly its e.m.f.

Solution 3.3 (e)

Worked steps

- a real voltmeter draws a small current, so there is a small p.d. Ir inside the cell and the reading is slightly **less** than the e.m.f. B1