Exercise walk-through

Errors and uncertainties ▪ 1.3

eXercise

You must be able to

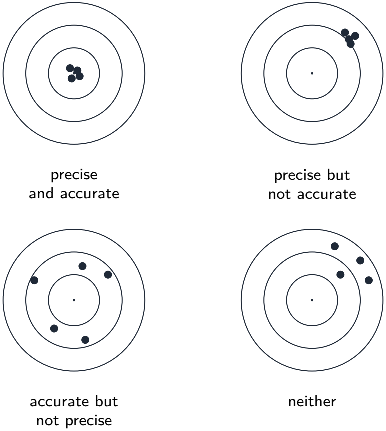
- explain **systematic** (incl. **zero**) and **random** errors
- distinguish **precision 精密度** from **accuracy 准确度**
- combine **absolute** and **percentage** uncertainties in a derived quantity
- **justify** the number of significant figures in a calculated result
- decide, from the uncertainties, whether two results **agree**

Method — Errors, precision and accuracy

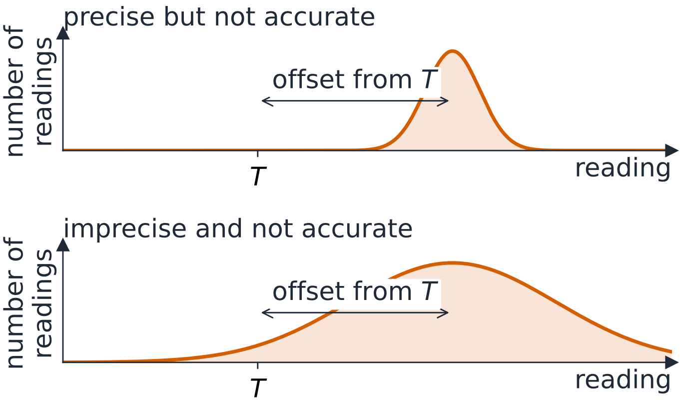
- A **systematic error** shifts every reading the **same way** (a **zero error**, a wrong calibration). Repeating does not reveal it; it harms **accuracy**.
- A **random error** scatters readings **both ways**. Taking the **mean** of many repeats reduces it; it harms **precision**.

Precision = how close repeats are to each other; **accuracy** = how close they are to the true value:

Method — Errors, precision and accuracy



Method — Errors, precision and accuracy



accuracy: how close readings are to the true value (systematic error reduces accuracy)

Precision contrasted with accuracy

Method — Uncertainty in one quantity

Write a measurement as $x \pm \Delta x$. The **percentage uncertainty** is $\dfrac{\Delta x}{|x|} \times 100\%$. E.g. (2.50 ± 0.05) m $\rightarrow \dfrac{0.05}{2.50} \times 100 = 2\%$.

Method — Combining uncertainties

- **Add / subtract** $\rightarrow$ add the **absolute** uncertainties: $y = a \pm b \Rightarrow \Delta y = \Delta a + \Delta b$.
- **Multiply / divide** $\rightarrow$ add the **percentage** uncertainties.
- **Power** $y = a^n$ $\rightarrow$ **multiply** the percentage by $|n|$.

Worked example. $R = V/I$ with $V = (6.0 \pm 0.1)$ V, $I = (2.0 \pm 0.1)$ A: $R = 3.0\ \Omega$; $\%V = 1.7\%$, $\%I = 5.0\%$, so $\%R = 6.7\%$, i.e. $R = (3.0 \pm 0.2)\ \Omega$.

Method — Significant figures in a calculated answer

Give a calculated result to the **same number of significant figures as the least precise measurement** it came from — no more. $\rho = \dfrac{m}{V}$ from a mass of 31.3 g (3 s.f.) and a side of 1.53 cm (3 s.f.) is quoted to **3 s.f.**; writing 8.7194×10^3 claims a precision the measurements do not have. When a question says **justify**, say which measurement set the limit.

Method — Do two results agree?

An answer with an uncertainty is a **range**, not a point. Two results **agree** when their ranges **overlap**:

$$\rho_{\mathsf{A}} = 8.7 \times 10^3 \text{ kg m}^{-3} \pm 4\% \Rightarrow 8.35 \times 10^3 \text{ to } 9.05 \times 10^3$$

$$\rho_{\mathsf{B}} = 9.2 \times 10^3 \text{ kg m}^{-3} \pm 6\% \Rightarrow 8.65 \times 10^3 \text{ to } 9.75 \times 10^3$$

The two ranges share the band $8.65–9.05 \times 10^3$, so the samples **could** be the same material. Write both ranges down — that comparison is where the marks are, not the verdict on its own.

Practice 2.1 ▪ 2 marks

State the difference between a **systematic** error and a **random** error.

Solution 2.1

Method: Errors, precision and accuracy

Worked steps

A **systematic** error shifts every reading the **same amount/direction** (not reduced by repeating) **B1**; a **random** error scatters readings **both ways** and is reduced by taking the **mean** of repeats **B1**.

Practice 2.2 ▪ 2 marks

A balance reads 0.3 g with **nothing** on it. Name this error and state how to correct a reading.

Solution 2.2

Method: Errors, precision and accuracy

Worked steps

A **zero error** (systematic) B1; **subtract** the 0.3 g from every reading B1.

Practice 2.3 ▪ 1 mark

A length is (2.50 ± 0.05) m. Find its **percentage uncertainty**.

Solution 2.3

Method: Uncertainty in one quantity

Worked steps

$$\frac{0.05}{2.50} \times 100 = 2\% \quad \boxed{\textbf{B1}}.$$

Practice 2.4 ▪ 2 marks

State the difference between **precision** and **accuracy**.

Solution 2.4

Method: Errors, precision and accuracy

Worked steps

Precision — how close repeated readings are to **each other**; **accuracy** — how close a reading (or the mean) is to the **true value** B1 B1.

Practice 2.5 ▪ 2 marks

A student measures a length with a metre rule as 84.2 cm. **Estimate** the absolute uncertainty in this reading, and explain your choice.

Solution 2.5

Worked steps

Half the smallest division: the rule is marked in millimetres, so a single reading is ± 0.05 cm (± 0.5 mm) **B1**; a length needs **two** readings (both ends), so ± 0.1 cm is also accepted if that reasoning is given **B1**.

Practice 2.6 ▪ 2 marks

A speed is calculated as 3.472 m s^{-1} from a distance of 12.5 m and a time of 3.6 s. State how many significant figures the answer should be given to, and **justify** your answer.

Solution 2.6

Method: Significant figures in a calculated answer

Worked steps

2 significant figures B1 — the time, 3.6 s, is given to only 2 s.f., and a calculated answer cannot be more precise than the least precise measurement it uses, so 3.5 m s^{-1} B1.

Exam-style 3.1 ▪ 1 mark

Repeated readings are **close together** but **far from** the true value. They are:

A accurate but not precise

B precise but not accurate

C both accurate and precise

D neither

Solution 3.1

Method: Errors, precision and accuracy

A accurate but not precise

B precise but not accurate ✓

C both accurate and precise

D neither

Worked steps

B — precise but not accurate.

Exam-style 3.2 ▪ 2 marks

Two lengths are $a = (40 \pm 1)$ mm and $b = (25 \pm 1)$ mm. **Determine** $a - b$ with its **absolute uncertainty**.

Solution 3.2

Worked steps

$a - b = 40 - 25 = 15$ mm **M1**; $\Delta = 1 + 1 = 2$ mm, so (15 ± 2) mm **A1**.

Exam-style 3.3 ▪ 4 marks

A resistance is found from $R = V/I$, with $V = (6.0 \pm 0.1)$ V and $I = (2.0 \pm 0.1)$ A. Find R and its **absolute uncertainty**.

Solution 3.3

Worked steps

$R = 6.0/2.0 = 3.0\ \Omega$ M1; $\%V = \frac{0.1}{6.0} \times 100 = 1.7\%$, $\%I = \frac{0.1}{2.0} \times 100 = 5.0\%$ M1; $\%R = 1.7 + 5.0 = 6.7\%$ A1; $\Delta R = 0.067 \times 3.0 = 0.2\ \Omega$, so $R = (3.0 \pm 0.2)\ \Omega$ A1.

Exam-style 3.4 ▪ 2 marks

A cube has side $L = (2.00 \pm 0.02)$ cm. Its volume is $V = L^3$. Find the **percentage uncertainty** in V.

Solution 3.4

Method: Uncertainty in one quantity

Worked steps

$\%L = \frac{0.02}{2.00} \times 100 = 1\%$ **M1**; $V = L^3$ so $\%V = 3 \times 1\% = 3\%$ **A1**.

Exam-style 3.5 ▪ 1 mark

Two samples of metal, A and B, are tested to find the density of the material each is made from.

(a) **Explain** what is meant by the **accuracy** of a measured value.

(b) Sample A is a sphere. Its diameter is $d = (2.40 \pm 0.02)$ cm and its mass is $m = (68.5 \pm 0.5)$ g. The volume of a sphere of diameter d is $V = \dfrac{\pi d^3}{6}$.

Show that the density of the material of sample A is about 9.5×10^3 kg m^{-3}.

(c) **Justify** the number of significant figures you have given for the density in **(b)**.

(d) **Calculate** the percentage uncertainty in the density of the material of sample A.

(e) **Determine** the absolute uncertainty in the density of sample A, and write the density as $\rho \pm \Delta\rho$.

(f) The density of the material of sample B is measured as 9.0×10^3 kg m^{-3} $\pm 3\%$.

State and explain whether A and B could be made from the same material.

Solution 3.5

Method: Significant figures in a calculated answer

Worked steps

(a) How close the measured value is to the **true value** of the quantity **B1**.

(b) $V = \dfrac{\pi d^3}{6} = \dfrac{\pi(2.40 \times 10^{-2})^3}{6} = 7.24 \times 10^{-6}\ \text{m}^3$ **M1**; $\rho = \dfrac{m}{V} = \dfrac{68.5 \times 10^{-3}}{7.24 \times 10^{-6}}$ **M1** $= 9.46 \times 10^3 \approx 9.5 \times 10^3\ \text{kg m}^{-3}$ **A1**. (Convert **both** the gram and the centimetre before dividing.)

(c) **3 significant figures** — both measurements are given to 3 s.f., so the answer cannot be quoted more precisely **B1**.

(d) $\%m = \dfrac{0.5}{68.5} \times 100 = 0.73\%$ **M1**; $\%d = \dfrac{0.02}{2.40} \times 100 = 0.83\%$, and $V \propto d^3$ so it

Solution 3.5

contributes $3 \times 0.83 = 2.5\%$ M1; $\%\rho = 0.73 + 2.5 = 3.2\%$ A1.
(e) $\Delta\rho = 0.032 \times 9.46 \times 10^3 = 0.30 \times 10^3$ M1, so $\rho = (9.5 \pm 0.3) \times 10^3$ kg m^{-3} A1.
(f) A is $9.46 \times 10^3 \pm 3.2\%$, i.e. 9.16×10^3 to 9.76×10^3; B is $9.0 \times 10^3 \pm 3\%$, i.e. 8.73×10^3 to 9.27×10^3 B1. The two ranges **overlap**, so the samples could be made from the same material B1.

Exam-style 3.6 ▪ 2 marks

A student times **20** complete swings of a pendulum with a stopwatch and records 31.4 s. Her reaction time makes each timing uncertain by about 0.2 s.

(a) **Estimate** the percentage uncertainty in her value for the **period** of one swing.

(b) Explain why timing 20 swings gives a smaller percentage uncertainty than timing one.

Solution 3.6

Method: Uncertainty in one quantity

Worked steps

(a) The period is $T = \dfrac{31.4}{20} = 1.57$ s. The uncertainty of 0.2 s is in the **total** time, so it is shared over 20 swings: $\% T = \dfrac{0.2}{31.4} \times 100 = 0.6\%$ **M1** **A1**.

(b) The absolute uncertainty stays about 0.2 s however many swings are timed (it comes from her reaction), but the **measured time is 20 times larger** **B1**, so the same absolute uncertainty is a much smaller **fraction** of it **B1**.