Exercise walk-through

SI units ▪ 1.2

eXercise

You must be able to

- recall the **five SI base units** (kg, m, s, A, K) and complete an exam table of them
- express a **derived unit** in SI base units, and find the units of an unknown constant
- check the **homogeneity** of an equation, and use it to find an unknown **power**
- convert a **compound unit** that carries **prefixes**

Method — Base and derived units

The **base quantities**: mass (kg), length (m), time (s), current (A), temperature (K).
Build a **derived unit** from the equation that defines it:
So 1 N $= 1$ kg m s^{-2}; the **joule** $=$ kg m^2 s^{-2}; the **watt** $=$ kg m^2 s^{-3}; the **pascal** $=$ kg m^{-1} s^{-2}.

Method — Base and derived units

$$
\begin{array}{ccccc}
\text{force} & = & \text{mass} & \times & \text{acceleration} \\
\downarrow & & \downarrow & & \downarrow \\
\text{kg m s}^{-2} & = & \text{kg} & \times & \text{m s}^{-2}
\end{array}
$$

so $1\ N = 1\ kg\ m\ s^{-2}$

Method — Units of an unknown constant

Rearrange for the constant, then substitute base units. For $F = kAv$, where A is an area and v a speed:

$$k = \frac{F}{Av} = \frac{\text{kg m s}^{-2}}{\text{m}^2 \times \text{m s}^{-1}} = \text{kg m}^{-2}\,\text{s}^{-1}$$

Write the fraction first, then cancel one line at a time — that first line is the method mark.

Method — Checking homogeneity

An equation is **homogeneous** when **every term** has the **same base units**. Check $v^2 = u^2 + 2as$:

- LHS: $(\mathrm{m\ s}^{-1})^2 = \mathrm{m}^2\,\mathrm{s}^{-2}$
- u^2: $\mathrm{m}^2\,\mathrm{s}^{-2}$; $2as$: $(\mathrm{m\ s}^{-2})(\mathrm{m}) = \mathrm{m}^2\,\mathrm{s}^{-2}$

All three are $\mathrm{m}^2\,\mathrm{s}^{-2}$ — homogeneous. Matching units do **not** prove an equation fully correct (a wrong number, like the 2, would still match), but **mismatched** units prove it wrong. Two terms that are **added** must have the same units, so $F = kv^2 + m$ can never be right.

Method — Finding an unknown power

If the equation is homogeneous, the powers must balance. In $T = 2\pi \left(\frac{L}{g}\right)^n$, with T a time, L a length and g an acceleration:

$$\left[\frac{L}{g}\right] = \frac{\mathrm{m}}{\mathrm{m\ s^{-2}}} = \mathrm{s}^2 \quad \Rightarrow \quad (\mathrm{s}^2)^n = \mathrm{s}^1 \quad \Rightarrow \quad 2n = 1 \quad \Rightarrow \quad n = \tfrac{1}{2}$$

The 2π has no units, so it never enters the check.

Method — Prefixes, and compound units

p	n	μ	m	c	d	k	M	G	T
10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^{-1}	10^{3}	10^{6}	10^{9}	10^{12}

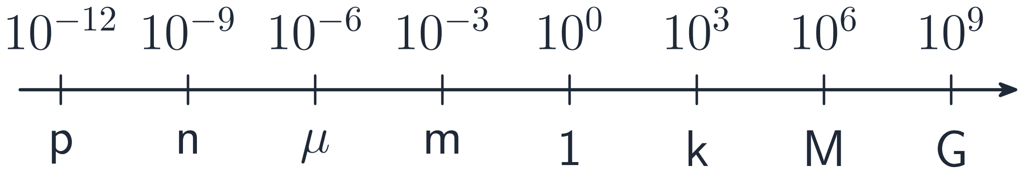
To convert a **compound** unit, replace **each** prefix by its power of ten — and remember a prefix inside a power is raised to that power too:

Method — Prefixes, and compound units

$$0.25\ \mathsf{kN\ mm}^{-2} = 0.25 \times \frac{10^3\ \mathsf{N}}{(10^{-3}\ \mathsf{m})^2} = 0.25 \times \frac{10^3}{10^{-6}}\ \mathsf{N\ m}^{-2} = 2.5 \times 10^8\ \mathsf{N\ m}^{-2}$$

$(10^{-3})^2 = 10^{-6}$, not 10^{-3} — that squaring is where most marks are lost.

Method — Prefixes, and compound units



each step is $\times 10^3$

Practice 2.1 ▪ 2 marks

Name the **five** SI base quantities and give each unit.

Solution 2.1

Method: Base and derived units

Worked steps

mass — kg; length — m; time — s; current — A; temperature — K **B1** **B1** *for three correct, for all five*.

Practice 2.2 ▪ 2 marks

Write the SI **base units** of **force**, and of **pressure**.

Solution 2.2

Worked steps

force: kg m s^{-2} **B1**; pressure: kg m^{-1} s^{-2} **B1**.

Practice 2.3 ▪ 2 marks

Write 5 mA in **amperes** and 2.0 GHz in **hertz**, both in standard form.

Solution 2.3

Method: Units of an unknown constant

Worked steps

5 mA $= 5 \times 10^{-3}$ A **B1**; 2.0 GHz $= 2.0 \times 10^{9}$ Hz **B1**.

Practice 2.4 ▪ 2 marks

Use $W = Fs$ to express the **joule** in SI base units.

Solution 2.4

Method: Base and derived units

Worked steps

$W = Fs$, so $\mathrm{J} = (\mathrm{kg\ m\ s^{-2}})(\mathrm{m})$ **M1** $= \mathrm{kg\ m^2\,s^{-2}}$ **A1**.

Practice 2.5 ▪ 2 marks

A density is 3.0 g cm^{-3}. Express it in kg m^{-3}.

Solution 2.5

Worked steps

$$3.0\ \frac{\mathrm{g}}{\mathrm{cm}^3} = 3.0 \times \frac{10^{-3}\ \mathrm{kg}}{(10^{-2}\ \mathrm{m})^3} = 3.0 \times \frac{10^{-3}}{10^{-6}}\ \boxed{\textbf{M1}} = 3.0 \times 10^3\ \mathrm{kg\ m^{-3}}\ \boxed{\textbf{A1}}.$$

Practice 2.6 ▪ 2 marks

In the equation $P = k\rho v^3$, P is a power, ρ a density and v a speed. **Determine** the SI base units of k.

Solution 2.6

Worked steps

$$k = \frac{P}{\rho v^3} = \frac{\text{kg m}^2\,\text{s}^{-3}}{(\text{kg m}^{-3})(\text{m}^3\,\text{s}^{-3})} \quad \textbf{M1} \quad = \frac{\text{kg m}^2\,\text{s}^{-3}}{\text{kg s}^{-3}} = \text{m}^2 \quad \textbf{A1}.$$

Exam-style 3.1 ▪ 1 mark

Which of these is **not** an SI base unit?

A ampere

B coulomb

C kelvin

D kilogram

Solution 3.1

- **A** ampere
- **B** coulomb ✓
- **C** kelvin
- **D** kilogram

Worked steps

B — the coulomb is a derived unit (1 C $= 1$ A s).

Exam-style 3.2 ▪ 1 mark

Which quantity has the same SI base units as **force**?

A $\dfrac{\text{energy}}{\text{distance}}$

B $\dfrac{\text{energy}}{\text{time}}$

C momentum $\times$ distance

D momentum $\times$ time

Solution 3.2

A $\dfrac{\text{energy}}{\text{distance}}$ ✓

B $\dfrac{\text{energy}}{\text{time}}$

C momentum $\times$ distance

D momentum $\times$ time

Worked steps

A — $\dfrac{\text{energy}}{\text{distance}} = \dfrac{\text{kg m}^2\,\text{s}^{-2}}{\text{m}} = \text{kg m s}^{-2}$, the units of force. (B is power; C is kg $\text{m}^2\,\text{s}^{-1}$; D is kg m.)

Exam-style 3.3 ▪ 1 mark

What is 6.0 kN cm^{-2} expressed in N m^{-2}?

A 6.0×10^{1}

B 6.0×10^{5}

C 6.0×10^{7}

D 6.0×10^{9}

Solution 3.3

A 6.0×10^1

B 6.0×10^5

C 6.0×10^7 ✓

D 6.0×10^9

Worked steps

C — $6.0 \times \dfrac{10^3}{(10^{-2})^2} = 6.0 \times \dfrac{10^3}{10^{-4}} = 6.0 \times 10^7$ N m^{-2}.

Exam-style 3.4 ▪ 1 mark

The speed v of a wave on the surface of a liquid is given by $v = k\left(\frac{p}{\rho}\right)^n$, where p is a pressure, ρ is a density and k has no units. The equation is homogeneous. What is the value of n?

A -2

B $-\frac{1}{2}$

C $\frac{1}{2}$

D 2

Solution 3.4

Method: Finding an unknown power

A -2

B $-\frac{1}{2}$

C $\frac{1}{2}$ ✓

D 2

Worked steps

C — $\left[\frac{p}{\rho}\right] = \frac{\mathrm{kg\ m^{-1}\ s^{-2}}}{\mathrm{kg\ m^{-3}}} = \mathrm{m^2\,s^{-2}}$, so $(\mathrm{m^2\,s^{-2}})^n = \mathrm{m\ s^{-1}}$ gives $2n = 1$ and $n = \frac{1}{2}$.

Exam-style 3.5 ▪ 3 marks

An engineering team releases an unmanned **submersible** (an underwater craft) from rest at the surface of a lake. Its volume is 0.25 m^3 and its weight is 2.1 kN. The density of the lake water is 1030 kg m^{-3}.

(a) The team records three quantities during the dive. **Complete** the table by writing the SI base units of each quantity, and ticking whether it is a scalar or a vector.

quantity	**SI base units**	**scalar**	**vector**
water temperature	____________	☐	☐
water pressure	____________	☐	☐
drag force on the submersible	____________	☐	☐

Exam-style 3.5 ▪ 3 marks

(b) **Show that** the upthrust on the fully submerged submersible is about 2.5 kN.
(Upthrust $=$ weight of the water displaced.)
(c) **Determine** the magnitude and the direction of the initial acceleration of the submersible when it is released.
(d) The drag force F on the submersible is modelled as $F = kv^2$, where v is its speed. **Determine** the SI base units of k.
(e) A student writes the drag equation as $F = kv^2 + m$, where m is the mass of the submersible. Use **units** to explain why this equation cannot be correct.

Solution 3.5

Method: Base and derived units

Worked steps

(a) water temperature — K, scalar; water pressure — kg m^{-1} s^{-2}, scalar; drag force — kg m s^{-2}, vector **B1** **B1** **B1** *for all three units, for the two scalars, for the vector.*

(b) mass of water displaced $= \rho V = 1030 \times 0.25 = 257.5$ kg; upthrust $= \rho V g = 1030 \times 0.25 \times 9.81$ **M1** $= 2526$ N ≈ 2.5 kN **A1**.

(c) resultant force $= 2526 - 2100 = 426$ N upwards **M1**; mass $= \dfrac{2100}{9.81} = 214$ kg **M1**; $a = \dfrac{426}{214} = 2.0$ m s^{-2}, directed **upwards** **A1** — *the direction is part of the answer.*

(d) $k = \dfrac{F}{v^2} = \dfrac{\text{kg m s}^{-2}}{\text{m}^2\,\text{s}^{-2}}$ **M1** $=$ kg m^{-1} **A1**.

Solution 3.5

(e) kv^2 has the units of force, kg m s^{-2}, but m has the unit kg. Terms that are **added** must have the same units, so the equation is not homogeneous and cannot be correct **B1**.