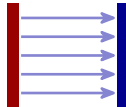


Capacitance

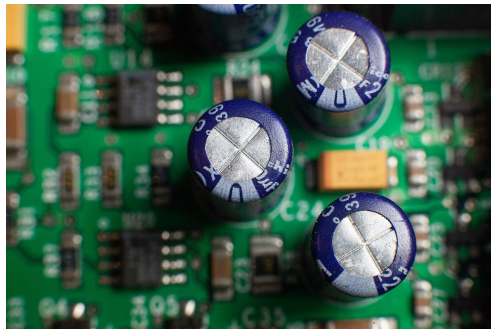
A2 Physics ■ Topic 19

A-Level Physics



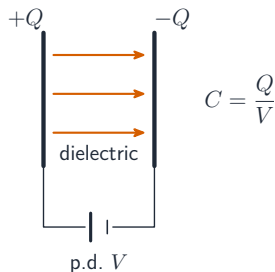
What we will cover

- 1 Capacitance $C = Q/V$
- 2 Combining capacitors
- 3 Energy stored
- 4 Discharge through a resistor
- 5 The time constant RC
- 6 Recap





What a capacitor does



A **capacitor** stores charge: two **conductor** plates with an **insulator** between them – a **dielectric**, or just a **vacuum** – with a **potential difference** V across the gap.

- connected to a battery, $+Q$ builds on one plate, $-Q$ on the other

The two charges are **equal and opposite** – the capacitor stores charge **separation**, not net charge.

Capacitance $C = Q/V$

$$C = \frac{Q}{V}, \quad [C] = \text{F} = \text{C V}^{-1}$$

- set by size and dielectric, so C is **constant**
- double Q and V doubles too – the **ratio** holds
- an isolated sphere: $C = 4\pi\epsilon_0 r$

A farad is **huge** – real capacitors run from pF to mF.



Worked example – charge stored

**A $100\ \mu\text{F}$ capacitor is charged to $12\ \text{V}$.
Find the charge stored.**

$$\begin{aligned}Q &= CV = (100 \times 10^{-6})(12) \\&= 1.2 \times 10^{-3}\ \text{C} = 1.2\ \text{mC}\end{aligned}$$

Put the μF in as $\times 10^{-6}\ \text{F}$.
A stored mC is a **lot** of separated charge for such a small device.



Parallel: the charges add

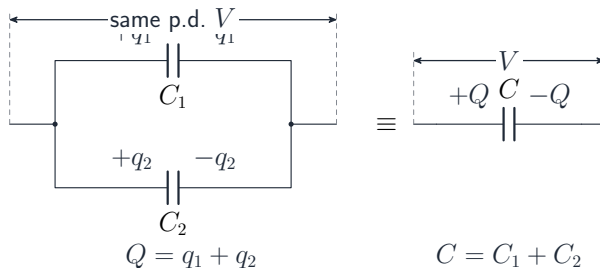
Same p.d. V across each, so the charges sum:

$$C_{\text{parallel}} = C_1 + C_2 + \dots$$

A parallel bank has **larger** capacitance than any one.

More plate area $\Rightarrow$ more room for charge – adding in parallel is like widening the plates.

Parallel: the charges add, in detail

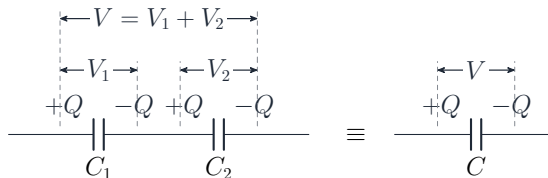


Same p.d. V across each, so the charges sum:

$$C_{\text{parallel}} = C_1 + C_2 + \dots$$

A parallel bank has **larger** capacitance than any one.

Series: the p.d.s add



Same charge Q through each, so the p.d.s sum:

$$\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

A series chain has **smaller** capacitance than any one.

The **opposite** of resistors – because $C = Q/V$ has V on the bottom, while $R = V/I$ has I on the bottom.



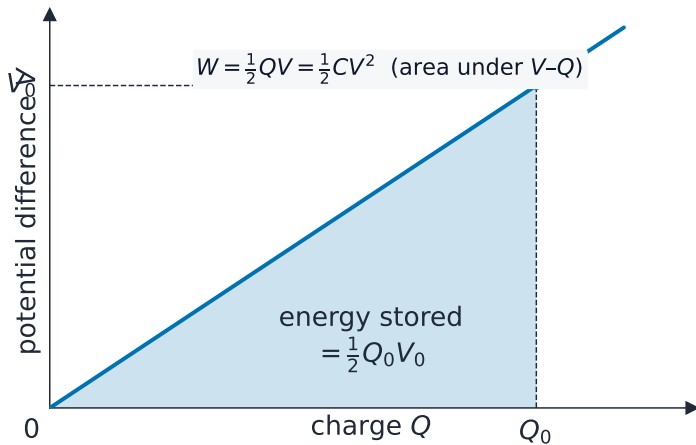
Energy stored in a capacitor

Each extra dq is pushed against the p.d. q/C already there, so

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$$

On the V - Q line the energy is the **area** beneath – the triangle $\frac{1}{2} QV$. The $\frac{1}{2}$ is the **average** p.d. during charging.

Energy stored in a capacitor, in detail



The energy stored is the area under the potential–charge line (the triangle $\frac{1}{2}QV$)

Reading the Q - V graph

A plot of V against Q is a straight line through the origin with gradient $1/C$.

The energy stored is the **area** under the line up to a given charge Q , which is the triangle $\frac{1}{2}QV$.

Worked example – energy stored

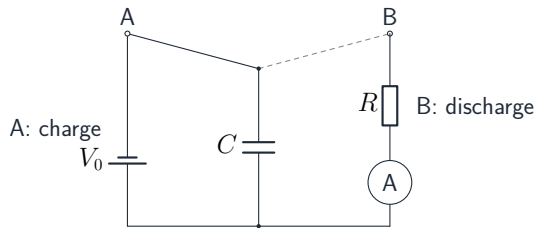
Find the energy in a $100\ \mu\text{F}$ capacitor charged to $12\ \text{V}$.

$$\begin{aligned} W &= \frac{1}{2} CV^2 = \frac{1}{2} (100 \times 10^{-6}) (12)^2 \\ &= 7.2 \times 10^{-3} \text{ J} = 7.2 \text{ mJ} \end{aligned}$$

Charging through a wire wastes the **other half**:
the battery gives out $QV = CV^2$, the capacitor keeps $\frac{1}{2} CV^2$, and the rest heats the wire – whatever its resistance.



Setting up the discharge



Close the switch and C drives current through R . By **Kirchhoff's second law** around the loop $V_C = V_R$; using $V_C = Q/C$, $V_R = IR$ and $I = -dQ/dt$:

$$\frac{Q}{C} = -R \frac{dQ}{dt}$$

The rate of loss is **proportional** to what is left – and that always gives an **exponential decay** .

The discharge equations

Charge, p.d. and current all decay with the **same** time constant RC :

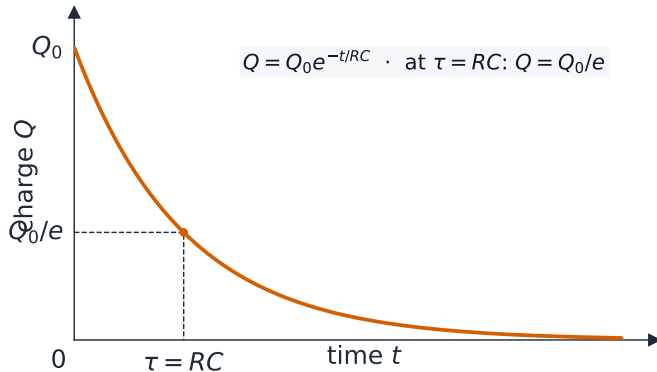
$$Q = Q_0 e^{-t/RC}$$

$$V = V_0 e^{-t/RC}, \quad I = I_0 e^{-t/RC}$$

with $I_0 = V_0/R$.

After one time constant RC the charge is down to $Q_0/e \approx 37\%$ of the start.

The discharge equations, in detail



Charge decays exponentially during discharge, falling to Q_0/e after one time constant RC

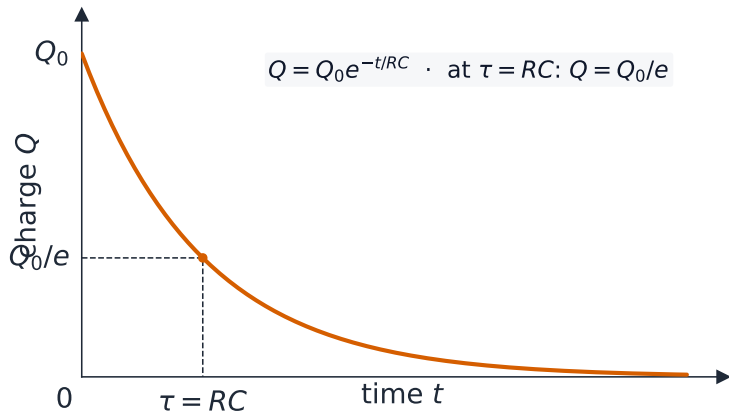
The time constant and half-life

$$\tau = RC, \quad t_{1/2} = RC \ln 2 \approx 0.69 RC$$

- units check: $\Omega \times \text{F} = \text{s}$
- bigger R or $C \Rightarrow$ slower discharge
- every RC , the charge falls by the same factor $1/e$

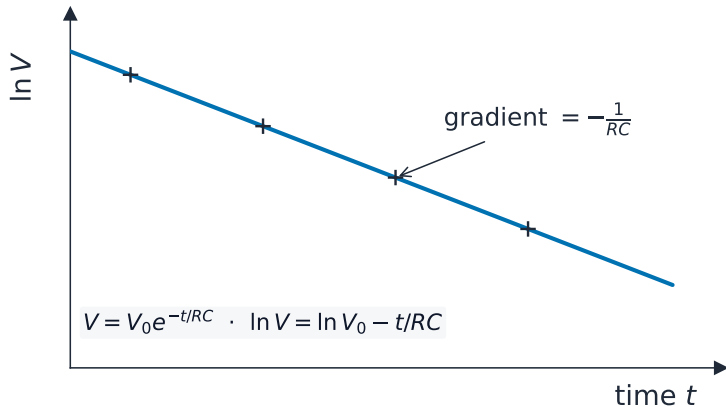
ln turns the exponential into a straight line: plotting $\ln Q$ against t gives a gradient of $-1/RC$.

The time constant and half-life, in detail



$$\tau = RC, \quad t_{1/2} = RC \ln 2 \approx 0.69 RC$$

Time constant



A graph of $\ln V$ against time is a straight line of gradient $-1/(RC)$

Reading a discharge curve in the exam

Start value

Read V_0 or Q_0 off the graph at $t = 0$.

Time constant

Read the time taken to fall to V_0/e (about 37% of the start): that time *is* $\tau = RC$.

The missing component

Given R , find $C = \tau/R$ – or given C , find R .

A later value

Predict any later value with $V = V_0 e^{-t/RC}$; or plot $\ln V$ against t for a straight line of gradient $-1/RC$.

Every discharge question is one of these four. Identify which before you touch the algebra.

Exam tips

- $C = Q/V$; energy stored $W = \frac{1}{2}QV = \frac{1}{2}CV^2$ (the $\frac{1}{2}$ is the area under the Q - V graph).
- Combine capacitors the **opposite way to resistors** (parallel add, series reciprocal).
- Discharge is exponential: $Q = Q_0 e^{-t/RC}$; the time constant $\tau = RC$ is when the charge falls to 37%.



Topic 19 – key takeaways

1 Capacitance

$C = Q/V$, in farads – a constant of the device

2 Combining

parallel $C_1 + C_2$; series
 $1/C_1 + 1/C_2$

3 Energy

$\frac{1}{2} QV = \frac{1}{2} CV^2$ – area under V - Q

4 Discharge

$Q = Q_0 e^{-t/RC}$ – time constant
 $\tau = RC$

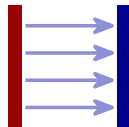
Check yourself

1 Two $2\ \mu\text{F}$ caps in parallel: total C ?

2 Same two in series: total C ?

3 Why the factor $\frac{1}{2}$ in the energy?

4 What fraction is left after one RC ?



Answers 1. $4\ \mu\text{F}$ 2. $1\ \mu\text{F}$ 3. average p.d. is $V/2$ 4. $1/e \approx 37\%$