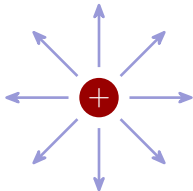


Electric Fields

A2 Physics ■ Topic 18

A-Level Physics



What we will cover

- 1 Fields & field lines
- 2 Uniform fields: $E = V/d$
- 3 Coulomb's law
- 4 The field of a point charge
- 5 Electric potential
- 6 Recap





Electric field strength



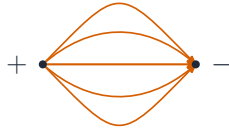
An **electric field** is a region where a charge feels a force. Its **field strength** is the force per unit **positive** charge:

$$E = \frac{F}{q} \Rightarrow F = qE, \quad [E] = \text{N C}^{-1}$$

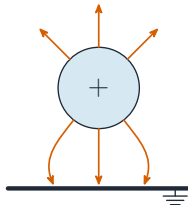
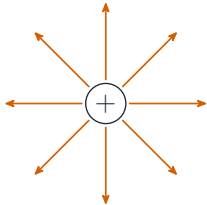
Defined on a **positive** test charge – so a **negative** charge feels a force **against** the field.

Field-line patterns

a) parallel plates (uniform field) b) dipole (+ and -)



- they run **from + to -** (or off to infinity)
- they **never cross**
- **closer lines** = stronger field



c) point charge (radial) d) sphere above earthed plate

Two crossing lines would give the force at that point **two** directions at once – impossible, which is why it never happens.

Four field patterns worth recognising

Point charge

Radial lines pointing *out* from a positive **point charge** , and *in* towards a negative one.

A dipole

Two opposite charges – a **dipole** – give lines curving from $+$ to $-$, densest in the gap between them.

Parallel plates

Two parallel charged plates a distance d apart with **potential difference** V give a **uniform field** of size $E = V/d$, apart from edge effects.

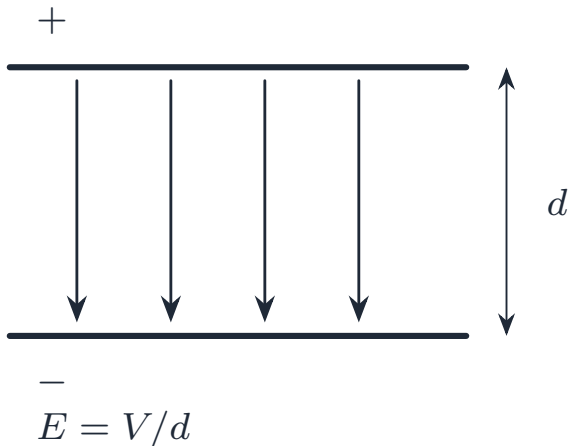
Several charges

For more than one charge, add the separate fields as a **vector sum** at each point.

A charge in a uniform field feels a constant force $F = qE$ and so a constant acceleration $a = qE/m$ – exactly like a mass in a uniform **gravitational field** .



The uniform field between plates



Plates a distance d apart with p.d. V across them:

$$E = \frac{V}{d}$$

pointing from the **high**-potential plate to the low.

This is where $V \text{ m}^{-1}$ comes from – and it is the **same unit** as N C^{-1} .

Worked example – plates and an electron

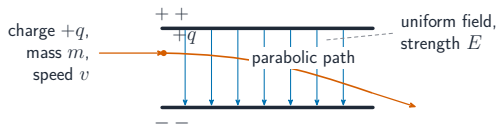
Plates 5.0 mm apart carry a p.d. of 200 V. Find E , and the force on an electron between them. ($e = 1.6 \times 10^{-19}$ C)

$$E = \frac{V}{d} = \frac{200}{5.0 \times 10^{-3}} = 4.0 \times 10^4 \text{ V m}^{-1}$$

$$F = qE = (1.6 \times 10^{-19})(4.0 \times 10^4) = 6.4 \times 10^{-15} \text{ N}$$

Convert **mm to m** first – forgetting it scales E by a thousand. The force is the **same everywhere** in the gap: that is what uniform means.

A charge crossing a uniform field



Constant force $\Rightarrow$ constant acceleration

$$a = qE/m:$$

- released **at rest**: speeds up along the field, gaining KE
- entering **at right angles**: a **parabolic** path

Exactly a **projectile** problem – steady velocity across, uniform acceleration down. This is how a **cathode-ray tube** steered its beam.

Two ways a charge is set moving

Released at rest

It accelerates along the field (positive charge) or against it (negative), gaining **kinetic energy** qV after falling through a potential difference V .

Fired across

Constant velocity along the plates, constant acceleration across them – the same parabola as a projectile.

In an atom

An **electron** orbiting a **nucleus** of charge $+Ze$ at radius r : the Coulomb attraction supplies the **centripetal force**, $\frac{Ze^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r}$.

Energy method or force method – both work. Use energy when the question gives a potential difference, force when it gives a field.



Coulomb's law

like charges



repel — pushed apart

opposite charges



attract — pulled together

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$$

with the **permittivity of free space** $\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$
and $\frac{1}{4\pi\epsilon_0} \approx 9.0 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$.

Inverse square, along the joining line – **like repels, opposite attracts**. That sign choice is the one thing gravity cannot do.

Worked example – two point charges

Find the force between $+2.0\text{ nC}$ and $+3.0\text{ nC}$ set 4.0 cm apart.

$$F = \frac{(9.0 \times 10^9)(2.0 \times 10^{-9})(3.0 \times 10^{-9})}{(0.040)^2}$$

$\approx 3.4 \times 10^{-5}\text{ N}$, and **repulsive** (both +)

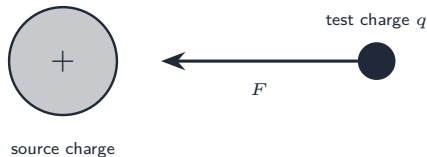
State the **direction** as well as the size – “repulsive” is part of the answer. And r goes in as **metres**, squared.

Spheres as point charges

A charged spherical **conductor** of total charge Q :

- **outside**: identical to a point charge Q at its centre (measure r from the **centre**)
- **inside** a hollow one: the field is **zero**

Zero field inside makes the whole conductor one **equipotential** – and is why a car shields you from lightning.



$$E = F/q \quad (\text{NC}^{-1})$$



The field of a point charge

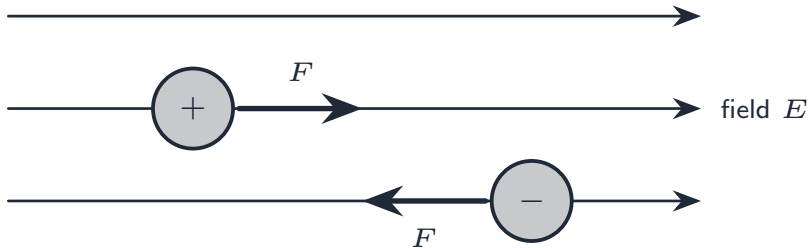
Coulomb's law gives the force between **two** charges. Divide by the test charge and only the **source** is left:

$$E = \frac{F}{q} = \frac{Q}{4\pi\epsilon_0 r^2}$$

Out of a positive Q , into a negative one, falling as $1/r^2$.

The same shape as $g = GM/r^2$ – but charges can **cancel**, so for several of them add the fields as a **vector** sum.

The field of a point charge, in detail



The field of a point charge Coulomb's law gives the force between two charges.



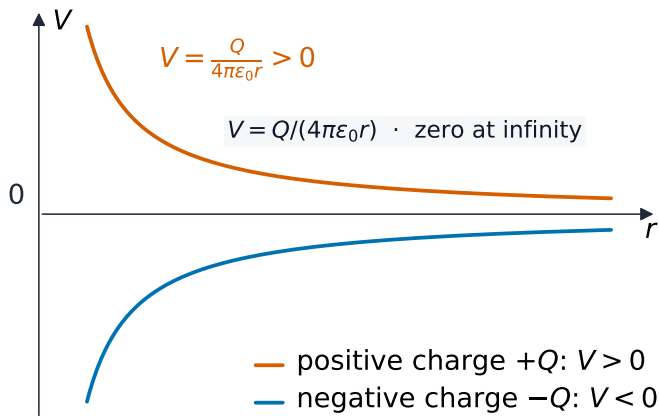
Electric potential

Electric potential is the work done per unit **positive** charge bringing a test charge in from **infinity** :

$$V = \frac{W}{q} = \frac{Q}{4\pi\epsilon_0 r}$$

Note $1/r$, **not** $1/r^2$. V is a **scalar** and takes the sign of Q – so several charges just **add**, signs and all, with no directions to resolve.

Electric potential, in detail



Potential falls as $1/r$, **not** $1/r^2$, and takes the sign of its charge – so several charges simply add, signs and all.

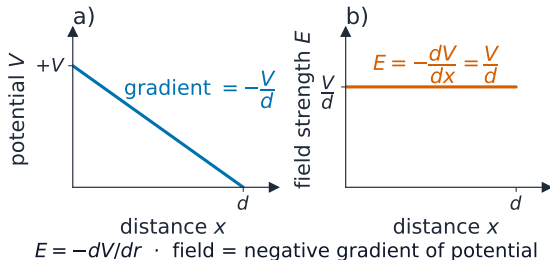
Worked example – potential near a charge

Find V at 4.0 cm from a point charge of +3.0 nC.

$$V = \frac{Q}{4\pi\epsilon_0 r} = \frac{(9.0 \times 10^9)(3.0 \times 10^{-9})}{0.040}$$
$$\approx 6.8 \times 10^2 \text{ V}$$

Same charge, same distance as the Coulomb example – but r appears to the **first** power here. Mixing up $1/r$ and $1/r^2$ is the standard slip.

Field is the potential gradient



The field is the **negative potential gradient**:

$$E = -\frac{dV}{dx}$$

- between plates V falls evenly $\Rightarrow E = V/d$, constant
- for a point charge

$$-\frac{dV}{dr} = \frac{Q}{4\pi\epsilon_0 r^2} = E$$

The minus sign says the field points **downhill** in potential – from high V to low.

Electric potential energy

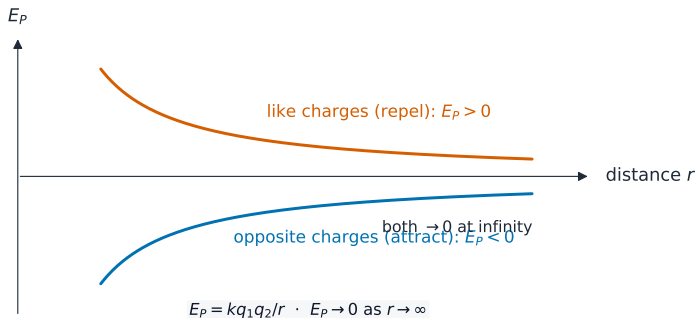
A charge q where the potential is V has $E_P = qV$; for two point charges

$$E_P = \frac{Qq}{4\pi\epsilon_0 r}$$

- **like**: $E_P > 0$ – energy released if they fly apart
- **opposite**: $E_P < 0$ – a **bound** well

Both tend to **zero** as $r \rightarrow \infty$ – that is the choice of zero, exactly as for gravity.

Electric potential energy, in detail



A charge q sitting where the potential is V has $E_P = qV$, so for two point charges

$$E_P = \frac{Qq}{4\pi\epsilon_0 r}.$$

Like charges give $E_P > 0$, opposite charges $E_P < 0$; both tend to zero at infinity.

Gravitational vs electric

Quantity	Gravitational	Electric
Source	mass M (always +)	charge Q (either sign)
Field strength	$g = GM/r^2$	$E = Q/(4\pi\epsilon_0 r^2)$
Force	$F = mg$	$F = qE$
Gravitational potential vs potential	$\phi = -GM/r$	$V = Q/(4\pi\epsilon_0 r)$
PE of two	$-GMm/r$	$Qq/(4\pi\epsilon_0 r)$
Nature	always attractive	attract <i>or</i> repel

Identical machinery, one difference: gravity's minus sign is **permanent** (mass only attracts), while the electric sign is inherited from the **charge**.

Exam tips

- **Coulomb's law** $F = Q_1 Q_2 / 4\pi\epsilon_0 r^2$ (inverse-square, but can attract or repel).
- Distinguish a **uniform field** ($E = V/d$, between plates) from a **radial field** ($E = Q/4\pi\epsilon_0 r^2$).
- Electric potential $V = Q/4\pi\epsilon_0 r$ (sign follows the charge); the field points from high to low potential.



Topic 18 – key takeaways

1 Coulomb

$F = Q_1 Q_2 / 4\pi\epsilon_0 r^2$ – attract or repel

2 Two fields

uniform $E = V/d$; radial $E = Q/4\pi\epsilon_0 r^2$

3 Potential

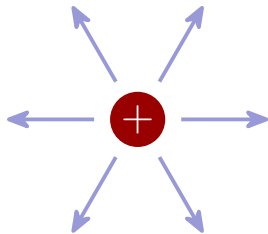
$V = Q/4\pi\epsilon_0 r$ – scalar, signed, $1/r$

4 The link

$E = -dV/dx$ – the field points downhill in V

Check yourself

- 1 Which way does a field line point?
- 2 Halve the plate gap at fixed V – what happens to E ?
- 3 Is potential a vector or a scalar?
- 4 What is the field inside a hollow charged sphere?



Answers 1. the force on a + charge 2. doubles 3. scalar 4. zero