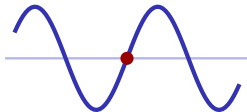


Oscillations

A2 Physics ■ Topic 17

A-Level Physics



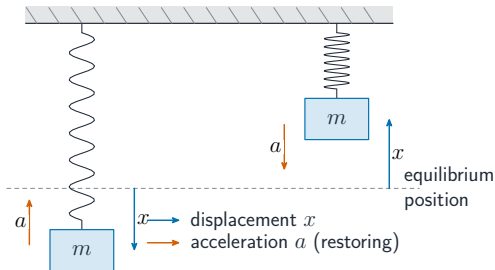
What we will cover

- 1 SHM: $a = -\omega^2 x$
- 2 Displacement, velocity, acceleration
- 3 Graphs
- 4 Energy
- 5 Damping & resonance
- 6 Recap





Simple harmonic motion



A body moves with **simple harmonic motion** when its acceleration is

- **proportional to its displacement** from a fixed **equilibrium** point, and
- **directed back towards that point.**

Many systems do this near a stable equilibrium: a mass on a **spring**, a **pendulum** (small swing), a floating block, the charge on a **capacitor** in an LC circuit, atoms in a solid.

The words you must use precisely

displacement x – from equilibrium, signed

amplitude x_0 – the largest $|x|$, always +

period T – one full oscillation

frequency $f = 1/T$, in Hz

angular frequency
 $\omega = 2\pi f = 2\pi/T$

phase difference – a fraction of a cycle, in rad

Given any one of ω , f , T you can reach the other two – start every question by getting ω .

Worked example – maximum acceleration

A mass oscillates with amplitude 0.050 m at 2.5 Hz. Find its maximum acceleration.

$$\omega = 2\pi f = 2\pi \times 2.5 = 15.7 \text{ rad s}^{-1}$$

$|a|$ is largest at the **extremes**, where $|a| = \omega^2 x_0$:

$$a_{\text{max}} = 15.7^2 \times 0.050 \approx 12 \text{ m s}^{-2}$$

Fastest at the **centre**, most accelerated at the **ends** – the two extremes are opposite. Confusing them is the classic error.



Displacement, velocity, acceleration

Starting from $x = 0$ moving positive:

$$x = x_0 \sin \omega t$$

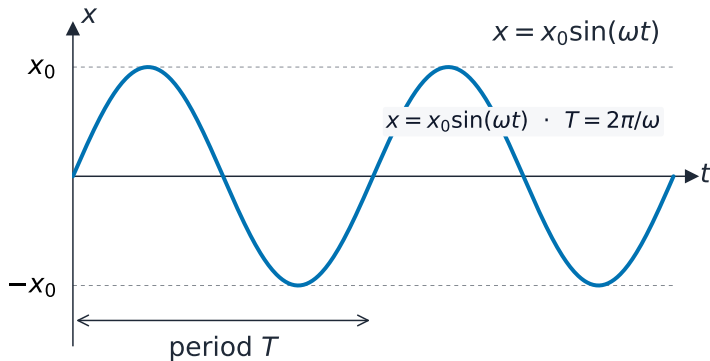
$$v = x_0 \omega \cos \omega t = v_0 \cos \omega t$$

$$a = -x_0 \omega^2 \sin \omega t = -\omega^2 x$$

so $v_0 = x_0 \omega$ and $a_{\max} = \omega^2 x_0$.

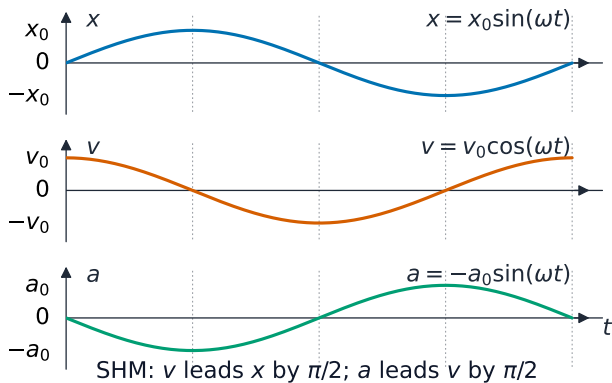
Starting at the **extreme** instead? Use $x = x_0 \cos \omega t$. Pick the one that matches $t = 0$.

Displacement, velocity, acceleration, in detail



Displacement is a **sine** curve of amplitude x_0 when the motion starts at the centre, and a **cosine** when it starts at the extreme. Pick the one that matches $t = 0$.

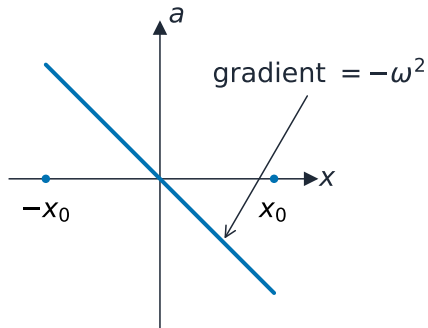
The three graphs together



- x : sine, amplitude x_0
- v : cosine, **leads x by $\pi/2$** , amplitude ωx_0
- a : **−sine, π out of phase** with x , amplitude $\omega^2 x_0$

Each differentiation **ad-**
vances the phase by $\pi/2$
and multiplies the ampli-
tude by ω – that is the
whole pattern.

Reading ω off the a - x graph



$a = -\omega^2 x$ is a straight line through the origin with gradient $-\omega^2$. So

$$\omega = \sqrt{|\text{gradient}|}, \quad T = \frac{2\pi}{\omega}$$

A straight line through the origin with **negative** gradient is the test for SHM – and a very common exam route to T .

SHM: $a = -\omega^2 x$ · gradient = $-\omega^2$

Velocity without time

A useful relation that does not use *time*:

$$v = \pm \omega \sqrt{x_0^2 - x^2}$$

- at $x = 0$: $v = \pm \omega x_0$ – the **maximum** speed
- at $x = \pm x_0$: $v = 0$ – at rest for an instant

The $\pm$ is real: the particle crosses equilibrium **twice** a cycle, once each way.

$$x_0 = 0.050 \text{ m}, \omega = 15.7 \text{ rad s}^{-1}.$$

Find v at $x = 0.030 \text{ m}$.

$$\begin{aligned} v &= 15.7 \sqrt{0.050^2 - 0.030^2} \\ &= 15.7 \times 0.040 \approx 0.63 \text{ m s}^{-1} \end{aligned}$$



Energy in SHM

An oscillator keeps swapping energy between two forms: **kinetic energy** $\frac{1}{2}mv^2$, and potential energy (elastic for a spring, gravitational for a pendulum).

$$E_K = \frac{1}{2}m\omega^2(x_0^2 - x^2)$$

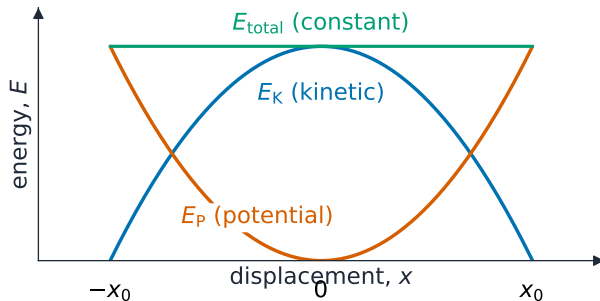
$$E_P = \frac{1}{2}m\omega^2x^2$$

E_K is a downward parabola, E_P an upward one – and their **sum is flat**:

$$E_{\text{total}} = \frac{1}{2}m\omega^2x_0^2$$

Kinetic energy and potential energy trade places while the total stays put – **conservation of energy** for a mass on a **spring**. $E \propto x_0^2$: **double** the amplitude for **four times** the energy.

Energy in SHM, in detail



$$\text{SHM: } E_k + E_p = \text{const} \cdot \max E_k \text{ at centre}$$

Kinetic and potential energy swap over a cycle while the total energy stays constant

Worked example – total energy

A 0.20 kg mass oscillates with $x_0 = 0.050$ m and $\omega = 15.7 \text{ rad s}^{-1}$. Find the total energy.

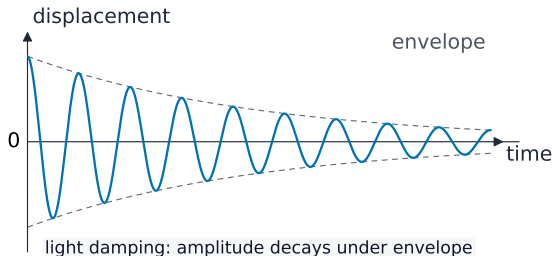
The total equals the **maximum** KE, as it passes $x = 0$ at $v_{\text{max}} = \omega x_0$:

$$E = \frac{1}{2} m \omega^2 x_0^2 = \frac{1}{2} (0.20) (15.7)^2 (0.050)^2 \\ \approx 0.062 \text{ J}$$

Evaluate the total **where one form vanishes** – at $x = 0$ it is all kinetic, at $x = x_0$ all potential. Either gives the same number.



Damped oscillations

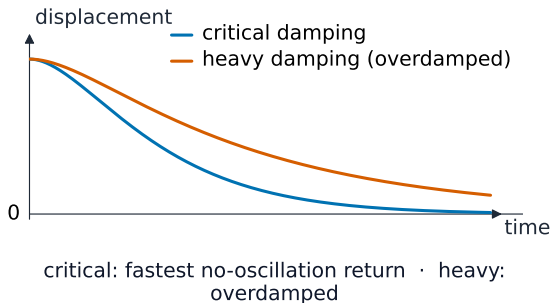


A resistive force (**air resistance** , **friction**) removes energy as heat, so the amplitude **shrinks**. **Light**

damping : the amplitude dies away slowly over **many** cycles, still near the natural frequency.

A car's suspension is light-to-medium damped – bumps die away, but the ride stays smooth.

Critical and heavy damping



- **critical damping** : back to equilibrium in the **shortest** time, **no** overshoot, no oscillation
- **heavy damping** : no oscillation either, but **slower** than critical

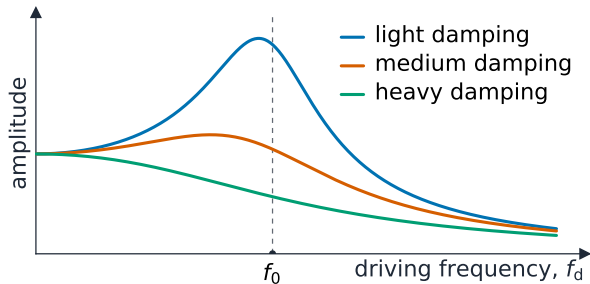
A **galvanometer** needle is **critically** damped so it settles at once; a strong door closer is **heavily** damped.

Forced oscillations and resonance

A **forced oscillation** runs at the **driving** frequency f_d , not its own. **Resonance** : $f_d = f_0$, the **natural frequency** – **largest** amplitude, most efficient transfer.

Damping shapes the **resonance curve** : **lighter** → sharper and higher; **heavier** → broader, lower, and nudged to a lower frequency.

Forced oscillations and resonance, in detail



resonance: max amplitude near f_0 · lighter damping
⇒ sharper peak

Forced oscillations and resonance A forced oscillation runs at the driving frequency f_d , not its own.

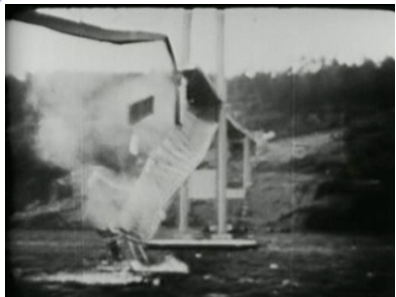
Resonance in the world



- a **swing** pushed at the right rate builds a big amplitude
- a **wine glass** shattered by its own ringing note
- a **building** shaken by an earthquake at a matching frequency

Engineers design a building's natural frequencies to **avoid** the main earthquake range – resonance is as often something to dodge as to exploit.

Resonance that brought a bridge down



Tacoma Narrows, 1940: a driving frequency close to a natural frequency, with too little damping to limit the amplitude.

Exam tips

- The **SHM condition** is $a = -\omega^2 x$ (acceleration proportional to displacement, directed back to equilibrium).
- Learn $x = x_0 \sin \omega t$ (or $\cos$), $v_{\max} = \omega x_0$, $a_{\max} = \omega^2 x_0$; KE and PE interchange with the **total energy constant**.
- Velocity is **zero at the extremes** and maximum at the centre.
- **Resonance** occurs when the driving frequency equals the natural frequency; damping lowers and broadens the peak.



Topic 17 – key takeaways

1 The condition

$a = -\omega^2 x$ – proportional, and back to equilibrium

2 Maxima

$v_{\max} = \omega x_0$ at the centre;
 $a_{\max} = \omega^2 x_0$ at the ends

3 Energy

$E = \frac{1}{2} m \omega^2 x_0^2$, constant; $E \propto x_0^2$

4 Resonance

peak at $f_d = f_0$; damping lowers and broadens it

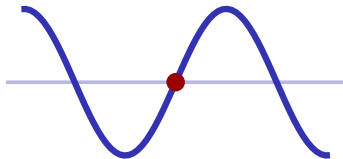
Check yourself

1 Where in the cycle is the speed greatest?

2 What is the gradient of an a - x graph?

3 Triple the amplitude – what happens to E ?

4 Which damping settles fastest with no overshoot?



Answers 1. at equilibrium, $x = 0$ 2. $-\omega^2$ 3. $\times 9$ 4. critical